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AIRBUS GENERIC CONTROL LAWS

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Abstract When designing the control laws for a new program, the aircraft manufacturer has to face to numerous constraints:

New hardware, new system architecture, new structural specificities, new functions, new certification basis.

More, automatic control theory improves continuously.

Then the engineers develop, each time, a new set of control laws. It needs some time, in development phase, but also during the flight test phase.

To save time, reduce cost and to minimize risk of such new developments, a new (once again, but breaking the rules) concept has been designed: G*, the Generic Control laws.

This concept proposes a new way of designing and computing the control laws, absolutely generic. The same set of laws is applied to all AIRBUS family members, covering almost all the functions, from take-off to landing, in manual and automatic modes, including all the flight domain protections.

The interests are numerous:

- drastic reduction of development lead time, before and during flight test campaign
- strong family behavior
- best performance and safety level
- certification easiness
- technical synergy with other disciplines

GSTAR is already partially applied on A350. It will be used on any new model, and is also introduced on legacy programs depending on opportunities.

This paper explains the equation cascade method to compute in real time the linear part of the control laws.

1- INTRODUCTION

To design control laws, the classical approach consists in creating a mathematical model of the aircraft on a flight domain grid. The control laws are designed on each point (state and vector placement, $H\infty$, predictive control, …), and the resulting gains (feedback, precommands, filters…) are tabulated along the flight domain, inside the computer.

The generic approach needs only a model of a subset of the aerodynamic characteristics of the aircraft: tabulation along the flight domain, or equations, depending on the parameter.

Then the complete set of control laws is a cascade of equations, computing in real time all the terms of these functions. The cascade is big and complex, but once validated and certified, it doesn't need any future (further) modification.

Almost all the functions from take-off to landing are considered, interacting with the others. Only some specific features are not solved by G* equations, such as vibratory comfort augmentation functions, or Load alleviation functions, that have to deal with very specific structural characteristics or sizing constraints, and then are not necessarily the same on each family member.

The different control laws are available as some bricks, that can be activated or not on any member. These bricks can also evolve in the future, without jeopardizing the global efficiency, because the other bricks adapt automatically – if needed- to the modification.

The lonely features that are coded specifically to the family member are the aerodynamic characteristics, the actuators characteristics, the structural filtering acting on the sensors, and the time delay of the computing chain, that can be different when the hardware architecture is not the same.

G* targets some objectives, defined to control precisely the aircraft, improve safety and optimize the structural sizing. Due to the high level of accuracy of G* concept ,the objectives or the equations don't need any tuning in flight tests, then the only tests that subsist are the ones to tune precisely the pilot feeling, linked mainly to the structural aircraft behavior.

G* is designed to cope with this particular objective, then some specific tuning keys, tunable in real time during flight tests, impact directly the cascade of equations.

2- AIRCRAFT MODEL

The necessary model is composed of 3 elements:

- Flight mechanic equations representing the aircraft
- Actuator model
- Hardware filtering chain and calculation or transmission delays

Classically, the flight mechanic linear equations are written on longitudinal axis, or on lateral axis, around an equilibrium:

On longitudinal axis: **(system 1 equations)**

$$q = pitchrate - r\sin(\varphi)$$
$$Nz = vertical\ load\ factor - \frac{\cos(\theta)}{\cos(\varphi)}$$

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}p_\alpha & 1\\ m_\alpha & m_q\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix} + \begin{bmatrix}0\\ m_{\delta q}\end{bmatrix}\delta q$$

$$Nz = \frac{V}{g}\frac{\pi}{180}(q - \dot{\alpha})$$

And on lateral axis: **(system 2 equations)**

$$r = yawrate - \frac{g}{V}\sin(\varphi)$$

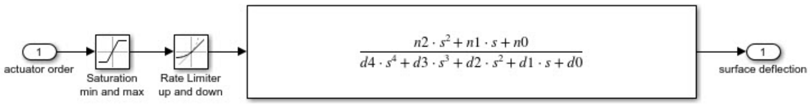
$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\\ \dot{p}\\ \dot{\varphi}\end{bmatrix} = \begin{bmatrix}\frac{Y_\beta}{V} & -\cos\alpha & \sin\alpha & g\frac{\cos\theta}{V}\\ n_\beta & n_r & n_p & 0\\ l_\beta & l_r & l_p & 0\\ 0 & \tan\theta & 1 & 0\end{bmatrix}\begin{bmatrix}\beta\\ r\\ p\\ \varphi\end{bmatrix} + \begin{bmatrix}\frac{Y_{\delta l}}{V} & \frac{Y_{\delta n}}{V}\\ n_{\delta l} & n_{\delta n}\\ l_{\delta l} & l_{\delta n}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta l\\ \delta n\end{bmatrix}$$

Where matrix coefficients are obtained from aerodynamic coefficients, dynamic pressure, and aircraft weight, inertias and geometric characteristics.

For instance, $p_\alpha = \frac{SP_{dyn}}{mV}Cz_\alpha$, or $m_{\delta q} = \frac{SLP_{dyn}}{I_{yy}}Cm_{\delta q}$

The actuator model can be very complex when representing all the non-linear effects that affect its behavior. To design a control law, only some of them are taken into account, because the most extreme ones (stall due to load, free plays, dead band zone…) are managed via some specific functions or protections, that are not part of the control laws we are dealing with in this paper.

Thus, the model useful for control laws design is composed with a limiter, a rate limiter and a linear transfer function. The linear TF is usually a 4th order filter, to cope with the frequency characteristics of the actuator in a [0:10Hz] range.



The limiter and RL are used to manage integral terms saturation. For GSTAR equations as presented below, only the linear Transfer Function is used.

The complete calculation chain considers sensors – that can be IRS, IMU, accelerometers,…-, the information transmission to the control laws host, and the transmission of control laws orders to actuation servo-loop. This numeric chain can be represented by the sampling periods of each component, the nominal duration and jitters of the transmission channels, and the asynchronisms between the components. Finally, this complete chain is represented with a global nominal delay, and a global jitter margin.

The information chain from sensors to the laws considers also the filtering of the informations: internal filter inside sensors, or specific filtering added into the control laws computer, for different purposes: anti-aliasing filter, noise suppression, structural damping filter,… On a modern aircraft, whose primary structure is optimized , the natural flexible modes are low frequency with a low damping ratio, then the structural filtering chain is “complex”, to deal with the vibratory information, whatever for passive information attenuation or active damping increase. Finally, the complete filtering chain, without actuator Transfer function, reaches easily a 8th or 10th order, that implies a phase lag fast evolving with frequency, in the range of aircraft rigid modes.

3- GSTAR PRINCIPLE

When considering a simple rigid aircraft motion, the aerodynamic equations can be linearized in a small state space model: 2X2 in longitudinal, and 4X4 in lateral. Then the equations to calculate a control law are quite simple, and can be solved manually or coded into the computer. But, when considering the numerous delays and filters of the system, its size grows drastically, and the equations become very complex.
Moreover the resulting control law is far from the one obtained on the “simple” aircraft.

The classical solving is done numerically with a mathematical software, for each tabulated point of the flight domain, and the resulting control law gains are tabulated into the computer.

The GSTAR methodology proposes to use a fixed order equivalent filter to represent all the delays and filters of the chain, then to consider this equivalent filter in the aircraft model. Thus the size of the model keeps manageable, and the equations of the control law can be written. The solving of this system is done in cascade, giving first the closed loop placement of the equivalent filter, before giving all the gains of the control law.
Once obtained, this equations system can be coded into the computer, and calculate the control law gains in real time, based on the tabulated aerodynamic coefficients and the objectives of the law.

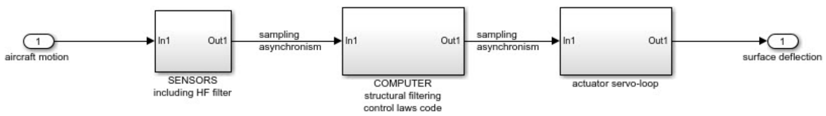
The equivalent filter used by AIRBUS has a frozen structure, that allows to adapt in real time to hardware modifications, and that suits to every family member.

The second step is to design the control laws.
Aircraft control laws can be seen in three layers:
The inner layer shapes the aircraft behavior. It is composed with two families of elements: some non-linear controllers whose target is to bring back the aircraft on the linear model, and some linear controllers that shape the dynamic behaviour of the system. This inner control loop makes up the “manual” control laws.
The second layer is the guidance loop, and is based on the augmented aircraft, shaped by the inner loop. This layer is the core of the Auto-pilot functions.
The upper layer is the navigation loop, based on the augmented aircraft shaped by the second layer. This third level represents the auto-pilot navigation loop.
Flight domain protections can be seen as second layer functions, taking control on human pilot or AP orders.

4- TEACHING EXAMPLE

Let us consider **system 1 equations**.
This system represents the longitudinal aircraft, once the possible non-linear controllers have managed the non-linear -or not considered- aerodynamic coefficients.

The inner loop controller will rely on Nz accelerometer and a pitch rate gyrometer. These 2 sensors are high frequency filtered. The information provided by the sensors is sent to a computer, hosting the control law. Some structural filtering is added on these sensor informations. The computer will send its order to elevator actuator computer ,which hosts the elevator servo-loop .



The global delay of the chain, called Tg, can be represented by a 2nd order pade filter, $TFdel(s) = pade(Tg,2) = \frac{\frac{Tg^2}{12}s^2 - \frac{Tg}{2}s + 1}{\frac{Tg^2}{12}s^2 + \frac{Tg}{2}s + 1}$

The global filter applied to the chain, including actuator transfer function, is called Tfilt(s).
Then the resulting filter to consider is
$TF_{global}(s) = pade(Tg,2) * Tfilt(s)$

We can design a 4th order equivalent filter, called TFequi(s), that has the following structure:

$TF_{equi}(s) = B(s).pade(T_{eq},2)$ where $B(s) = \frac{1}{\frac{s^2}{{\omega_0}^2} + 2\frac{\xi}{\omega_0}s + 1}$ and T_{eq} is an equivalent delay.

To design it we establish a theorem:

> Given a frequency f1, and 2 values g1 and g2.
> We note $\omega 1 = 2\pi f1,$ and $\omega 2 = 2\pi \frac{f1}{2}$
> A 2nd order butterworth filter $B(s) = \frac{1}{\frac{s^2}{\omega_0^2} + 2\xi\frac{s}{\omega_0} + 1}$
> verifies $g1 = abs\big(B(\omega 1)\big),\ and\ g2 = abs\big(B(\omega 2)\big)$
> If and only if $\frac{1}{g_1^2} - \frac{4}{g_2^2} = 3\frac{X^2}{4} - 3$ has a real solution X^2.
> If yes, $X = \frac{\omega_1^2}{\omega_0^2}$,
> And $\xi^2 = \frac{\frac{1}{g_1^2} - (1-X)^2}{4X}$

We then can fix B(s) to equal $abs(TF_{global}(s))$ on ω_1 and ω_2, and fix T_{eq} to cope with $\arg(TF_{global})$ $below$ ω_2

We can write from system 1 equations

$$\left(s^2-(p_\alpha+m_q)s+m_q p_\alpha-m_\alpha\right)Nz=-\frac{V}{g}\frac{\pi}{180}p_\alpha m_{\delta q}\delta q$$

Let us define the inner control law:

$$\delta q=pade(T,2)*B(s)*(K_D Nz_c+K_{Nz}Nz+K_q q+\frac{K_i}{s}(Nzc-Nz))$$

Then we have:

$$\left(s^2-(p_\alpha+m_q)s+m_q p_\alpha-m_\alpha\right)Nz =-\frac{V}{g}\frac{\pi}{180}p_\alpha m_{\delta q}*pade(T,2)*B(s)*(K_D Nz_c+K_{Nz}Nz +K_q q+\frac{K_i}{s}(Nzc-Nz))$$

If we consider the following general system of 7th order whose controlled variable is u:

$$(\theta_2 s^2+\theta_1 s+\theta_0)(as^2+bs+d)(K_2 s^2+K_1 s+K_0)u+R.u =(\theta_2 s^2-\theta_1 s+\theta_0)(K_{uc}u_c+K_u u+K_{udot}su+\frac{K_{ui}}{s}(u_c-u))$$

Then the closed loop system is:

(v)

$$(T_7 s^7+T_6 s^6+T_5 s^5+(T_4-\theta_2 K_{udot})s^4+(T_3-\theta_2 K_u+\theta_1 K_{udot})s^3 +(T_2+\theta_2 K_{ui}+\theta_1 K_u-\theta_0 K_{udot})s^2+(T_1-\theta_1 K_{ui}-\theta_0 K_u)s +\theta_0 K_{ui})u=(\theta_2 s^2-\theta_1 s+\theta_0)(K_{uc}s+K_{ui})u_c$$

Whose terms can be calculated in a specific function $coef\_equation\_BF$

$$[T_1,T_2,T_3,T_4,T_5,T_6,T_7,\theta_0,\theta_1,\theta_2]=coef\_equationBF(a,b,d,T,K_0,K_1,K_2,R)$$

Content:

$$\theta_2=\frac{T^2}{12}$$

$$\theta_1=\frac{T}{2}$$

$$\theta_0=1.0$$

$$T_7=a\theta_2K_2$$

$$T_6=a\theta_2K_1+(b\theta_2+a\theta_1)K_2$$

$$T_5=a\theta_2K_0+(b\theta_2+a\theta_1)K_1+(d\theta_2+b\theta_1+a\theta_0)K_2$$

$$T_4=(b\theta_2+a\theta_1)K_0+(d\theta_2+b\theta_1+a\theta_0)K_1+(d\theta_1+b\theta_0)K_2$$

$$T_3=(d\theta_2+b\theta_1+a\theta_0)K_0+(d\theta_1+b\theta_0)K_1+d\theta_0K_2$$

$$T_2=(d\theta_1+b\theta_0)K_0+d\theta_0K_1$$

$$T_1=d\theta_0K_0+R$$

The system has 7 dynamics. In closed loop, 3 of these dynamics are placed by the control law on the objectives,

$$(s^2 + 2\varepsilon\omega s + \omega^2)\left(s + \frac{1}{\tau}\right) = \mu_3 s^3 + \mu_2 s^2 + \mu_1 s + \mu_0$$

The 4 last ones correspond to the equivalent filter pole placement.

$$(x_4 s^4 + x_3 s^3 + x_2 s^2 + x_1 s + x_0)$$

So, the left part of (v) can be identified to:

$$(x_4 s^4 + x_3 s^3 + x_2 s^2 + x_1 s + x_0).\left(\mu_3 s^3 + \mu_2 s^2 + \mu_1 s + \mu_0\right).u$$

This identification can be written via an equations cascade, written in a specific function

$$[K_u, K_{ui}, K_{udot}] = equation\_loi\_ordre7(\mu_0, \mu_1, \mu_2, \mu_3, T_1, T_2, T_3, T_4, T_5, T_6, T_7, \theta_0, \theta_1, \theta_2)$$

Content:

$$x_4 = \frac{T_7}{\mu_3}$$

$$x_3 = \frac{T_6 - x_4\mu_2}{\mu_3}$$

$$x_2 = \frac{T_5 - (x_4\mu_1 + x_3\mu_2)}{\mu_3}$$

$$C_1 = \frac{T_4 - (x_4\mu_0 + x_3\mu_1 + x_2\mu_2)}{\mu_3}$$

$$C_2 = \frac{T_3 - x_3\mu_0 - x_2\mu_1}{\mu_3}$$

$$C_3 = C_2 - \frac{\mu_2 C_1}{\mu_3}$$

$$K_{11} = \theta_1 + \frac{\mu_2\theta_2}{\mu_3}$$

$$K_{12} = \frac{\mu_1\theta_2}{\mu_3} - \theta_0 - \frac{\mu_2(\theta_1 + \frac{\mu_2}{\mu_3}\theta_2)}{\mu_3}$$

$$K_{13} = \theta_2$$

$$D_{11} = x_2\mu_0 + \mu_1 C_1 + \mu_2 C_3 - T_2$$

$$K_{21} = \theta_0 - \frac{\mu_1\theta_2}{\mu_3}$$

$$K_{22} = \frac{\mu_1\left(\theta_1 + \frac{\mu_2\theta_2}{\mu_3}\right) - \mu_0\theta_2}{\mu_3}$$

$$K_{23} = \theta_1$$

$$D_{22} = T_1 - \mu_0 C_1 - \mu_1 C_3$$

$$K_{31} = \frac{-\mu_0\theta_2}{\mu_3}$$

$$K_{32} = \frac{\mu_0(\theta_1 + \frac{\mu_2\theta_2}{\mu_3})}{\mu_3}$$

$$K_{33} = -\theta_0$$

$$D_{33} = -\mu_0 C_3$$

Thus we obtain a 3 equations system with 3 unknowns K_u, K_{udot}, K_{ui}

$$\begin{aligned} K_{11}K_u + K_{12}K_{udot} + K_{13}K_{ui} &= D_{11} \\ K_{21}K_u + K_{22}K_{udot} + K_{23}K_{ui} &= D_{22} \\ K_{31}K_u + K_{32}K_{udot} + K_{33}K_{ui} &= D_{33} \end{aligned}$$

Whose determinants are:

$$D = K_{11}\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix} - K_{21}\begin{vmatrix} K_{12} & K_{13} \\ K_{32} & K_{33} \end{vmatrix} + K_{31}\begin{vmatrix} K_{12} & K_{13} \\ K_{22} & K_{23} \end{vmatrix}$$

$$Rq = D_{11}\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix} - D_{22}\begin{vmatrix} K_{12} & K_{13} \\ K_{32} & K_{33} \end{vmatrix} + D_{33}\begin{vmatrix} K_{12} & K_{13} \\ K_{22} & K_{23} \end{vmatrix}$$

$$Rz = K_{11}\begin{vmatrix} D_{22} & K_{23} \\ D_{33} & K_{33} \end{vmatrix} - K_{21}\begin{vmatrix} D_{11} & K_{13} \\ D_{33} & K_{33} \end{vmatrix} + K_{31}\begin{vmatrix} D_{11} & K_{13} \\ D_{22} & K_{23} \end{vmatrix}$$

$$Ri = K_{11}\begin{vmatrix} K_{22} & D_{22} \\ K_{32} & D_{33} \end{vmatrix} - K_{21}\begin{vmatrix} K_{12} & D_{11} \\ K_{32} & D_{33} \end{vmatrix} + K_{31}\begin{vmatrix} K_{12} & D_{11} \\ K_{22} & D_{22} \end{vmatrix}$$

D cannot be zero if the aircraft is controllable.
And we finally obtain:

$$K_u = \frac{R_q}{D}$$

$$K_{udot} = \frac{R_z}{D}$$

$$K_{ui} = \frac{R_i}{D}$$

and:

$$x_1 = C_1 - \frac{\theta_2 K_{udot}}{\mu_3}$$

$$x_0 = C_3 - \frac{\theta_2 K_u}{\mu_3} + \frac{\left(\theta_1 + \frac{\mu_2\theta_2}{\mu_3}\right)K_{udot}}{\mu_3}$$

This equation cascade is easily applied to our system to compute the gains of the inner loop.

Considering $pade(T,2) = \frac{\frac{T^2}{12}s^2 - \frac{T}{2}s + 1}{\frac{T^2}{12}s^2 + \frac{T}{2}s + 1}$

And $B(s) = \frac{1}{as^2 + bs + d}$

we note: $A = -\frac{V}{g}\frac{\pi}{180}p_\alpha m_{\delta q}$, $K_0 = m_q p_\alpha - m_\alpha$, $K_1 = -(p_\alpha + m_q), K_2 = 1.0$

Then we have:
$[T_1, T_2, T_3, T_4, T_5, T_6, T_7, \theta_0, \theta_1, \theta_2] = coef\_equationBF(a, b, d, T, K_0, K_1, K_2, 0.0)$
We note now the objectives:

$$\mu_3 = 1.0$$

$$\mu_2 = 2\xi\omega + \frac{1}{\tau}$$

$$\mu_1 = \omega^2 + \frac{2\xi\omega}{\tau}$$

$$\mu_0 = \frac{\omega^2}{\tau}$$

We obtain:

$$[K_u, K_{ui}, K_{udot}] = equation\_loi\_ordre7(\mu_0, \mu_1, \mu_2, \mu_3, T_1, T_2, T_3, T_4, T_5, T_6, T_7, \theta_0, \theta_1, \theta_2)$$

and

$$K_{Nz} = \frac{K_u + p_\alpha K_{udot}}{A}$$

$$K_q = \frac{K_{udot}}{m_{\delta q}}$$

$$K_i = \frac{K_{ui}}{A}$$

The precommand term is written from (v) to compensate the real mode,

$$K_D = \tau K_i$$

The augmented aircraft transfer function becomes

$$\left(\frac{x_4}{x_0}s^4 + \frac{x_3}{x_0}s^3 + \frac{x_2}{x_0}s^2 + \frac{x_1}{x_0}s + 1\right).\left(\frac{s^2}{\omega^2} + \frac{2\varepsilon s}{\omega} + 1\right).N_z = (\frac{T^2}{12}s^2 - \frac{T}{2}s + 1)Nzc$$

Thus we can have a second layer controller to define Nzc, and use the same equations cascade method to compute in real time the components of this second layer.

For example, $Nzc = K_{vzg}(K_{zg}(Zc - Z) - DV_z) + K_{nz}Nz + K_{dnz}\dot{N}z$ allows to define a “altitude holder” control law.

5- GSTAR ADVANTAGES

This method brings many advantages for our control laws design.

Technically:

- the equations cascades define precisely where all the aircraft dynamics are placed, and allow to modify the objectives of the law to cope with some stability margins constraints.
- The efficiency of the law is guaranteed, because every “perturbating” parameter is taken into account. The equivalent delay can be automatically adapted when the hardware path is varying. Thus, objectives of the law are the only tuning parameters to be adjusted during flight test campaign. It simplifies drastically the way to perform the flight tests.
- A specific aerodynamic model can be developed and identified in flight test for the control laws design need. It is simpler and faster than building a complete aerodynamic model.

Strategically:

- All family members have the same control laws, that guarantee the homogeneity, and the same level of safety.
- No regression risk when developing a new model.
- An innovative function can easily be retrofitted on any member.