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Composite Adaptive Control for Robot Manipulator Systems

Hongyang Dong, Qinglei Hu and Maruthi R. Akella

Abstract This paper presents a new class of adaptive controllers for robot manipulators under parameter uncertainties. The core design structure of this method is the employment of a special adaptive algorithm, in which both instantaneous state data and past measurements (historical data) are introduced into the adaptation process. The main contribution of the overall control scheme is that parameter estimation errors are ensured to exponentially converge to zero subject to the satisfaction of a finite excitation condition, which is a relaxation when compared to the persistent excitation condition that is typically required for these classes of problems regarding parameter convergence. Numerical simulations are illustrated to show the effectiveness of the proposed method.

1 Introduction

With the rapid development of space industry, robot manipulator systems play an increasingly pominent role due to applications such as in on-orbit refueling and assembly [1]. The control problems of robot manipulator systems have received wide attention in the literature in recent decades [2, 3, 4]. In particular, various adaptive control methods have been extensively investigated to address trajectory tracking problems of robot maniplator systems in the presence of parameter uncertainties [5, 6, 7]. Most of the existing results are based on the certainty equivalence (CE) principle, which enables adap-

Hongyang Dong, Qinglei Hu
Beihang University, Beijing, 100191, China. e-mail: hdong@buaa.edu.cn, huql_buaa@buaa.edu.cn

Maruthi R. Akella
The University of Texas at Austin, Austin, Texas, 78712, USA. e-mail: makella@utexas.mail.edu

tive controllers to retain same structures as the deterministic-case controllers (no uncertainties in system dynamics) while replacing the unknown parameters with their instantaneous estimates, and then further introduce stable adaptive laws to cancel out undesirable terms resulting from the parameter estimation errors in the closed-loop dynamics. Given this design philosophy, the performance of CE-based controllers can typically only at best match the performance of the corresponding controllers for deterministic cases, that too, only if parameter estimates rapidly converge to their corresponding true values (unknown). This is due to the fact that adaptive controllers synthesized through the CE principle are intrinsically designed to ensure tracking error elimination instead of precise parameter estimation. Moreover, it is well understood that CE-based adaptive controllers cannot guarantee the convergence of parameter estimation errors to zero unless reference signals additionally satisfy certain persistent excitation (PE) conditions [8]. These facts result in potential performance degradation of CE-based adaptive controllers in many applications when compared with original deterministic controllers. Thus, research efforts in the adaptive control field started exploring deviations from the CE philosophy and by seeking the introduction of judiciously designed parameter update laws into the adaptation process. A notable example of such an effort is the immersion and invariance (I&I) formulation [9, 10], which has been shown to address many limitations arising from the original CE design structure. However, even though I&I-based controllers have been demonstrated to have potentially improved parameter estimation and state tracking performance (for example, Ref. [11]), the convergence of estimation errors under these new schemes are still subject to the satisfaction of PE conditions.

Aiming to address the possible absence of persistence of excitation, the main contribution of this paper is the introduction of a new adaptive control scheme for tracking control problems in robot manipulator systems, which has the ability to ensure the convergence of not only state tracking errors but also parameter estimation errors while relaxing the strict PE condition. It should be emphasized that the results presented in this paper are partially inspired by the concurrent learning adaptive control (CLAC) theory [12, 13], but maintain certain crucial distinctions. The CLAC design innovatively uses specially selected and online recorded state data concurrently with instantaneous state data for adaptation, Under the CLAC framework, if system states could be providing sufficient excitation over a finite interval (which is formally referred to as a finite excitation (FE) condition), the control algorithm can be designed such that rich enough historical data could be recorded to ensure the convergence of parameter estimation errors. However, to acquire the historical data, smoothers or observers (such as the optimal fixed point smoother given in Ref. [12]) need to be employed to approximate state derivatives, a process that is usually vulnerable to multiple sources of measurement noise and approximation errors. This requirement thus lays a great theoretical barrier for the applications of CLAC to mechanical systems due to the inevitable coupling that exists between state derivatives and unknown parameters. In this paper, we significantly extend the original CLAC framework, and for the first time (to the best knowledge of the authors), a novel adaptive tracking control algorithm ensuring precise parameter estimation under a FE condition is developed for robot manipulator systems. To be specific, low-pass filtered regressor matrices and states are first introduced into the formulation, which could not only circumvent the state derivative estimation requirements of the classical CLAC formulation within the adaptation scheme but also renders the resulting parameter-adaptation dynamics reside within a stable and attracting manifold. Subsequently, a special information matrix is designed to continuously record historical data and introduce new information into adaptation process, and a judiciously designed non-CE term is also introduced into the adaptive algorithm to help the resulting closed-loop system overcome the uniform detectability obstacle if FE condition cannot be satisfied. Since the adaptive law is intrinsically the combination of an information-based part and a non-CE part, we label the proposed controller a composite adaptive controller. Under this design framework, system states could asymptotically track the desired trajectories, and if the reference signal further satisfies a FE condition (a much weaker condition than PE), state tracking errors as well as parameter estimation errors are guaranteed to exponentially converge to zero.

The remainder of this paper is organized as follows. The robot manipulator dynamics is introduced in Sec. II, and the control problem is also formulized. The main results of this paper are presented along with theoretical stability proof in Sec. III. Subsequently, prototypical simulation results are illustrated in Sec. IV to show the effectiveness and advantages of the proposed method. And then this paper ends with some conclusions in Sec. V.

2 Preliminaries and Problem Formulation

2.1 Preliminaries

Before presenting the main results of this paper, the definitions of FE and PE conditions are first introduced as follows.

Definition 1. [12] A bounded signal $f(\cdot):\mathbb{R}\to\mathbb{R}^{n\times m}$ is said to be finite exciting (FE) over an interval $[t,t+T]$, where $t\geq 0$ is finite, if there exist finite constants $T>0$ and $c>0$ such that

$$\int_t^{t+T} f^{\mathrm{T}}(\tau)f(\tau)\mathrm{d}\tau \geq c\boldsymbol{I}_{m\times m} \tag{1}$$

where $\boldsymbol{I}_{m\times m}$ is the m dimensional identity matrix.

Definition 2. [14] A bounded signal $f(\cdot):\mathbb{R}\rightarrow\mathbb{R}^{n\times m}$ is said to be persistently exciting (PE) if there exist finite positive constants c and T such that for arbitrary $t\geq 0$, one has

$$\int_{t}^{t+T} f^{\mathrm{T}}(\tau)f(\tau)\mathrm{d}\tau \geq c\boldsymbol{I}_{m\times m} \tag{2}$$

The contrast between FE and PE conditions are clearly indicated by their definitions, the former requires the signal to be exciting just over a finite time interval, whereas, qualitatively speaking, PE implies the satisfaction of FE throughout the whole timeline.

2.2 Problem Formulation

In this paper, the dynamics of n-degree-of-freedom rigid robot manipulator systems is employed, which could be described by the following model:

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 \tag{3}$$

$$M(\boldsymbol{x}_1)\dot{\boldsymbol{x}}_2 + C(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{x}_2 + g(\boldsymbol{x}_1) = \boldsymbol{u} \tag{4}$$

where $\boldsymbol{x}_1,\boldsymbol{x}_2\in\mathbb{R}^n$ respectively denote the generalized position and velocity vectors, $M(\boldsymbol{x}_1)$ is the generalized mass matrix, $C(\boldsymbol{x}_1,\boldsymbol{x}_2)$ denotes the Coriolis matrix, $\boldsymbol{u}$ is the control input to be designed, and the gravity vector is denoted by $g(\boldsymbol{x}_1)$. The dynamics given in Eqs. (3)-(4) satisfies several well-known structural properties as follows [15]:

1) $M(\boldsymbol{x}_1)$ is positive definite for all $\boldsymbol{x}_1\in\mathbb{R}^n$, furthermore, there exists positive constants $\lambda_{\min}$ and $\lambda_{\max}$ such that

$$\lambda_{\min} \leq \|M(\boldsymbol{x}_1)\| \leq \lambda_{\max},\ \forall \boldsymbol{x}_1 \in \mathscr{L}_\infty \tag{5}$$

2) For arbitrary vectors $\boldsymbol{y},\boldsymbol{z}\in\mathbb{R}^n$, the governing equations given in the following form permit a parameter affine representation

$$M(\boldsymbol{x}_1)\boldsymbol{y} + C(\boldsymbol{x}_1,\boldsymbol{x}_2)\boldsymbol{z} + g(\boldsymbol{x}_1) = Y(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{z})\boldsymbol{\theta}^* \tag{6}$$

where $Y(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{z})\in\mathbb{R}^{n\times m}$ is a regressor matrix, and $\boldsymbol{\theta}^*\in\mathbb{R}^m$ contains the information of all unknown constant parameters.
3) $M(\boldsymbol{x}_1)$ and $C(\boldsymbol{x}_1,\boldsymbol{x}_2)$ satisfy the following skew-symmetric property:

$$\boldsymbol{a}^{\mathrm{T}}\left[\dot{M}(\boldsymbol{x}_1) - 2C(\boldsymbol{x}_1,\boldsymbol{x}_2)\right]\boldsymbol{a} = 0,\ \forall \boldsymbol{a}\in\mathbb{R}^n \tag{7}$$

We consider a reference signal $\boldsymbol{x}_m$ for the trajectory tracking problem, which is smooth and satisfies $\boldsymbol{x}_m,\dot{\boldsymbol{x}}_m,\ddot{\boldsymbol{x}}_m\in\mathscr{L}_\infty$. The tracking error signals are further defined by $\boldsymbol{e}_1 = \boldsymbol{x}_1 - \boldsymbol{x}_m$ and $\boldsymbol{e}_2 = \boldsymbol{x}_2 - \dot{\boldsymbol{x}}_2$. Substituting $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ into

Eqs. (3) and (4) yields

$$\dot{\boldsymbol{e}}_1 = \boldsymbol{e}_2 \tag{8}$$

$$M(\boldsymbol{x}_1)\dot{\boldsymbol{e}}_2 = W(\boldsymbol{x}_1, \boldsymbol{x}_2, \ddot{\boldsymbol{x}}_m)\boldsymbol{\theta}^* + \boldsymbol{u} \tag{9}$$

wherein $W(\boldsymbol{x}_1, \boldsymbol{x}_2, \ddot{\boldsymbol{x}})\boldsymbol{\theta}^* = -M(\boldsymbol{x}_1)\ddot{\boldsymbol{x}}_m - C(\boldsymbol{x}_1, \boldsymbol{x}_2) - g(\boldsymbol{x}_1)$. Hereafter, for the sake of brevity, arguments of matrix functions will be ignored except for additional explanations.

Given the tracking error dynamic model and assuming full state feedback (i.e., perfect measurements of signals $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$), the control objective is to design control signal $\boldsymbol{u}$, such that $\lim_{t\to\infty}\{\boldsymbol{e}_1(t), \boldsymbol{e}_2(t)\} = \mathbf{0}_{n\times 1}$.

3 Composite Adaptive Control Scheme Development and Stability Analysis

In this section, a composite adaptive control method is presented to address the tracking control problem of robor manipulators in the presence of parameter uncertainties. First, as an essential part of the whole control method, a low-pass filtered model is introduced in next subsection.

3.1 Filtered System Model

Define the following filtered states and regressor matrix,

$$\dot{\boldsymbol{e}}_{f1}(t) = -\alpha \boldsymbol{e}_{f1}(t) + \boldsymbol{e}_1(t), \ \boldsymbol{e}_{f1}(0) \in \mathbb{R}^n \tag{10}$$

$$\dot{\boldsymbol{e}}_{f2}(t) = -\alpha \boldsymbol{e}_{f2}(t) + \boldsymbol{e}_2(t), \ \boldsymbol{e}_{f2}(0) \in \mathbb{R}^n \tag{11}$$

$$\dot{W}_f(t) = -\alpha W_f(t) + W_r(t), \ W_f(0) \in \mathbb{R}^{n\times m} \tag{12}$$

where $\alpha > 0$ is any user-defined filter gain, $\boldsymbol{W}_r$ is defined by

$$W_r\boldsymbol{\theta}^* = W\boldsymbol{\theta}^* + M(k_v\boldsymbol{e}_2 + k_p\boldsymbol{e}_1) + \dot{M}[(k_v - \alpha)\boldsymbol{e}_{f2} + k_p\boldsymbol{e}_{f1} + \boldsymbol{e}_2] \tag{13}$$

and here $k_p, k_v \in \mathbb{R}$ are positive constants. Further define $\dot{\boldsymbol{u}}_f = -\alpha\boldsymbol{u}_f + \boldsymbol{u}$ just for analysis purposes, and substitute Eqs. (10)-(12) into Eqs. (3) and (4), to result in

$$\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\boldsymbol{e}}_{f1} - \boldsymbol{e}_{f2}) = -\alpha(\dot{\boldsymbol{e}}_{f1} - \boldsymbol{e}_{f2}) \tag{14}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}[M(\dot{\boldsymbol{e}}_{f2} + k_p\boldsymbol{e}_{f1} + k_v\boldsymbol{e}_{f2}) + W_f\boldsymbol{\theta}^* + \boldsymbol{u}_f] = \\ -\alpha[M(\dot{\boldsymbol{e}}_{f2} + k_p\boldsymbol{e}_{f1} + k_v\boldsymbol{e}_{f2}) + W_f\boldsymbol{\theta}^* + \boldsymbol{u}_f]\end{aligned} \tag{15}$$

which renders

$$\dot{\boldsymbol{e}}_{f1} = \boldsymbol{e}_{f2} + \boldsymbol{\gamma}_1,\ \boldsymbol{\gamma}_1(t) = \boldsymbol{\gamma}_1(0)\mathrm{e}^{-\alpha t} \tag{16}$$

$$M(\dot{\boldsymbol{e}}_{f2} + k_p\boldsymbol{e}_{f1} + k_v\boldsymbol{e}_{f2}) = W_f\boldsymbol{\theta}^* + \boldsymbol{u}_f + \boldsymbol{\gamma}_2,\ \boldsymbol{\gamma}_2(t) = \boldsymbol{\gamma}_2(0)\mathrm{e}^{-\alpha t} \tag{17}$$

If we take $\boldsymbol{u}_f = -W_f\hat{\boldsymbol{\theta}}$, where $\hat{\boldsymbol{\theta}} \in \mathbb{R}^m$ denotes the estimation of $\boldsymbol{\theta}^*$, and further choosing $W_f(0) = \mathbf{0}_{n\times m}$, $\boldsymbol{e}_{f1}(0) = [(\alpha - k_v)\boldsymbol{e}_1(0) - \boldsymbol{e}_2(0)]/[\alpha(\alpha - k_v) + k_p]$ and $\boldsymbol{e}_{f2}(0) = [k_p\boldsymbol{e}_1(0) + \alpha\boldsymbol{e}_2(0)]/[\alpha(\alpha - k_v) + k_p]$, then we have $\boldsymbol{\gamma}_1 \equiv \mathbf{0}_{n\times 1}$ and $\boldsymbol{\gamma}_2 \equiv \mathbf{0}_{n\times 1}$, thus the filtered system model can finally be given as follows.

$$\dot{\boldsymbol{e}}_{f1} = \boldsymbol{e}_{f2} \tag{18}$$

$$M(\dot{\boldsymbol{e}}_{f2} + k_p\boldsymbol{e}_{f1} + k_v\boldsymbol{e}_{f2}) = W_f\boldsymbol{\theta}^* + \boldsymbol{u}_f \tag{19}$$

3.2 Composite Adaptive Controller Design

Before designing the adaptive controller, the following discussion is in order.

1) An auxiliary variable $\boldsymbol{\beta}$ which is defined by the following equation will be employed in the adaptive law design.

$$\boldsymbol{\beta} = -\dot{\boldsymbol{e}}_{f2} - k_p\boldsymbol{e}_{f1} - k_v\boldsymbol{e}_{f2} \tag{20}$$

From Eq. (19) and recall that $\boldsymbol{u}_f$ follows the form $\boldsymbol{u}_f = W_f\hat{\boldsymbol{\theta}}$, it can be readily verified that $\boldsymbol{\beta} = \boldsymbol{M}^{-1}(\boldsymbol{x}_1)W_f\tilde{\boldsymbol{\theta}}$, where $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}^*$. Thus the attainable variable $\boldsymbol{\beta}$ explicitly contains the information concerning the parameter estimation error.

2) Recalling Eq. (19), one has

$$W_a\boldsymbol{\theta}^* = \boldsymbol{u}_f \tag{21}$$

where

$$W_a\boldsymbol{\theta}^* = M(\dot{\boldsymbol{e}}_{f2} + k_p\boldsymbol{e}_{f1} + k_v\boldsymbol{e}_{f2}) - W_f\boldsymbol{\theta}^* \tag{22}$$

These results indicate that, by introducing filtered states and regressor matrices, the information of the unknown parameter vector $\boldsymbol{\theta}^*$ (coupled with W_a) can be acquired through the auxiliary control input $\boldsymbol{u}_f$.

Then the main result of this paper, a composite adaptive tracking control scheme for the rigid robot manipulator application, is summarized in the following theorem.

Theorem 1. Consider the tracking error dynamics of robot manipulator systems in Eqs. (3)-(4), and the filtered states and regressor matrices introduced in Eqs. (10)-(12). Suppose the controller $\boldsymbol{u}(t)$ and the update law for the parameter estimator $\hat{\boldsymbol{\theta}}(t)$ are respectively given by

$$\boldsymbol{u}(t) = \dot{\boldsymbol{u}}_f(t) + \alpha\boldsymbol{u}_f(t), \quad \boldsymbol{u}_f(t) = W_f(t)\hat{\boldsymbol{\theta}}(t) \tag{23}$$

$$\dot{\hat{\boldsymbol{\theta}}}(t) = -(\frac{1}{2k_v} + k_a\mu)\boldsymbol{W}_f^{\mathrm{T}}(t)\boldsymbol{\beta}(t) - k_a k_c \boldsymbol{\Omega}_a(t) \tag{24}$$

where

$$\boldsymbol{\Omega}_a(t) = \begin{cases} \boldsymbol{C}(t)[\boldsymbol{A}(t)\hat{\boldsymbol{\theta}}(t) - \boldsymbol{B}(t)] & \text{if } \forall t_a \in [0,t],\ \mathrm{rank}(\boldsymbol{A}(t_a)) < m \\ \boldsymbol{A}^{-1}(t_a)[\boldsymbol{A}(t_a)\hat{\boldsymbol{\theta}}(t) - \boldsymbol{B}(t_a)] & t_a = \min\{\arg_{\tau\in[0,t]}[\mathrm{rank}(\boldsymbol{A}(\tau)) = m]\} \end{cases} \tag{25}$$

with $\boldsymbol{C}(t) = \boldsymbol{A}(t)[\boldsymbol{A}(t)\boldsymbol{A}(t) + a\boldsymbol{I}_{m\times m}]^{-1}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined by

$$\dot{\boldsymbol{A}}(t) = -\sigma\boldsymbol{A}(t) + W_a^{\mathrm{T}}(t)W_a(t), \quad \boldsymbol{A}(0) = \boldsymbol{0}_{m\times m} \tag{26}$$

$$\dot{\boldsymbol{B}}(t) = -\sigma\boldsymbol{B}(t) + W_a^{\mathrm{T}}(t)\boldsymbol{u}_f(t), \quad \boldsymbol{B}(0) = \boldsymbol{0}_{m\times 1} \tag{27}$$

and μ, k_a, k_c, a, σ are user-defined positive constants. Then for arbitrary $\boldsymbol{e}_1(0), \boldsymbol{e}_2(0) \in \mathbb{R}^n$ and $\hat{\boldsymbol{\theta}}(0) \in \mathbb{R}^m$, it can be guaranteed that

$$\lim_{t\to\infty}\{\boldsymbol{e}_1(t), \boldsymbol{e}_2(t), W_f(t)\tilde{\boldsymbol{\theta}}(t)\} = \boldsymbol{0}_{n\times 1} \tag{28}$$

Furthermore, if $W_a(t)$ satisfies a FE condition as given in Definition 1, the tracking errors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, and also the parameter estimation error $\tilde{\boldsymbol{\theta}}$ exponentially converge to zero.

Proof. Consider the following storage function,

$$V = \frac{\rho}{2}(k_p\boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f1} + \boldsymbol{e}_{f2}^{\mathrm{T}}\boldsymbol{e}_{f2} + \frac{1}{k_a b}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}) + \boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f2} \tag{29}$$

where ρ and b are positive constants introduced just for stability analysis purposes, such that

$$\rho \geq \max\{\frac{2}{k_p}, 2, \frac{4}{k_v} + \frac{4k_v}{k_p}, \frac{2}{\mu k_p}\}, \quad b \leq \lambda_{\min}(M) \tag{30}$$

and here $\lambda_{\min}(M)$ denotes the minimum eigenvalue of M. From Eq. (29), one has

$$V \geq (\frac{k_p\rho}{2} - \frac{1}{2})\boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f1} + (\frac{\rho}{2} - \frac{1}{2})\boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f1} + \frac{\rho}{2k_a b}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}} \tag{31}$$

Thus V is a valid Lyapunov function candidate when ρ satisfies the condition given in Eq. (30).

Substituting Eqs. (18) and (19) into the time derivative of V yields

$$\begin{aligned}\dot{V} = &- k_p\boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f1} - (\rho k_v - 1)\boldsymbol{e}_{f2}^{\mathrm{T}}\boldsymbol{e}_{f2} \\ &- k_v\boldsymbol{e}_{f1}^{\mathrm{T}}\boldsymbol{e}_{f2} + (\boldsymbol{e}_{f1} + \rho\boldsymbol{e}_{f2})^{\mathrm{T}}\boldsymbol{M}^{-1}(\boldsymbol{x}_1)\boldsymbol{W}_f\tilde{\boldsymbol{\theta}} + \frac{\rho}{bk_a}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}\end{aligned} \tag{32}$$

According to Cauchy-Schwarz inequality, one has

$$k_v \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f2} \leq \frac{k_p}{4} \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f1} + \frac{k_v^2}{k_p} \boldsymbol{e}_{f2}^{\mathrm{T}} \boldsymbol{e}_{f2} \tag{33}$$

$$\begin{aligned} (\boldsymbol{e}_{f1} + \rho \boldsymbol{e}_{f2}) \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \boldsymbol{W}_f \tilde{\boldsymbol{\theta}} \leq & \frac{k_p}{4} \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f1} + \frac{\rho k_v}{2} \boldsymbol{e}_{f2}^{\mathrm{T}} \boldsymbol{e}_{f2} \\ & + (\frac{1}{bk_p} + \frac{\rho}{2bk_v}) \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{W}_f^{\mathrm{T}} \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \boldsymbol{W}_f \tilde{\boldsymbol{\theta}} \end{aligned} \tag{34}$$

Then, substituting Eqs. (33) and (34) into Eq. (32), one has

$$\begin{aligned} \dot{V} \leq & -\frac{k_p}{2} \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f1} - \frac{\rho k_v}{4} \boldsymbol{e}_{f2}^{\mathrm{T}} \boldsymbol{e}_{f2} \\ & + (\frac{1}{bk_p} + \frac{\rho}{2bk_v}) \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{W}_f^{\mathrm{T}} \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \boldsymbol{W}_f \tilde{\boldsymbol{\theta}} + \frac{\rho}{bk_a} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \dot{\hat{\boldsymbol{\theta}}} \end{aligned} \tag{35}$$

Further substituting adaptive law Eq. (24) into Eq. (35), one can obtain

$$\dot{V} \leq -\frac{k_p}{2} \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f1} - \frac{\rho k_v}{4} \boldsymbol{e}_{f2}^{\mathrm{T}} \boldsymbol{e}_{f2} - \frac{\rho \mu}{2b} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{W}_f^{\mathrm{T}} \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \boldsymbol{W}_f \tilde{\boldsymbol{\theta}} - \frac{\rho k_c}{b} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{\Omega}_a \tag{36}$$

It should be noted that Eq. (30) is employed in Eqs. (35) and (36). The remainder of proof is separated into two parts given different conditions.

1) W_a fails to satisfy the FE condition. From the definition of $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ in Eqs. (26) and (27), one can readily verify that $\boldsymbol{A}(t)$ is a positive semi-definite matrix for all $t \geq 0$, and $\boldsymbol{B}(t) = \boldsymbol{A}(t)\boldsymbol{\theta}$. Thus

$$\begin{aligned} \dot{V} \leq & -\frac{k_p}{2} \boldsymbol{e}_{f1}^{\mathrm{T}} \boldsymbol{e}_{f1} - \frac{\rho k_v}{4} \boldsymbol{e}_{f2}^{\mathrm{T}} \boldsymbol{e}_{f2} - \frac{\rho \mu}{2b} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{W}_f^{\mathrm{T}} \boldsymbol{M}^{-1}(\boldsymbol{x}_1) \boldsymbol{W}_f \tilde{\boldsymbol{\theta}} \\ & - \frac{\rho k_c}{b} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{A} [\boldsymbol{A}\boldsymbol{A} + a\boldsymbol{I}_m]^{-1} \boldsymbol{A} \tilde{\boldsymbol{\theta}} \\ \leq & -\frac{k_p}{2} \|\boldsymbol{e}_{f1}\|^2 - \frac{\rho k_v}{4} \|\boldsymbol{e}_{f2}\|^2 - \frac{\rho \mu}{2b^2} \|\boldsymbol{W}_f \tilde{\boldsymbol{\theta}}\|^2 \end{aligned} \tag{37}$$

Accordingly, we have $\boldsymbol{e}_{f1}, \boldsymbol{e}_{f2}, \tilde{\boldsymbol{\theta}} \in \mathscr{L}_\infty$, recall $\boldsymbol{x}_m, \dot{\boldsymbol{x}}_m, \ddot{\boldsymbol{x}}_m \in \mathscr{L}_\infty$, we can obtain $W_f \in \mathscr{L}_\infty$. Further from Eqs. (18) and (19), one has $\dot{e}_{f1}, \dot{e}_{f2}, \ddot{e}_{f1}, \ddot{e}_{f2} \in \mathscr{L}_\infty$. Since Eq. (37) also indicates $\boldsymbol{e}_{f1}, \boldsymbol{e}_{f2}, W_f \tilde{\boldsymbol{\theta}} \in \mathscr{L}_2$. According to Barbalat's lemma, these results ensure

$$\lim_{t \to \infty} \{\boldsymbol{e}_{f1}(t), \boldsymbol{e}_{f2}(t), \dot{\boldsymbol{e}}_{f1}(t), \dot{\boldsymbol{e}}_{f2}(t), W_f(t) \tilde{\boldsymbol{\theta}}(t)\} = \boldsymbol{0}_{n \times 1} \tag{38}$$

Finally, recall Eqs. (10) and (11), we have

$$\lim_{t \to \infty} \{\boldsymbol{e}_1(t), \boldsymbol{e}_2(t), W_f(t) \tilde{\boldsymbol{\theta}}(t)\} = \boldsymbol{0}_{n \times 1} \tag{39}$$

2) W_a satisfies the FE condition. By the defnition of FE, there exist finite constants $t^* \geq 0$, $T > 0$ and $c > 0$, such that

$$\int_{t^*}^{t^*+T} W_a^{\mathrm{T}}(\tau) W_a(\tau) \geq c\boldsymbol{I}_{m\times m} \tag{40}$$

Thus, from Eq. (26), one has

$$\begin{aligned}\boldsymbol{A}(t^*+T) =& \mathrm{e}^{-\sigma(t^*+T)} \int_0^{t^*+T} \mathrm{e}^{\sigma\tau} W_a^{\mathrm{T}}(\tau) W_a(\tau)\mathrm{d}\tau \\ \geq& \mathrm{e}^{-\sigma(t^*+T)} \mathrm{e}^{\sigma t^*} \int_{t^*}^{t^*+T} W_a^{\mathrm{T}}(\tau) W_a(\tau)\mathrm{d}\tau \\ \geq& c\mathrm{e}^{-\sigma T} \boldsymbol{I}_{m\times m}\end{aligned} \tag{41}$$

This result indicates if $\boldsymbol{W}_a$ satisfies the FE condition, then there always exists a finite time t_a, such that $\mathrm{rank}(\boldsymbol{A}(t_a)) = m$. Within this context, the storage function V satisfies

$$\dot{V} \leq -\frac{k_p}{2}\|\boldsymbol{e}_{f1}\|^2 - \frac{\rho k_v}{4}\|\boldsymbol{e}_{f2}\|^2 - \frac{\rho\mu}{2b^2}\|\boldsymbol{W}_f\tilde{\boldsymbol{\theta}}\|^2 - \frac{\rho k_c}{b}\|\tilde{\boldsymbol{\theta}}\|^2 \leq -\omega V \tag{42}$$

after a finite time t_a, where $\omega = 2/\rho$. Eq. (42) ensures the exponential convergence of $\boldsymbol{e}_{f1}$, $\boldsymbol{e}_{f2}$ and $\tilde{\boldsymbol{\theta}}$. Then owing to Eqs. (10), (11), (18) and (19), we can finally guarantee that the tracking errors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and the parameter estimation error $\tilde{\boldsymbol{\theta}}$ could exponentially converge to zero.

The proof is complete.

4 Numerical Simulation

To study the effectiveness of the proposed controller, the two-link robot manipulator example presented in Ref. [16] is employed. The robot model is defined by

$$M(\boldsymbol{x}_1) = \begin{bmatrix} p_1 + 2p_3\cos(x_{12}) & p_2 + p_3\cos(x_{12}) \\ p_2 + p_3\cos(x_{12}) & p_2 \end{bmatrix} \tag{43}$$

$$C(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{bmatrix} -p_3 x_{22}\sin(x_{12}) & -p_3(x_{21}+x_{22})\sin(x_{12}) \\ p_3 x_{21}\sin(x_{12}) & 0 \end{bmatrix} \tag{44}$$

where $\boldsymbol{\theta}^* = [p_1, p_2, p_3]^{\mathrm{T}} = [3.6, 0.2, 0.15]^{\mathrm{T}}$ is the unknown parameter vector. The initial conditions are set to be $\boldsymbol{x}_1(0) = \boldsymbol{x}_2(0) = \boldsymbol{0}_{2\times 1}$ and $\hat{\boldsymbol{\theta}}(0) = [1, 0, 2]^{\mathrm{T}}$. The reference signal is defined by $\boldsymbol{x}_m = [\mathrm{e}^{-0.2t}\cos(t) + 2, \mathrm{e}^{-0.2t}\sin(t) + 2]^{\mathrm{T}}$. And choosing the control parameters to be $k_p = 10$, $k_v = 5$, $\alpha = 5$, $k_a = 1$, $\mu = 0.2$, $k_c = 20$, $\sigma = 0.05$, $a = 0.1$. To show the advantages of the proposed method (denoted as "CAC"), the CE-based adaptive controller in Ref. [17] and the I&I-based non-CE adaptive controller in Ref. [18] (denoted as "CE" and "NCE", respectively) are also employed to make comparison, and their various control parameters have been carefully adjusted to achieve satisfactory transient performance.

Under all these settings, simulation results are given in Figs. (1)-(3). The state tracking errors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are illustrated in Figs. 1 and 2, respectively. One can see that all the three control methods could ensure the convergence of tracking errors, and it is remarkable that the proposed method renders a superior transient performance and steady precision when compared to the other two controllers. Fig. 3 further shows the responses of parameter estimation errors under different controllers. With the weakening of excitation of the reference signal, CE-based controller cannot guarantee the convergence of $\hat{\boldsymbol{\theta}}$ at its true value, the non-CE method presented in Ref. [18] has a better performance than CE-based method regarding parameter estimation, but still renders residual errors. On the other hand, the adaptive method proposed in this paper ensures precise parameter estimation, $\hat{\boldsymbol{\theta}}$ ripidly converges to $\boldsymbol{\theta}^*$.

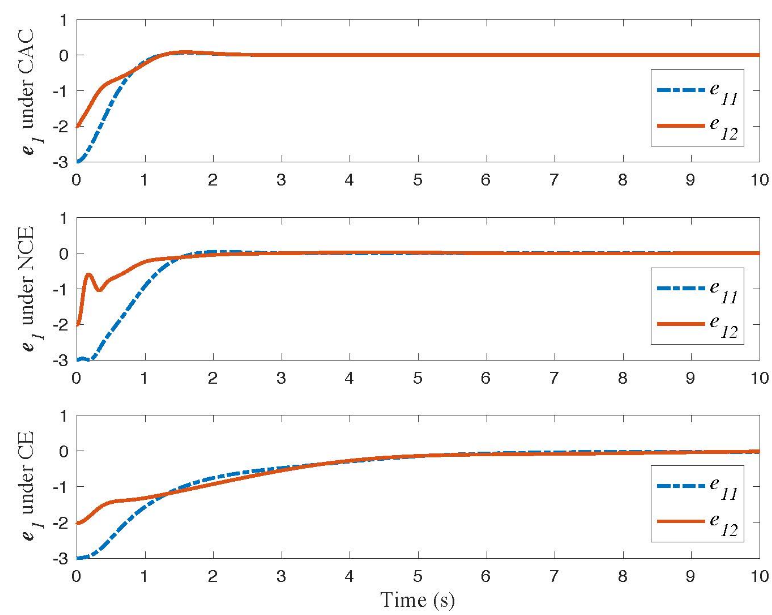

Fig. 1 Time responses of $\boldsymbol{e}_1$ under different controllers

5 Conclusion

A novel adaptive control scheme with robot manipulator application is proposed in this paper. The most important feature of this new approach is that it could potentially guarantee the precision convergence of parameter

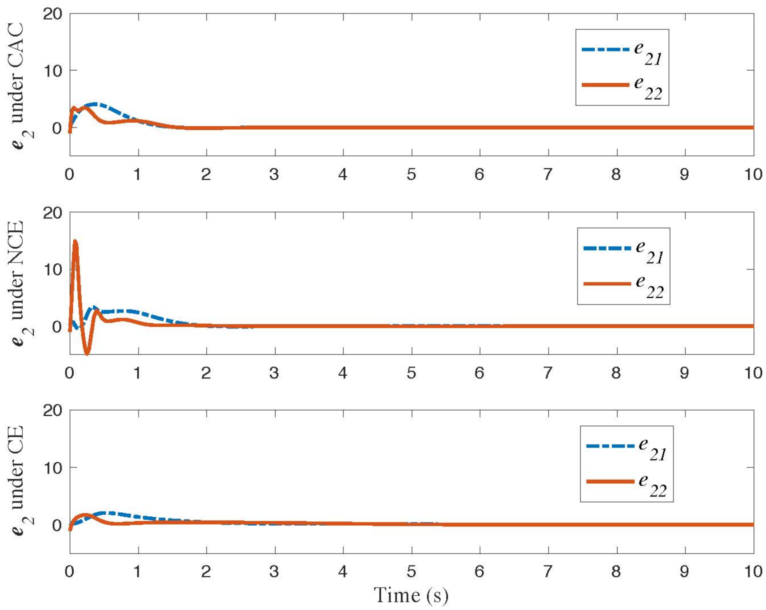

Fig. 2 Time responses of $\boldsymbol{e}_2$ under different controllers

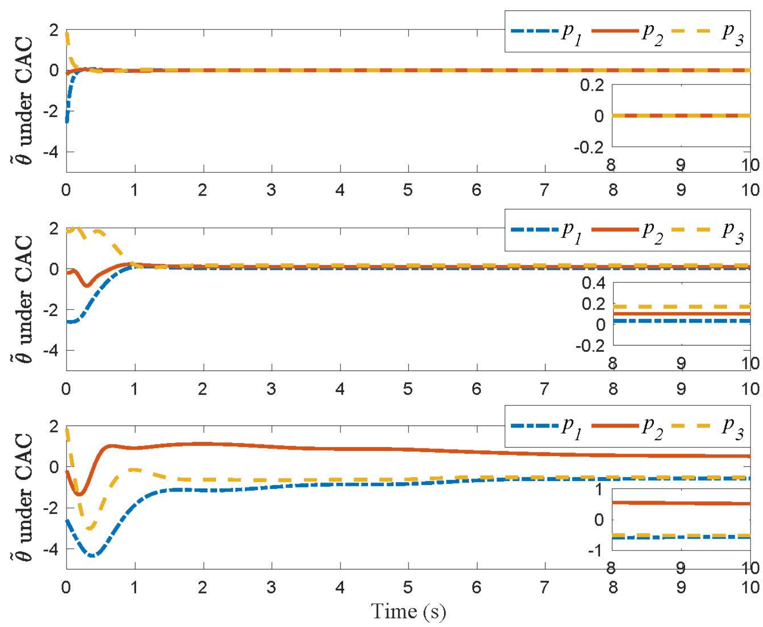

Fig. 3 Time responses of $\tilde{\boldsymbol{\theta}}$ under different controllers

estimation errors if system states satisfy a finite excitation condition. The overall implication of this result is the improved closed-loop performance of the adaptive controller at the additional cost of increased memory allocation for the adaptive controller due to the use of the information matrix in the feedback structure. Numerical simulation results are presented to illustrate the various features of the proposed method. Further work in this direction would consider robustness modifications to account of imperfect measurements and possible presence of unmodeled dynamics.

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References

1. Flores, A. A., Ma, O., Pham K., et al., "A review of space robotics technologies for on-orbit servicing," Progress in Aerospace Sciences, 2014, 68(6): 1-26.
2. Kolhe, J. P., Shaheed, M., Chandar, T. S., et al., "Robust control of robot manipulators based on uncertainty and disturbance estimation," International Journal of Robust and Nonlinear Control, 2013, 23(1): 104-122.
3. Jin, M., Kang, S. H., and Chang, P. H., "Robust control of robot manipulators using inclusive and enhanced time delay control," IEEE-ASME Transactions on Mechatronics, 2017, 22(5): 2141-2152.
4. Refoufi, S., and Benmahammed, K., "Control of a manipulator robot by neuro-fuzzy subsets form approach control optimized by the genetic algorithms," ISA Transactions, 2018, 77(Jun.): 133-145.
5. Mai, T., and Wang, Y., "Adaptive force/motion control system based on recurrent fuzzy wavelet CMAC neural networks for condenser cleaning crawler-type mobile manipulator robot," IEEE Transactions on Control Systems Technology, 2014 22(5): 1973-1982.
6. Yang, C., Jiang, Y., He, W., et al., "Adaptive parameter estimation and control design for robot manipulators with finite-time convergence," IEEE Transactions on Industrial Electronics, 2018, 65(10): 8112-8123.
7. Chen, D., Zhang, Y., and Li, Shuai., "Tracking control of robot manipulators with unknown models: a Jacobian-matrix-adaption method," IEEE Transactions on Industrial Informatics, 2018, 14(7): 3044-3053.
8. Ioannou, P., and Sun, J. Robust Adaptive Control. Prentice-Hall, 1996.
9. Astolfi, A., and Ortega R., "Immersion and invariance: A new tool for stabilization and adaptive control of nonlinear systems," IEEE Transactions on Automatic Control, 2003, 48(4): 590-606.
10. Ortega, R., Hsu, L., and Astolfi, A., "Immersion and invariance adaptive control of linear multivariable systems," Systems & Control Letters, 2003, 49(1): 37-47.
11. Seo, D., and Akella, M. R., "High-performance spacecraft adaptive attitude-tracking control through attracting-manifold design", Journal of Guidance Control and Dynamics, 2008, 31(4): 884-891.
12. Chowdhary, G., Muhlegg, M., and Johnson, E., "Exponential parameter and tracking error convergence guarantees for adaptive controllers without persistency of excitation," International Journal of Control, 2014, 87(8): 1583-1603.

13. Kamalapurkar, R., Reish, B., Chowdhary G., et al., “Concurrent learning for parameter estimation using dynamic state-derivative estimators,” IEEE Transactions on Automatic Control, 2017, 62(7): 3594-3601.
14. Sastry, S. and Bodson, M., Adaptive Cotrol-Stability, Convergence and Robustness. Prentice-Hall, 1989.
15. Slotine, J., and Li, W., Applied Nonlinear Control. Prentice-Hall, 1991.
16. Pagilla, P. R., and Tomizuka, M., “An adaptive output feedback controller for robot arms: stability and experiments,” Automatica, 2001, 37: 983-995.
17. Panteley, E., Ortega, R. and Moya, P., “Overcoming the detectability obstacle in certainty equivalence adaptive control,” Automatica, 2002, 38(7): 1125-1132.
18. Seo, D., and Akella, M. R., “Non-certainty equivalence adaptive control for robot manipulator systems”, Systems & Control Letters, 2009, 58: 304-308.