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A Modified Re-entry Trajectory Planning Approach and Comparison with EAGLE Method

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Abstract In this paper, a continuous smooth drag acceleration profile is generated for the hypersonic entry phase of the winged entry vehicles. Evolved Acceleration Guidance Logic for Entry (EAGLE) method is used as the baseline, and a modified strategy is developed in this paper, to plan a reference drag-energy profile. In this work, a continuous smooth drag acceleration profile is created in the drag-energy plane using cubic spline interpolation, whereas in EAGLE method a three segment constant drag-energy profile with sharp corners is used. With this modified strategy, unlike EAGLE method, the actual trajectory follows the reference trajectory without many oscillations both in drag-energy space and altitude-velocity space. This method has been tested on SL-12 vehicle which has a medium L/D ratio. Simulation results show that all the path and terminal constraints are satisfied and the downrange requirement is also met. The terminal altitude and velocity errors are minimal compared to the EAGLE method and are within the permissible range. Robustness analysis is also carried out to show the effectiveness of the proposed method.

1 Introduction

Re-entry trajectory planning is a challenging task because the vehicle is subjected to high heat and structural loads during atmospheric entry. Therefore, the reference trajectory needs to be designed carefully such that it lies within the entry corridor. The entry corridor is constructed using the path constraints which are heat rate, dynamic

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pressure, maximum g-load, and equilibrium glide constraint. It can be illustrated either in altitude-velocity space or drag-energy space. The primary objective of entry guidance is to steer the vehicle to the desired target along a feasible trajectory which satisfies all the constraints and range requirement. There are two approaches for re-entry guidance; one is predictor-corrector approach and another one is based on a reference trajectory. In Predictor-Corrector approach[3], the predictor calculates the rough approximation for the desired control variable and the corrector corrects the prediction using predicted and previous values and update the control variable.

The guidance algorithm using reference trajectory is a classical approach for re-entry trajectory planning. In this category, the guidance algorithm consists of a reference trajectory planner to generate a feasible trajectory and tracker to track the planned reference trajectory. This approach has been widely used in literature, and the baseline for this type of re-entry guidance is shuttle entry guidance algorithm[2, 5]. In shuttle entry guidance, velocity is used as the independent variable and the reference trajectory is drag-velocity profile. The main disadvantage of using velocity as independent variable is in the prediction of range during flights with larger flight path angle. Evolved Acceleration Guidance Logic for Entry (EAGLE) method[4, 7] uses specific energy as independent variable and it is an extension of shuttle entry guidance approach. Several works have been presented in literature[4, 7, 9, 10] with drag acceleration based re-entry trajectory planning algorithm. An optimization based drag-energy profile planning and tracking are described in [1]. Trajectory tracking is done using Incremental Non-linear Dynamic Inversion.

The work presented in this paper uses EAGLE method as a baseline approach, and a continuous smooth reference drag acceleration profile is generated using cubic spline interpolation. EAGLE method uses a three segment linear drag-energy profile with sharp corner points. It has two linear splines which are functions of specific energy and a constant drag segment that can be adjusted to meet the desired downrange. Similarly, in [10] a five segment reference trajectory with sharp corners is planned using the trajectory planning algorithm. With these sharp corner points, accurate tracking of reference trajectory is normally not be possible and the actual trajectory deviates from the reference trajectory which leads to tracking errors and range dispersions. In order to overcome this disadvantage with present study, 3-segment or 5 segment drag-energy profiles are replaced with a continuous cubic spline function which has continuous first and second derivatives with respect to specific energy. The resultant reference trajectories are compared with the traditional EAGLE method. Then the number of intermediate points are increased to study the variation in terminal errors. Also, the robustness of the proposed method is analysed using different initial conditions with the same reference drag-energy trajectory. A medium L/D ratio vehicle SL-12 is used for the simulations[6].

This paper is organized as follows. Section 2 briefly describes the 3-degree of freedom translational equations of motion for a re-entry vehicle. The concept of path constraints and entry corridor are presented in section 3. It also explains two methods for reference trajectory planning. They are Evolved Acceleration Guidance Logic for Entry (EAGLE) method and a modified version of EAGLE method using

Cubic Spline interpolation approach. The simulations results and its analyses are explained in section 4. Section 5 concludes the paper.

2 Re-entry Dynamics

A three degree of freedom (3-DOF) translational equations of the vehicle representing the point mass trajectory movements over a spherical non rotating Earth are as follows.

$$\dot{r} = V \sin\gamma \tag{1}$$

$$\dot{\theta} = \frac{V \cos\gamma \sin\psi}{r \cos\phi} \tag{2}$$

$$\dot{\phi} = \frac{V \cos\gamma \cos\psi}{r} \tag{3}$$

$$\dot{V} = -D - g \sin\gamma \tag{4}$$

$$\dot{\gamma} = \frac{1}{V}\left[L\cos\sigma - \left(g - \frac{V^2}{r}\right)\cos\gamma\right] \tag{5}$$

$$\dot{\psi} = \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} + \frac{V^2\cos\gamma\sin\psi\tan\phi}{r}\right] \tag{6}$$

where r is the radial distance from Earth's centre to the vehicle, θ is the geodetic longitude, ϕ is the geodetic latitude, V is the relative velocity of the vehicle, γ is the flight path angle and ψ represents the heading angle of the vehicle measured positively from North. The control variables are $u = [\sigma \quad \alpha]$. The bank angle σ appears explicitly in the equations of motion, whereas angle of attack α appears implicitly through lift and drag. In the above equations $r = R + h$, where, R is the radius of Earth, h is the altitude of the vehicle from the surface of the Earth and g is the acceleration due to gravity. The aerodynamic lift and drag accelerations can be expressed as follows.

$$L = \frac{1}{2m}\rho V^2 S_{ref} C_L \tag{7}$$

$$D = \frac{1}{2m}\rho V^2 S_{ref} C_D \tag{8}$$

where m is the mass of the vehicle, ρ is the atmospheric density, S_{ref} is the reference area of the vehicle and C_L and C_D represents lift and drag coefficients which are functions of angle of attack and Mach number.

Instead of time, specific energy is used as the independent variable, which eliminates the need to select a final time. The total mechanical energy is the sum of the potential and the kinetic energy. The specific energy can be defined as

$$e = V^2/2 + gh \tag{9}$$

$$\frac{de}{dt} = -DV \tag{10}$$

The energy is normalized using the equation $\frac{e-e_i}{e_f-e_i}$, so that the normalized energy varies form 0 to 1, where e is the instantaneous energy per unit mass and e_i and e_f are the initial and final energies calculated from the entry and TAEM (Terminal Area Energy Management) phase conditions.

3 Reference Trajectory Planning

Drag acceleration is used as the reference parameter in the entry guidance because it is directly related to the trajectory length and it can be accurately measured on board. If the drag acceleration is proper, the path constraints will be satisfied implicitly. The reference trajectory is constructed in a drag-energy plane, whose x-axis is normalized energy and y-axis is drag acceleration in m/s^2.

Path Constraints

For a safe atmospheric re-entry of the vehicle, the trajectory is limited by the inequality constraints on heat rate, dynamic pressure and load factor and the equilibrium glide condition [6, 8]. These are called path constraints and these constraints together form an entry corridor for the vehicle. The equations for path constraints are given in Eq. 11-14.

$$\dot{Q} = C\sqrt{\rho}V^{c_2} \leq \dot{Q}_{max} \tag{11}$$

$$q = \frac{1}{2}\rho V^2 \leq q_{max} \tag{12}$$

$$|L\cos\alpha + D\sin\alpha| \leq n_{max} \tag{13}$$

$$L\cos\sigma - g + \frac{V^2}{r} = 0 \tag{14}$$

where,

$$C = \frac{c_1}{\sqrt{R_n\rho_0}}\frac{1}{V_c^{c_2}} \tag{15}$$

where, R_n is the nose radius of the vehicle, ρ_0 is the sea level atmospheric density, atmospheric constants $c_1 = 1.06584 \times 10^8\ W/m^{3/2},\ c_2 = 3$ and circular velocity, $V_c = \sqrt{gr}$. The maximum boundaries of heat rate, dynamic pressure and aerodynamic overload are represented as $\dot{Q}_{max},\ q_{max}$ and n_{max} respectively. The equilibrium glide constraint is obtained by equating $\dot{\gamma}=0$ and it is known as soft constraint. This is used for reducing the phugoid oscillations and to maintain a control over the vehicle, therefore, slight violation of this constraint does not create much problem.

The reference trajectory is drag-energy profile therefore all the inequality constraints are transformed to drag-energy space which limits the magnitude of drag acceleration[10]. The drag acceleration constraints corresponding to heat rate, dynamic pressure and g-load are as follows.

$$D_{\dot{Q}} < \frac{\dot{Q}^2_{max} C_D S_{ref}}{2mC^2 V^{2(c_2-1)}} \tag{16}$$

$$D_n < \frac{n_{max} g}{(C_L/C_D)\cos\alpha + \sin\alpha} \tag{17}$$

$$D_q < \frac{q_{max} C_D S_{ref}}{m} \tag{18}$$

The lower boundary of the corridor is formed by the drag corresponding to minimum lift and is given by the zero bank-equilibrium glide condition (Eq. 19).

$$D_{eqm} > \left(g - \frac{V^2}{r}\right)\frac{C_D}{C_L} \tag{19}$$

3.1 Brief Description of EAGLE Method [6]

Evolved Acceleration Guidance Logic for Entry (EAGLE) method is an extension of shuttle entry guidance. In this method a three segment constant reference drag-energy (D-E) profile is created and control variable bank angle is generated from the second derivative of drag with respect to energy. The first and third segments are linear functions of energy and the second segment is a constant drag acceleration. By adjusting this constant drag segment we can achieve the downrange requirement. The relation between range travelled and drag acceleration can be expressed using the following equation.

$$S = -\int_{e_i}^{e_f} \frac{1}{D(e)} de \tag{20}$$

The constant drag acceleration D_c can be obtained by solving the following trajectory length equation using secant method.

$$S = \frac{e_1 - e_i}{D_i - D_c} ln\frac{D_c}{D_i} + \frac{e_1 - e_2}{D_c} + \frac{e_f - e_2}{D_c - D_f} ln\frac{D_f}{D_c} \tag{21}$$

where, e_1 and e_2 are energy corner points chosen according to the shape of the entry corridor and D_i and D_f are the initial and final drag accelerations which are calculated from entry and terminal conditions of the vehicle which are fixed apriori.

The reference drag-energy profile is of trapezoidal shape with sharp corner points which is shown in Fig. 1.

3.2 D-E Profile using Cubic Spline Interpolation

This is a modified form of EAGLE method in which the reference profile with sharp corner points are replaced with a continuous smooth profile which lies within the drag-energy corridor and satisfies all the path and terminal constraints. The objective of this method is to minimize the deviation of actual drag acceleration generated using simulations from the reference drag acceleration profile.

The initial and final energy points and drag accelerations are fixed using the entry and TAEM interface conditions. They are (e'_i, D_i) and (e'_f, D_f), where, $e'_i = 0$ and $e'_f = 1$ are the normalized initial and final energy respectively. There are three intermediate points selected in between the initial and final conditions. The second and fourth energy points e_1 and e_2 are chosen according to the shape of the drag-energy corridor. The corresponding drag acceleration are selected in such a way that it has to satisfy all the path constraints. The drag points can be slightly varied to achieve the downrange requirement. Then the other terminal conditions like altitude and velocity will be satisfied automatically.

The matlab command '`csapi`' is used for interpolating the selected points using cubic spline interpolation method. A set of energy points $[e'_i,\ e_1,\ \frac{e'_i+e'_f}{2},\ e_2,\ e'_f]$ and the drag acceleration points of equal length are selected. Then cubic spline interpolation is used to find the values of reference drag acceleration at each instantaneous energy. The resultant drag-energy profile is a continuous smooth trajectory as shown in Fig. 1. The reference parameters such as altitude (h_{ref}) and flight path angle (γ_{ref}) are extracted from the generated reference trajectory and using these parameters the magnitude of bank angle is derived from the second derivative of the reference drag profile which is a smooth and continuous function of energy. Cubic Spline function ensures the continuity of first and second derivatives of reference drag-energy profile.

The reference altitude can be obtained by solving the following equation using secant method. The density and velocity can be expressed as functions of energy and altitude, assuming C_D as a constant. Therefore for a particular energy and corresponding drag acceleration value, altitude is the only unknown parameter.

$$D_{ref}(e) - \frac{1}{2m}\rho(h)V^2(e,h)S_{ref}C_D = 0 \tag{22}$$

Similarly, reference flight path angle can be obtained from first derivative of drag with respect to energy (Eq.23). The magnitude of bank angle can be derived from the second derivative of drag with respect to energy which is given Eq. 24. β is the inverse scale height.

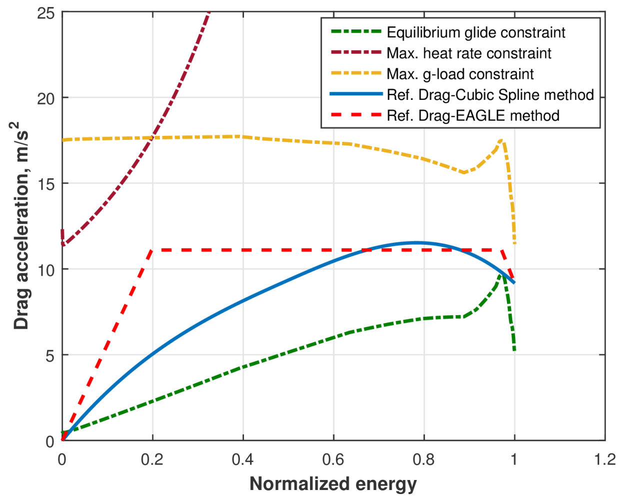

Fig. 1 Reference D-E profile using Cubic Spline interpolation and EAGLE method along with path constraints

$$D' = \frac{2D}{V^2} + \sin\gamma\left(\beta + \frac{2g}{V^2}\right) \tag{23}$$

$$D'' = a + b(L/D)\cos\sigma \tag{24}$$

$$a = \frac{2D'}{V^2} - \frac{4D}{V^4} + \frac{1}{DV^2}\left(\beta + \frac{2g}{V^2}\right)\left(g - \frac{V^2}{r}\right) \tag{25}$$

$$b = -\frac{1}{V^2}\left(\beta + \frac{2g}{V^2}\right) \tag{26}$$

4 Results and Discussion

The actual drag acceleration is plotted against normalized energy using the bank angle generated using both EAGLE method and Cubic Spline interpolation method. Simulation results are shown in Fig. 2 and 3. Both the reference profiles are satisfying the path constraints, but in EAGLE method there is a large deviation of the actual drag-energy profile from the reference profile can be noted. The constant drag segment is the adjustment parameter for meeting downrange requirement, but it should be adjusted carefully, otherwise due to the oscillations the actual drag may violate the path constraints. If the reference trajectory is having sharp corners, then accurate tracking may not be possible always. This creates considerable tracking error which leads to range dispersions.

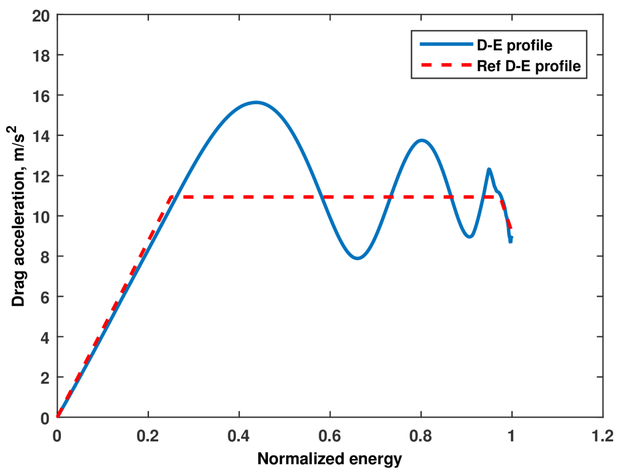

Fig. 2 Actual and reference D-E profile using EAGLE method

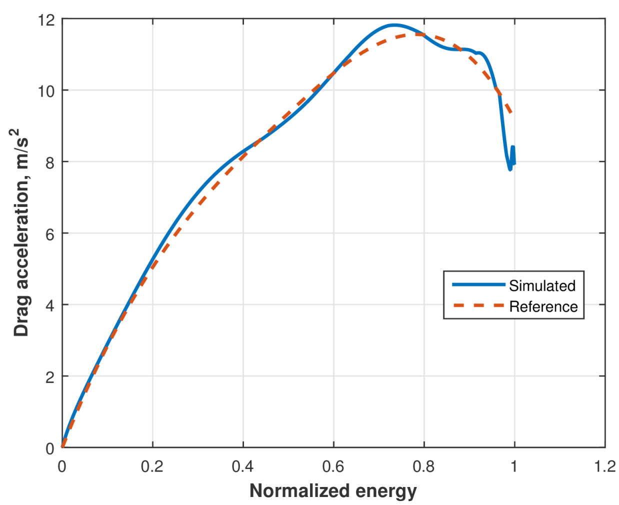

Fig. 3 Actual and reference D-E profile using Cubic Spline method

While in the other method, the deviation from the reference trajectory is comparatively less compared to EAGLE method. We can see that the actual drag acceleration profile is almost following the reference drag-energy profile created. Therefore, adjusting the trajectory to meet the desired downrange is easy compared to EAGLE method. The reference drag-energy trajectory constructed using this modified method satisfies all the path constraints and meeting the desired downrange.

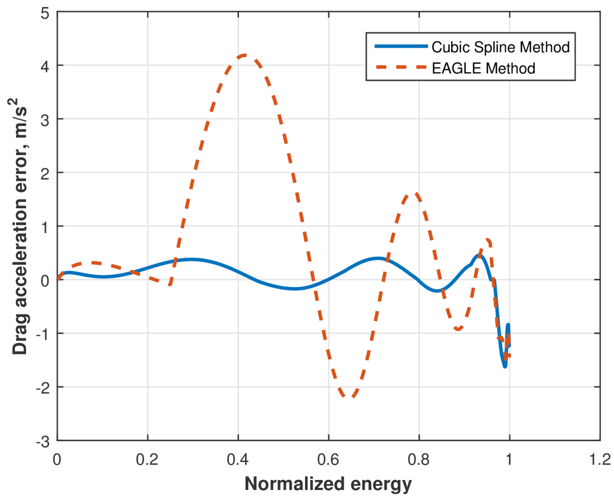

Fig. 4 Comparison of error in drag acceleration using two methods

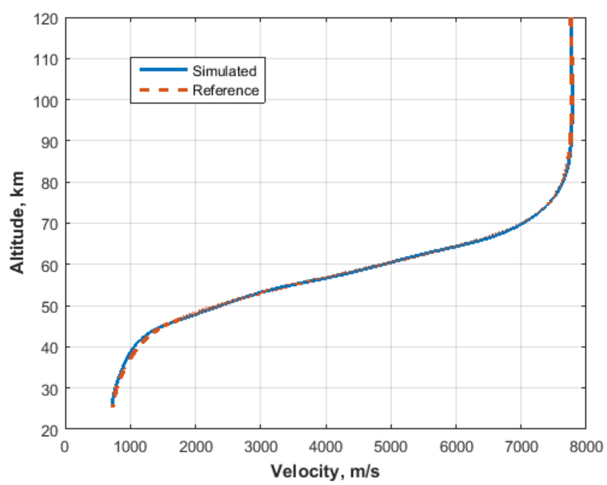

Fig. 5 Altitude-Velocity profile using cubic spline method

A quantitative measure of drag acceleration error using both the approaches is shown in Fig. 4. This figure indicates the deviation of actual drag acceleration from the reference drag acceleration for the cubic spline method as well as EAGLE method. The maximum drag deviation is 4.185 m/s^2 in EAGLE method whereas in cubic spline approach the maximum deviation of drag from the reference is 0.4543 m/s^2 which is very less compared to EAGLE method. Therefore, cubic spline interpolation approach is preferred over the other for generating reference drag-energy profile.

The altitude-velocity trajectory for the re-entry vehicle based on the cubic spline interpolation method is shown in Fig. 5. From the figure it is clear that the trajectory obtained through simulation is exactly following the reference trajectory and the deviation from the reference trajectory is very small. The final altitude error is also very less compared to EAGLE method. Figure 6 indicates that all the path constraints are satisfied with this modified reference trajectory planning.

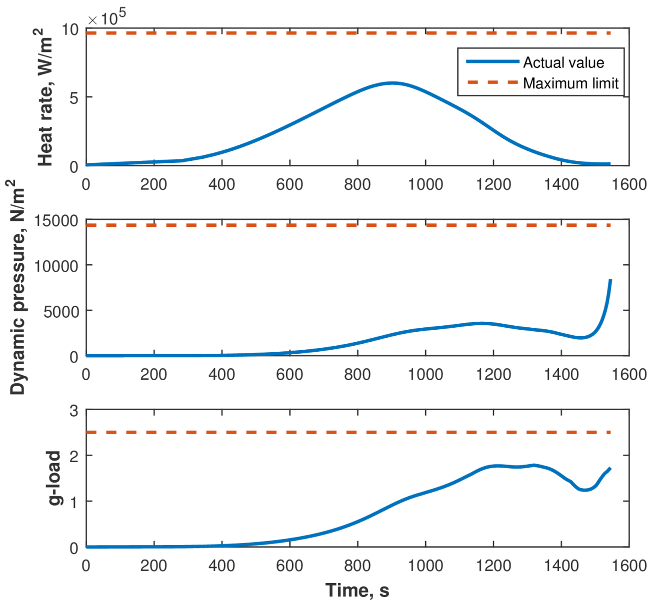

Fig. 6 Path constraints with their maximum limits

The terminal altitude error can be further reduced if the number of intermediate points increased. A drag-energy profile constructed using 7 intermediate points are shown in Fig. 7. Hence the terminal altitude error is further reduced from 0.8 km to 0.5 km. A comparison in terms of terminal altitude error is given in Table 1. The parameters used for simulations[6] are given in Table 2.

Further, robustness analysis is carried out with different initial conditions and for the same reference trajectory. Drag acceleration is a function of altitude,velocity and instantaneous energy, therefore, when the initial altitude and velocity changes, initial drag also changes. The drag-energy profiles corresponding to three different initial altitudes are plotted along with the same reference trajectory. The initial conditions chosen are $(125km, 7764m/s), (120km, 7764m/s)$, and $(100km, 7782m/s)$ and the corresponding initial drag varies as 0.0025 m/s^2, 3.6427×10^{-4} m/s^2, and 0.0877 m/s^2 respectively. Simulation results are given in Fig. 8, which shows that

Table 1 Comparison between EAGLE method and Cubic Spline method

Method	Terminal altitude error (km)
EAGLE method	±1.5
Cubic spline method (3 intermediate points)	±0.8
Cubic spline method (7 intermediate points)	±0.5

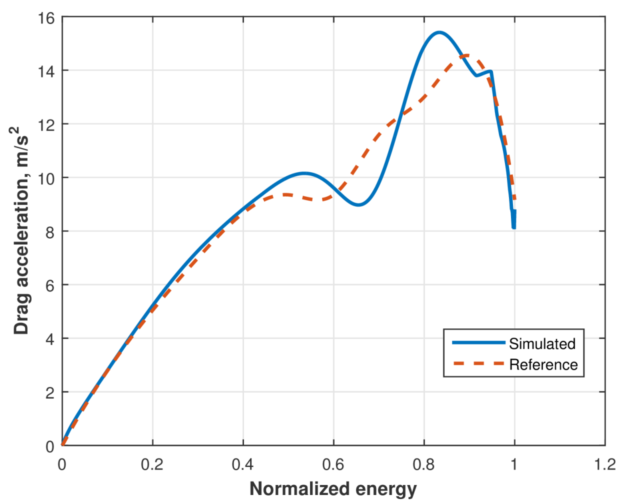

Fig. 7 D-E profile using cubic spline method with more intermediate points

Table 2 Simulation parameters

Parameter	Value	Unit
Mass	14186	kg
Reference area	52.71	m^2
Nose radius	1	m
Entry altitude	120	km
Entry velocity	7764	m/s
Entry flight path angle	0	deg
TAEM altitude	24.384	km
TAEM velocity	743	m/s
Maximum heat rate	964	kW/m^2
Maximum g-load	2.5g	-
Maximum dynamic pressure	14360	N/m^2
Maximum α	45	deg
Maximum σ	90	deg

for a large range of variation in altitude also, the actual drag-energy profiles follows the reference trajectory. Therefore, we can say that the cubic spline interpolation method is more robust with respect to initial drag. The deviation from reference profile for each trajectory is plotted in Fig. 9 and they are more or less same.

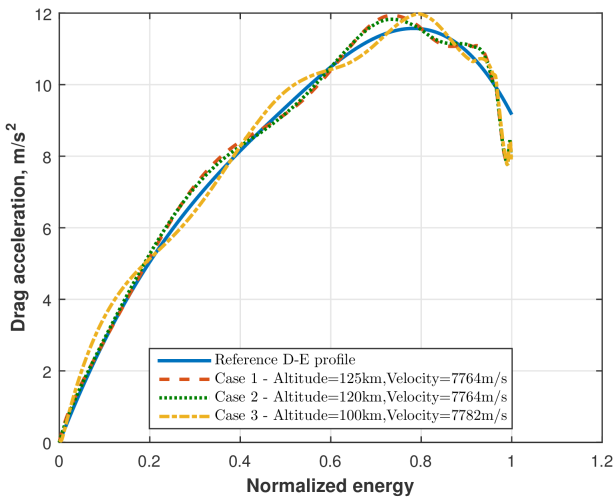

Fig. 8 D-E profiles with same reference trajectory and different initial drag (Cubic spline method)

Similarly, the same robustness analysis has been done for EAGLE method also, in order to compare the performance with the cubic spline approach and the simulation results are shown in Fig. 10 and 11. From the results we can observe that there are large deviations of drag acceleration profiles with different initial conditions from the same reference profile.

5 Conclusion

This paper describes a drag acceleration based trajectory planning method using cubic spline interpolation. This is a modified form of Evolved Acceleration Guidance Logic for Entry (EAGLE) method in which a continuous smooth drag-energy profile is created in place of a profile with sharp corners. The advantage of using a continuous smooth drag acceleration is that the first and second derivatives of drag can be expressed as cubic spline functions of specific energy. The main purpose of this modified approach is to reduce the deviation of simulated actual drag from the reference drag profile. From the simulations it is observed that the actual trajectory follows the reference trajectory with less deviations compared to EAGLE method. The initial and final points of the reference profile are fixed according to

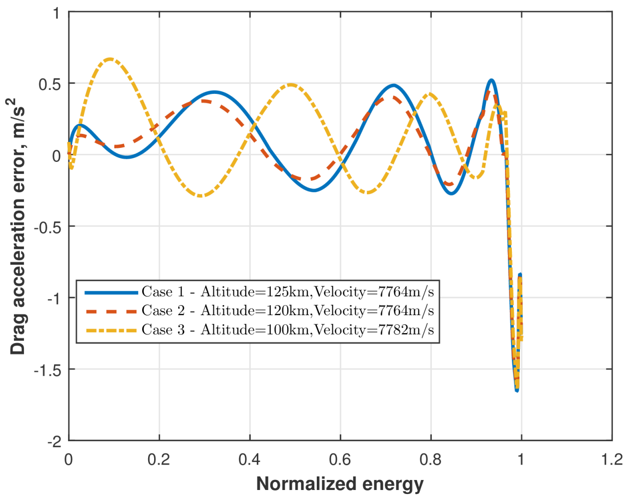

Fig. 9 Deviation of D-E profiles from reference trajectory for different initial conditions (Cubic spline method)

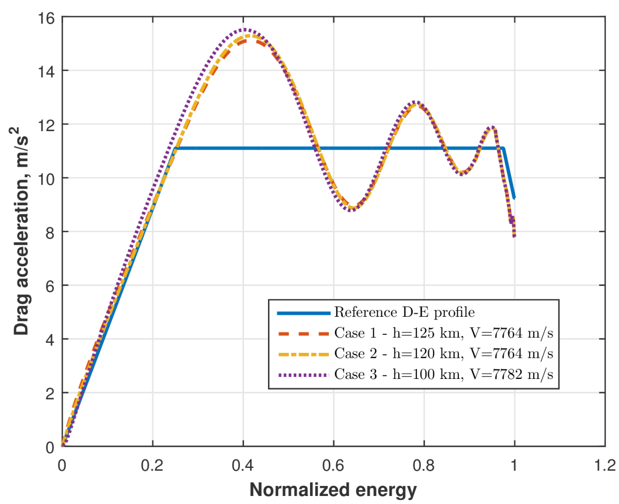

Fig. 10 D-E profiles with same reference trajectory and different initial drag (EAGLE method)

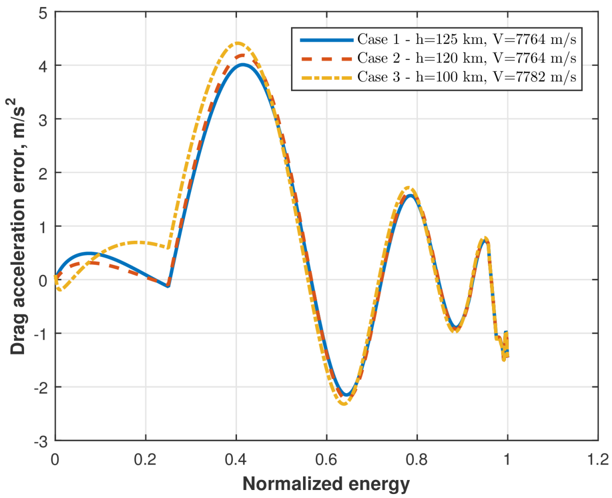

Fig. 11 Deviation of D-E profiles from reference trajectory for different initial conditions (EAGLE method)

the given entry and terminal conditions and intermediate points are varied to satisfy the path constraints as well as to meet the downrange requirement. It is seen that as the number of intermediate points increases, the terminal altitude error decreases. This method is tested in SL-12 vehicle which has a medium L/D ratio of 1.4. Simulation results show that all the path and terminal constraints are satisfied and the downrange requirement is also met for both the vehicles. The terminal altitude error is very small and are within the admissible range. Simulation results demonstrates that compared to EAGLE method, the cubic spline approach for trajectory planning is more robust with respect to different initial drag accelerations. These simulation results bring out the effectiveness of the proposed trajectory planning algorithm.

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