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Closed–Loop MRAC Augmented LQR with Integral Action for Quadrotor UAV under the Uncertainties

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ABSTRACT

In this paper, we propose a closed-loop model reference adaptive controller augmented Linear Quadratic Regulator to compensate for uncertainties from the payload and motor faults. When the quadrotor UAV conveys objects or transports people, parameters such as mass, moment of inertia, center of gravity, and aerodynamic coefficient significantly affect control performances. And, the actuator degradation is called loss of effectiveness, which can cause the quadrotor to crash during the mission. Firstly, we describe the mathematical model of quadrotor with the payload and motor faults. Secondly, the closed-loop model reference adaptive controller augmented Linear Quadratic Regulator enhances altitude and attitude tracking performances. Third, the reference model is designed using the Linear Quadratic Regulator (LQR) approach, and an observer-like term is added to eliminate the undesirable oscillations in the transient response. Fourth, the uniform ultimate boundedness of the proposed controller is proved by using the Lyapunov candidate function. Lastly, we compare the LQR with integral action and the proposed control scheme under the parametric uncertainties to analyze the overall performances of the proposed controller. The simulation results show that the proposed control scheme presents better performance when the payload is attached to the quadrotor than the LQR method.

Keywords: Nonlinear Control; Model Reference Adaptive Control(MRAC); Closed-loop Reference Model; Payload; Center of Gravity(CoG); Quadrotor;

Nomenclature

$\{I\}$	=	Inertial frame fixed to earth
$\{B\}$	=	Body-fixed frame fixed to quadrotor of center of gravity
x,y,z	=	Position measured in the inertial frame
ψ,θ,ϕ	=	Euler angles (yaw, pitch, roll) measured from the inertial frame
u,v,w	=	Linear velocity measured in the body-fixed frame
p,q,r	=	Angular velocity measured in the body-fixed frame
$e_{z,I},e_{phi,I},e_{theta,I},e_{psi,I}$	=	Integrated output tracking error
m	=	Mass
I	=	Inertia tensor
x_G,y_G,z_G	=	Center of gravity in the body-fixed frame
$K_{F,u},K_{F,v},K_{F,w}$	=	Aerodynamic force coefficient
$K_{M,p},K_{M,p},K_{M,p}$	=	Aerodynamic moment coefficient
F_g,M_g	=	Force and Moment from gravity exerted in the body-fixed frame
F_a,M_a	=	Force and Moment from aerodynamic exerted in the body-fixed frame
F_t,M_t	=	Force and Moment from thrust exerted in the body-fixed frame
Fx,Fz,Fz	=	Force exerted in the body-fixed frame
M_x,M_y,M_z	=	Moment exerted in the body-fixed frame
$A_p \in R^{n\times n}$	=	A matrix in the plant model
$B_p \in R^{n\times m}$	=	B matrix in the plant model
$A \in R^{n\times n}$	=	A matrix in the extended plant model
$B \in R^{n\times m}$	=	B matrix in the extended plant model
$A_{ref} \in R^{n\times n}$	=	A matrix in the reference model
$B_{ref} \in R^{n\times m}$	=	B matrix in the reference model
$\Lambda \in R^{m\times m}$	=	Unknown constant diagonal matrix
$L_v \in R^{n\times n}$	=	Error feedback gain
$P_v \in R^{n\times n}$	=	Unique solution of the Algebraic Ricatti Equation (ARE)
$R_v \in R^{n\times n}$	=	ARE weight matrix
$Q_v \in R^{n\times n}$	=	ARE weight matrix
K_{LQR}	=	Optimal LQR gain
R	=	LQR weight matrix
$Q \in R^{n\times n}$	=	LQR weight matrix
$e(t)$	=	Error vector
V	=	Lyapunov candidate function
$u(t)$	=	Control input vector
$f(x): R^n \to R^m$	=	Unknown nonlinear function matrix
$\Theta \in R^{N\times m}$	=	Constant matrix of the unknown coefficients
$\hat{\Theta}$	=	Estimated constant matrix of the unknown coefficients
Φ	=	Basis function
$\bar{\Theta}$	=	Modified constant matrix of the unknown coefficients
$\bar{\Phi}$	=	Modified basis function
K_x	=	Ideal feedback gain
$\hat{K}_x$	=	Estimated feedback gain
$\Gamma_{\bar{\Theta}}$	=	Adaptive update gain

1 Introduction

Unmanned Aerial Vehicle (UAV) has been widely used for military and civilian usage in recent times. The quadrotor type UAV is used for various tasks such as spraying pesticides [1] and quickly reconnaissance [2] because the UAV has a simple structure, can vertically take off and land on a narrow space, and hover at a certain altitude during a mission. In order to perform all of these missions safely and reliably, designing a stable controller for quadrotor is essential. One of the most used controller approaches is the Proportional-Integral-Derivative (PID) because this method structure is simple and easy to implement the quadrotor [3]. However, the person who has no knowledge of control theory and flight experience is difficult to tune the PID gains. The Linear-Quadratic-Regulator (LQR) is an alternative solution that is convenient to find the optimal gain because we only change the state and input weighted matrix depending on the results [4]. Moreover, LQR with integral action is well known that the integral action term reduces the steady-state error [5]. However, the quadrotor has the highly nonlinear and strong coupled term associated with several parametric uncertainties to deteriorate the flight performance while performing a mission. For example, when the payload like electronic devices, battery, and parcel is attached asymmetrically to the quadrotor, the model parameters such as mass, moment of inertia, and center of gravity are changed in real-time. The payload also causes the aerodynamic effect that deteriorates tracking performance because it is considered a flat plate with thick thickness. Moreover, the actuator degradation is called loss of effectiveness, which can cause the quadrotor to crash during the mission. Therefore, we need the adaptive controller to compensate for the uncertainties in real-time.

Various adaptive control techniques have been presented in the two decades to solve these parametric uncertainties. Among proposed adaptive control approaches, Model Reference Adaptive Control (MRAC) can estimate uncertain parameters and control gains with respect to the reference model online, tracking the the desired reference model is designed considering the overshoot, settling time, and rising time. Sadeghzadeh proposed the M.I.T rule, a kind of MRAC to change feedback and feedforward gain online automatically [6]. Schreier compared with MRAC to Model Identification Adaptive Control (MIAC) on the quadrotor platform [7]. Moreover, Ibarra suggested an adaptive attitude controller when the lengths of quadrotor arms are exactly unknown [8]. Using the decentralized MRAC method, Jurado proposed an attitude controller to overcome uncertainties in the moment of inertia [9]. In Ref. [10], Palunko tried to compensate for errors due to the CoG offset due to the slung payload using the feedback linearization adaptive tracking controller. Bakshi suggested the Indirect Model Reference Adaptive Control using a neural network to reject the effect from the variation of motor and aerodynamic coefficient [11]. In Ref. [12], when the motor efficiency degenerates, Combined Model Reference Adaptive Control (CMRAC) is used to achieve better performance than the traditional MRAC. In Ref. [13], Predictor-based Model Reference Adaptive Control (PMRAC) is proposed to compensate for the loss of effectiveness by using the state-predictor and reference model.

Previous research does not consider the entire parametric uncertainty from the mass property, aerodynamic, and loss of effectiveness because the quadrotor is not considered for transportation. When transporting the parcel, the quadrotor has uncertainties from center of gravity, aerodynamic, and loss of effectiveness that can not be ignored because they can affect a significant impact on control performance. Therefore, the closed-loop model reference adaptive control augmented LQR with integral action is proposed to compensate for the parametric uncertainties and guarantee stable performance. And, The closed-loop reference model also is used to enhance the robustness and transient response in MRAC architecture.

This paper is organized as follows. Section 2 represents the dynamic modelling under the parametric uncertainty such as mass and moment of inertia, center of gravity, aerodynamic effect, and loss of effectiveness. Section 3 gives the suggesting closed-loop model reference adaptive control augmented LQR with integral action. Section 4 provides simulation results, which compares LQR with integral action and the proposed controller. Section 5 summarizes this study and presents future research directions.

2 Quadrotor UAV Modeling with Parametric Uncertainties

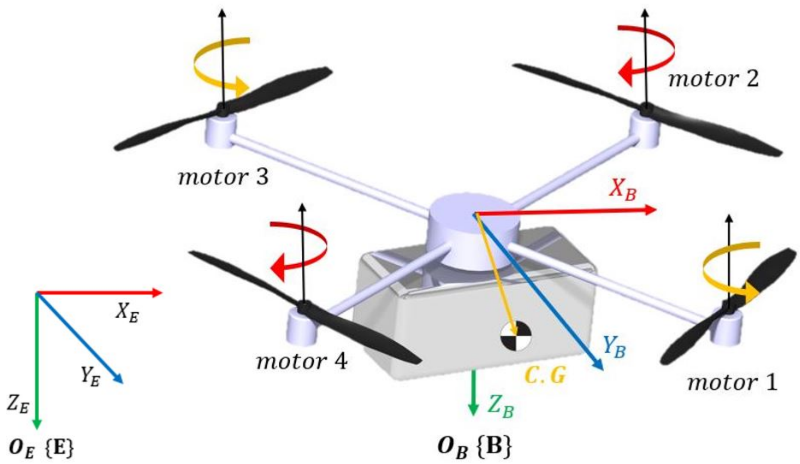

Fig. 1 Quadrotor with payload coordinate frame

In order to derive the dynamic model of quadrotor, we need to define the inertial coordinate system and body-fixed coordinate. The inertial coordinate frame,$\{I\}$, have the z-axis pointing the center of the earth where gravity is applied, and the x-axis pointing the north, y-axis facing east, as shown in Fig.1. In addition, the body-fixed coordinate frame,$\{B\}$, is located at center of quadrotor. Let, $[x,y,z]^T$ represent position measured in frame $\{I\}$, $[\phi,\theta,\psi]^T$ represent Euler angle of $\{B\}$ measured from $\{I\}$. $[u,v,w]^T$ represent velocity vector, and $[p,q,r]^T$ represent angular velocity both measured from the frame $\{B\}$.

2.1 Quadrotor UAV Kinematics

In order to obtain the quadrotor kinematics, we have to define a rotation matrix that transforms any vector expressed in the inertial frame to the body-fixed frame. (For the simplicity, the following notations is used, $\sin(x) \to s_x \quad \cos(x) \to c_x \quad \tan(x) \to t_x$.)

$$R_I^B = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ -c_\phi s_\psi + s_\phi s_\theta c_\psi & c_\phi c_\psi + s_\phi s_\theta s_\psi & s_\phi c_\theta \\ s_\phi s_\psi + c_\phi s_\theta c_\psi & -s_\phi c_\psi + c_\phi s_\theta s_\psi & c_\phi c_\theta \end{bmatrix} \tag{1}$$

The velocity in the inertial frame and velocity in body-fixed frame have the following relation.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ -c_\phi s_\psi + s_\phi s_\theta c_\psi & c_\phi c_\psi + s_\phi s_\theta s_\psi & s_\phi c_\theta \\ s_\phi s_\psi + c_\phi s_\theta c_\psi & -s_\phi c_\psi + c_\phi s_\theta s_\psi & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{2}$$

The time derivative of Euler angles in the inertial frame and the angular velocity in the body-fixed frame have the following relation.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{s}_\phi t_\theta & \mathrm{c}_\phi t_\theta \\ 0 & \mathrm{c}_\phi & -\mathrm{s}_\phi \\ 0 & \frac{\mathrm{s}_\phi}{\mathrm{c}_\theta} & \frac{\mathrm{c}_\phi}{\mathrm{c}_\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

2.2 Quadrotor UAV Dynamics

6 DOF dynamic quadrotor UAV model is derived using Newton's second law ($F = ma$). We will derive the model considering the offset of the center of gravity because the payload can cause the variation of center of gravity. Let, the actual center of gravity to the origin of body-fixed frame is expressed as follows: $r_G = \{x_G, y_G, z_G\}$. when the payload is mounted to the quadrotor, the non-diagonal moment of inertia is not zero. However, we assume that the non-diagonal term will be ignored because the diagonal moment of inertia is dominant. The inertia tensor is as follows:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{4}$$

The effects on gravity due to the centre of gravity are induced as:

$$F_g = \begin{bmatrix} mg\sin\theta \\ mg\cos\theta\sin\phi \\ mg\cos\theta\cos\phi \end{bmatrix}, M_g = \begin{bmatrix} -z_G mg\cos\theta\sin\phi + y_G mg\cos\theta\cos\phi \\ z_G mg\sin\theta - x_G mg\cos\theta\sin\phi \\ -y_G mg\sin\theta + x_G mg\cos\theta\sin\phi \end{bmatrix} \tag{5}$$

And, the payload causes the aerodynamic force and moment because this shape is similar to the flat plate, which has thick thickness. The aerodynamic force and moment of quadrotor are assumed as the lumped system.

$$F_a = \begin{bmatrix} -K_{F,u}\|u\|u \\ -K_{F,v}\|v\|v \\ -K_{F,w}\|w\|w \end{bmatrix}, M_a = \begin{bmatrix} -K_{F,p}\|p\|p \\ -K_{F,q}\|q\|q \\ -K_{F,r}\|r\|r \end{bmatrix} \tag{6}$$

The thrust force and moment are obtained through an allocation matrix, which converts force and moment into each motor thrust command. The loss of effectiveness in each motor is introduced as $0 \leq \gamma_i \leq 1$. And, the motor dynamics is assumed to be the first-order transfer function.

$$F_t = \begin{bmatrix} 0 \\ 0 \\ -(\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3 + \gamma_4 T_4) \end{bmatrix}, M_t = \begin{bmatrix} dy(-\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3 - \gamma_4 T_4) \\ dx(\gamma_1 T_1 + \gamma_2 T_2 - \gamma_3 T_3 - \gamma_4 T_4) \\ c_T(\gamma_1 T_1 - \gamma_2 T_2 + \gamma_3 T_3 - \gamma_4 T_4) \end{bmatrix} \tag{7}$$

$$G_{motor}(s) = \frac{\tau}{\tau s + 1} \tag{8}$$

where τ is the motor time constant.

The equations about linear velocity and angular velocity are based on body fixed frames. Unlike the traditional quadrotor dynamics model, dynamics is associated with the parametric uncertainty such as: mass, moment of inertia, center of gravity, aerodynamic, and loss of effectiveness. The force $F = F_g + F_a + F_t$ is exerted at frame B. The moment $M = M_g + M_a + M_t$ is exerted at frame B.

$$
\begin{aligned}
\dot{u} &= \frac{F_{x,t}}{m} + vr - wq + g\sin\theta + x_G(q^2+r^2) - y_G(pq-\dot{r}) - z_G(pr+\dot{q}) - \frac{1}{m}K_{F,u}\|u\|u \\
\dot{v} &= \frac{F_{y,t}}{m} + wp - ur + g\cos\theta\sin\phi - x_G(qp+\dot{r}) + y_G(p^2+r^2) - z_G(qr-\dot{p}) - \frac{1}{m}K_{F,v}\|v\|v \\
\dot{w} &= \frac{F_{z,t}}{m} + uq - vp + g\cos\theta\cos\phi - x_G(rp-\dot{q}) - y_G(rq-\dot{p}) + z_G(q^2+p^2) - \frac{1}{m}K_{F,w}\|w\|w \\
\dot{p} &= \frac{M_{x,t}}{I_{xx}} + \frac{I_{yy}-I_{zz}}{I_{xx}}qr - \frac{m}{I_{xx}}[y_G(\dot{w}-uq+vp-g\cos\theta\cos\phi) - z_G(\dot{v}-wp+ur-g\cos\theta\sin\phi)] - \frac{1}{I_{xx}}K_{F,p}\|p\|p \\
\dot{q} &= \frac{M_{y,t}}{I_{yy}} + \frac{I_{zz}-I_{xx}}{I_{yy}}rp - \frac{m}{I_{yy}}[z_G(\dot{u}-vr+wq+g\sin\theta) - x_G(\dot{w}-uq+vp-g\cos\theta\sin\phi)] - \frac{1}{I_{yy}}K_{F,q}\|q\|q \\
\dot{p} &= \frac{M_{z,t}}{I_{zz}} + \frac{I_{xx}-I_{yy}}{I_{zz}}pq - \frac{m}{I_{zz}}[x_G(\dot{v}-wp+ur-g\cos\theta\sin\phi) - y_G(\dot{u}-vr+wq+g\sin\theta)] - \frac{1}{I_{zz}}K_{F,r}\|r\|r
\end{aligned}
\tag{9}
$$

To control the z-axis velocity, the equation based on inertia frame is rewritten as:

$$
\begin{aligned}
\ddot{x} &= -\frac{F_z}{m}\left(s_\phi s_\psi + c_\phi c_\psi s_\theta\right) \\
\ddot{y} &= -\frac{F_z}{m}\left(c_\phi s_\psi s_\theta - s_\phi c_\psi\right) \\
\ddot{z} &= g - \frac{F_z}{m}\left(c_\phi c_\theta\right)
\end{aligned}
\tag{10}
$$

3 Closed-Loop Model Reference Adaptive Control Augmented LQR with Integral Action

The proposed controller is divided into baseline controller and adaptive controller. The baseline controller is designed using LQR with integral action to reduce the steady-state error. The overall structure of the controller is shown below. Our controller consists of an altitude controller and an attitude controller. The process of designing each controller is discussed in the following chapters.

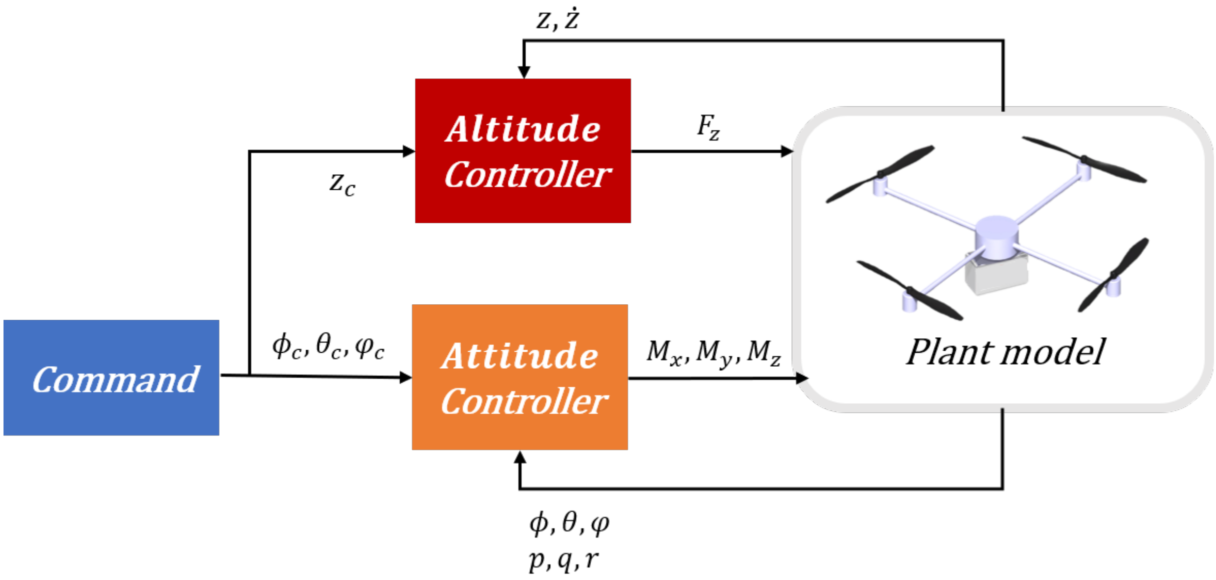

Fig. 2 Structure of closed-loop model reference adaptive control

3.1 Linear Quadratic Regulator with Integral Action

According to the Ref. [14], we can define the MIMO system including uncertainties as follows.

$$\begin{aligned}\dot{x}_p &= A_p x + B_p \Lambda(u + f(x_p))\\ y_p &= C_p x\\ f(x_p) &= \sum_{i=1}^{N} \theta_i \phi_i(x_p) = \Theta^T \Phi(x_p)\end{aligned} \tag{11}$$

The linear quadratic regulator method is used as baseline controller that have the Integral feedback connections to enhance the performance of the baseline controller. we add the new state called integrated output tracking error.

$$\begin{aligned}\dot{e}_y(t) &= y(t) - y_c(t)\\ e_{yI} &= \int_0^t e_y(\tau)d\tau\end{aligned} \tag{12}$$

Adding (11), the new state $x = (e_{yI}^T, x_p^T)$ is defined to obtain the extended system and open-loop dynamics. And, the linearized model of quadrotor is obtained by linearizing the nonlinear z-axis equation at the hover state and gathering the nonlinear term into the unknown function.

$$\begin{aligned}&\dot{x} = Ax + B\Lambda(u + \Theta^T \Phi(x)) + B_{ref} y_c(t),\\ &y = Cx\\ &A = \begin{bmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_p \end{bmatrix}, B = \begin{bmatrix} 0_{m\times m} \\ B_p \end{bmatrix}, B_{ref} = \begin{bmatrix} -I_{m\times m} \\ 0_{n_p\times m} \end{bmatrix}, C = \begin{bmatrix} 0_{m\times m} & C_p \end{bmatrix}\end{aligned} \tag{13}$$

Using (9) and 10), this dynamic model is arranged in the same form as the (13).

$$\begin{aligned}&\ddot{z} = g - \tfrac{F_z}{m}\left(c_\phi c_\theta\right)\\ &\dot{p} = \tfrac{M_{x,t}}{I_{xx}} + \tfrac{I_{yy}-I_{zz}}{I_{xx}} qr - \tfrac{m}{I_{xx}}[y_G(\dot{w} - uq + vp - g\cos\theta\cos\phi) - z_G(\dot{v} - wp + ur - g\cos\theta\sin\phi)] - \tfrac{1}{I_{xx}} K_{F,p}\|p\|p\\ &\dot{q} = \tfrac{M_{y,t}}{I_{yy}} + \tfrac{I_{zz}-I_{xx}}{I_{yy}} rp - \tfrac{m}{I_{yy}}[z_G(\dot{u} - vr + wq + g\sin\theta) - x_G(\dot{w} - uq + vp - g\cos\theta\sin\phi)] - \tfrac{1}{I_{yy}} K_{F,q}\|q\|q\\ &\dot{p} = \tfrac{M_{z,t}}{I_{zz}} + \tfrac{I_{xx}-I_{yy}}{I_{zz}} pq - \tfrac{m}{I_{zz}}[x_G(\dot{v} - wp + ur - g\cos\theta\sin\phi) - y_G(\dot{u} - vr + wq + g\sin\theta)] - \tfrac{1}{I_{zz}} K_{F,r}\|r\|r\end{aligned} \tag{14}$$

$$x = \begin{bmatrix} e_{z,I} & e_{\phi,I} & e_{\theta,I} & e_{\varphi,I} & z & \phi & \theta & \varphi & \dot{z} & p & q & r \end{bmatrix}^T$$

$$A_p = \begin{bmatrix} 0_{4\times4} & I_{4\times4} \\ 0_{4\times4} & 0_{4\times4} \end{bmatrix}, B_p = \begin{bmatrix} 0_{4\times4} \\ B_{p,(2,1)} \end{bmatrix}, B_{p,(2,1)} = \begin{bmatrix} \frac{1}{m} & 0 & 0 & 0 \\ 0 & \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix}, C_p = \begin{bmatrix} I_{8\times8} \end{bmatrix} \tag{15}$$

$$A = \begin{bmatrix} 0_{4\times4} & C_p \\ 0_{8\times4} & A_p \end{bmatrix}, B = \begin{bmatrix} 0_{4\times4} \\ B_p \end{bmatrix}, B_{ref} = \begin{bmatrix} -I_{4\times4} \\ 0_{8\times4} \end{bmatrix}, C = \begin{bmatrix} 0_{4\times4} & 0_{4\times8} \\ 0_{8\times4} & C_p \end{bmatrix}, \Lambda = \begin{bmatrix} I_{4\times4} \end{bmatrix}$$

Define the line-in-parameter state-dependent function and unknown coefficient matrix to compensate for nonlinear terms caused by the parametric uncertainties including mass, moment of inertia, center of gravity, and loss of effectiveness.

$$\Theta = \begin{bmatrix} \Delta m & 0 & 0 & 0 \\ 0 & 0_{1\times 4} & 0_{1\times 4} & K_{M,p} \\ 0 & 0_{1\times 4} & K_{M,q} & I_{yy}-I_{xx} \\ 0 & K_{M,r} & I_{zz}-I_{xx} & -my_g \\ 0 & I_{xx}-I_{yy} & -mz_g & mz_g \\ 0 & -mx_g & mx_g & 0_{1\times 4} \\ 0 & -my_g & 0_{1\times 4} & 0_{1\times 4} \end{bmatrix}, \Phi = \begin{bmatrix} g \\ \|p\|p \\ qr \\ \dot{w}-uq+vp-g\cos\theta\cos\phi \\ \dot{v}-wp+ur-g\cos\theta\sin\phi \\ \|q\|q \\ rp \\ \dot{u}-vr+wq+g\sin\theta \\ \dot{w}-uq+vp-g\cos\theta\cos\phi \\ \|r\|r \\ pq \\ \dot{v}-wp+ur-g\cos\theta\sin\phi \\ \dot{u}-vr+wq+g\sin\theta \end{bmatrix} \tag{16}$$

where $\Delta m = m - \hat{m}$ is the difference between true mass and known mass.

The Linear Quadratic Regulator with integral action is also an optimal control method. We obtain the optimal control input to minimize the cost function. This cost function consists of quadratic state and quadratic control input which is weighted by the Q matrix and R matrix to decide the desired response. Unlike the traditional LQR method, our state X includes the integrated output tracking error to overcome the limitation of LQR which is weak when the disturbance exists.

$$J(x,u) = \int_0^t x^T Qx + u^T Ru \, dt \tag{17}$$

$$Q = \begin{bmatrix} 350\cdot I_{4\times 4} & 0 & 0 \\ 0 & 10\cdot I_{4\times 4} & 0 \\ 0 & 0 & I_{4\times 4} \end{bmatrix}, R = \begin{bmatrix} I_{4\times 4} \end{bmatrix} \tag{18}$$

Finally, the control input of baseline controller is derived as follows.

$$u = -R^{-1}B^T Px = -K_{LQR}x = -\begin{bmatrix} e_{yI}, x_p \end{bmatrix} \begin{bmatrix} K_I \\ K_p \end{bmatrix} \tag{19}$$

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{20}$$

where P matrix is the soultion of algebraic riccati equation.

$$K_{LQR} = \begin{bmatrix} 17.3 & 0 & 0 & 0 & 15.7 & 0 & 0 & 0 & 5.4 & 0 & 0 & 0 \\ 0 & 5.3 & 0 & 0 & 0 & 2.61 & 0 & 0 & 0 & 0.425 & 0 & 0 \\ 0 & 0 & 5.3 & 0 & 0 & 0 & 2.61 & 0 & 0 & 0 & 0.425 & 0 \\ 0 & 0 & 0 & 5.3 & 0 & 0 & 0 & 2.61 & 0 & 0 & 0 & 0.68 \end{bmatrix}^T \tag{21}$$

3.2 Closed-Loop Model Reference Adaptive Control

Firstly, we can choose the reference model that considers the desired response. The classical MRAC has a trade-off in which increasing the update gain results in poor tracking performance, and decreasing the update gain results in poor transient response. According to the [15], we can remove undesirable oscillations in transients response and obtain fast tracking performance by adding the observer-like term to the reference model. A stable reference model is defined as follows.

$$\begin{aligned} \dot{x}_{ref} &= A_{ref}x + B_{ref}r(t) + L_v(x - x_{ref}) \\ y_{ref} &= C_{ref}x_{ref} \end{aligned} \tag{22}$$

Reference model is obtained using the optimal LQR gain. The A_{ref} is derived through the matching condition.

$$A_{ref} = A - BK_{LQR}{}^T \tag{23}$$

To find observer-like gain, we can select $v = 0.1$. And, we solve the Alegebra Ricati Equation numerically using appropriate Q_v matrix and R_v matrix to obtain the following L_v.

$$\begin{aligned} &L_v = P_v R_v{}^{-1}, Q_v = Q_0 + \tfrac{v+1}{v} I_{n\times n}, R_v = R_0 + \tfrac{v}{v+1} I_{n\times n} \\ &P_v A_{ref}{}^T + A_{ref} P_v{}^T - P_v R_v{}^{-1} P_v + Q_v = 0 \end{aligned} \tag{24}$$

where v is the parameter that control transient response.

$$L_v = \begin{bmatrix} 10\cdot \mathrm{I}_{4\times 4} & 0_{4\times 4} & 0_{4\times 4} \\ 0_{4\times 4} & 5\cdot \mathrm{I}_{4\times 4} & 0_{4\times 4} \\ 0_{4\times 4} & 0_{4\times 4} & 1\cdot \mathrm{I}_{4\times 4} \end{bmatrix} \tag{25}$$

Secondly, using the (16), we will compensate the parametric uncertainty of quadrotor to guarantee the tracking performance. we can describe the equation (13) in the following form.

$$\begin{aligned} \dot{x} &= A_{ref}x + B\Lambda(u_{ad} + (I_{m\times m} - \Lambda^{-1})u_{bl} + \Theta^T \Phi(x_p)) + B_{ref}y_{cmd} \\ \dot{x} &= A_{ref}x + B\Lambda(u_{ad} + \bar{\Theta}^T \bar{\Phi}(u_{bl}, x_p)) + B_{ref}y_{cmd} \end{aligned} \tag{26}$$

where $\bar{\Theta}^T = \begin{bmatrix} K_u{}^T, \Theta^T \end{bmatrix}$ and $\bar{\Phi}(u_{bl}, x_p) = \begin{bmatrix} u_{bl}{}^T, \Phi^T(x_p) \end{bmatrix}$ is estimated adaptive gain.

The adaptive control term is decided to compensate the modified uncertainty term.

$$u_{ad} = -\hat{\bar{\Theta}}^T \bar{\Phi}(u_{bl}, x_p) \tag{27}$$

To obtain the error dynamics, we change the open loop dynamics using (27).

$$\dot{x} = A_{ref}x - B\Lambda(\Delta\bar{\Theta}^T \bar{\Phi}(u_{bl}, x_p)) + B_{ref}y_{cmd} \tag{28}$$

where $\Delta\bar{\Theta}^T = \hat{\bar{\Theta}}^T - \bar{\Theta}^T$.

Let,

$$e = x - x_{ref} \tag{29}$$

gives following error dynamics by subtracting (23) from (28).

$$\dot{e} = (A_{ref} - L_v)e + B\Lambda(\Delta\bar{\Theta}^T \bar{\Phi}(u_{bl}, x_p)) \tag{30}$$

To prove Lyapunov Stability, we define A_v as follows and organize algebraic ricatti equation as follows using P_v^{-1}.

$$\begin{aligned} &A_v = A_{ref} - L_v, {P_v}^{-1} = \tilde{P}_v \\ &{A_v}^T \tilde{P}_v + A_v \tilde{P} = -{R_v}^{-1} - \tilde{P}_v Q_v \tilde{P}_v < 0 \end{aligned} \tag{31}$$

To check the stability of the proposed controller, we assume the following Lyapunov function cadidate as follows.

$$V(e, \Delta\bar{\Theta}) = e^T \tilde{P}_v e + trace(\Lambda\Delta\bar{\Theta}^T \Gamma_{\bar{\Theta}}^{-1} \Delta\bar{\Theta}) \tag{32}$$

where $\Gamma_{\bar{\Theta}=500 \cdot I_{15\times 15}}$ is update gain.

using this error dynamics(30) equation, the time differential of V is described applying the vector trace identity method.

$$\dot{V}(e, \Delta\bar{\Theta}) = e^T(\tilde{P}_v A_v + {A_v}^T \tilde{P}_v)e + 2e^T \tilde{P}_v B\Lambda(\Delta\bar{\Theta}^T \bar{\Phi}(u_{bl}, x_p)) + 2trace(\Delta\bar{\Theta}^T \Gamma_{\bar{\Theta}}^{-1} \Delta\dot{\bar{\Theta}}) \tag{33}$$

According to the property (31), we derive adaptive law and prove time derivative of V is semi-negative.

$$\begin{aligned} &\dot{\hat{\bar{\Theta}}} = \Gamma_{\bar{\Theta}} \bar{\Phi}(u_{bl}, x_p) e^T PB \\ &\dot{V}(e, \Delta\bar{\Theta}) = -e^T({R_v}^{-1} + \tilde{P}_v Q_v \tilde{P}_v)e \leq 0 \end{aligned} \tag{34}$$

since $\dot{x}_{ref} \in L_\infty \quad and \quad \dot{e} \in L_\infty$, we can guarantee second degree time derivative is uniformly lower bounded. using Barbalat's lemma, we prove time derivative of V become to zero, at t $\rightarrow \infty$.

$$\ddot{V}(e, \Delta K_x, \Delta\Theta) = -2e^T({R_v}^{-1} + \tilde{P}_v Q_v \tilde{P}_v)\dot{e} \tag{35}$$

Finally, we can conclude that the proposed controller is global asymptotically stable. In practice, the control inputs are: baseline controller + adaptive controller.

$$\begin{aligned} &u = u_{baseline} + u_{adaptive} \\ &u_{bl} = -K_{LQR} x \\ &u_{ad} = -\hat{\bar{\Theta}}^T \bar{\Phi}(u_{bl}, x_p) \end{aligned} \tag{36}$$

4 Simulation Results

The simulation results are analyzed whether the proposed control method follows the desired command correctly when the parametric uncertainties exist. The simulation was conducted for two scenarios: In scenario 1, we assume that parameter such as the mass, moment of inertia, center of gravity is inaccurate so that this effect causes the unstable behavior. Moreover, aerodynamic force and moment deteriorate the tracking performance. In scenario 2, the fault is injected into motor1 and motor2 to show the passive fault-tolerant performance. The true model parameters and initial model parameters are listed in Table 3. In order to compare the performance of our proposed controller, LQR with integral action was selected. We will compare the results using a graph of each state and a graph of error root-mean-square (RMSE) to compare the performance of the controller.

Table 1 Parameters of quadrotor model

Parameter	Initial Values	True Values	Units	Parameter	Initial Values	True Values	Units
I_{xx}	0.025	0.0325	$kg \cdot m/s^2$	$K_{M,q}$	0.0	0.25	m
I_{yy}	0.025	0.0325	$kg \cdot m/s^2$	$K_{M,p}$	0.0	0.25	m
I_{zz}	0.04	0.052	$kg \cdot m/s^2$	$K_{M,r}$	0.0	0.25	m
m	1.5	1.95	kg	k_t	$1.3e^{-6}$	$1.3e^{-6}$	$N \cdot s^2$
x_G	0	0.025	m	k_q	$4.5e^{-8}$	$4.5e^{-8}$	$N \cdot m \cdot s^2$
y_G	0	-0.025	m	τ_{motor}	0.03	0.03	$1/s$
z_G	0	0.025	m	l	0.25	0.25	m
γ_1	1.0	0.75	–	γ_3	1.0	1.0	–
γ_2	1.0	0.80	–	γ_4	1.0	1.0	–

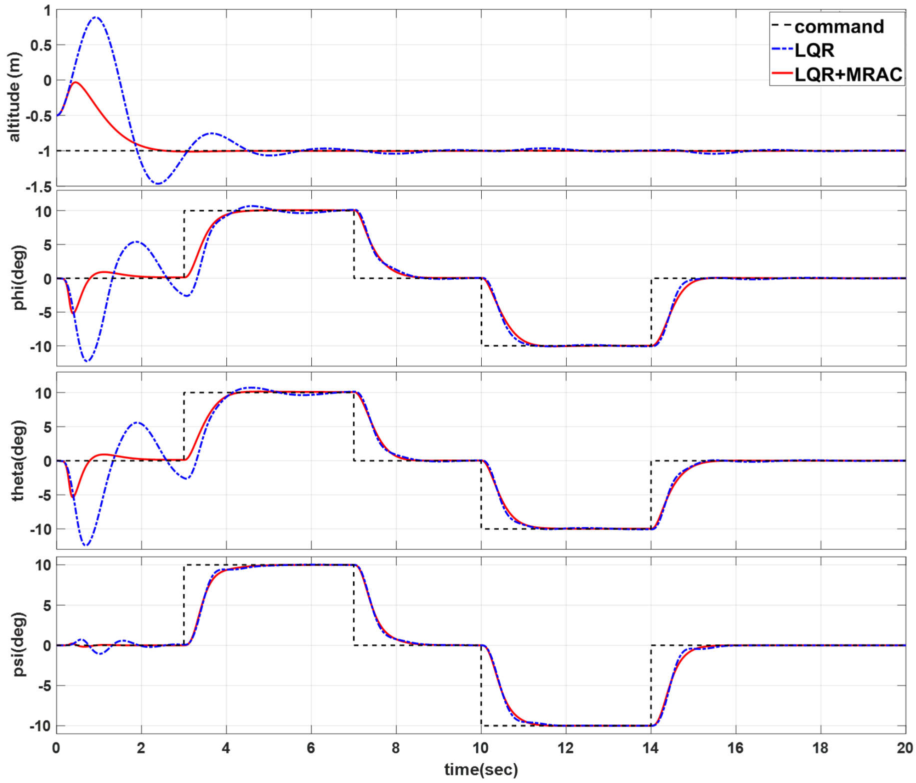

Fig. 3 Simulation result of Euler angle with LQR and LQR+MRAC in Scenario 1

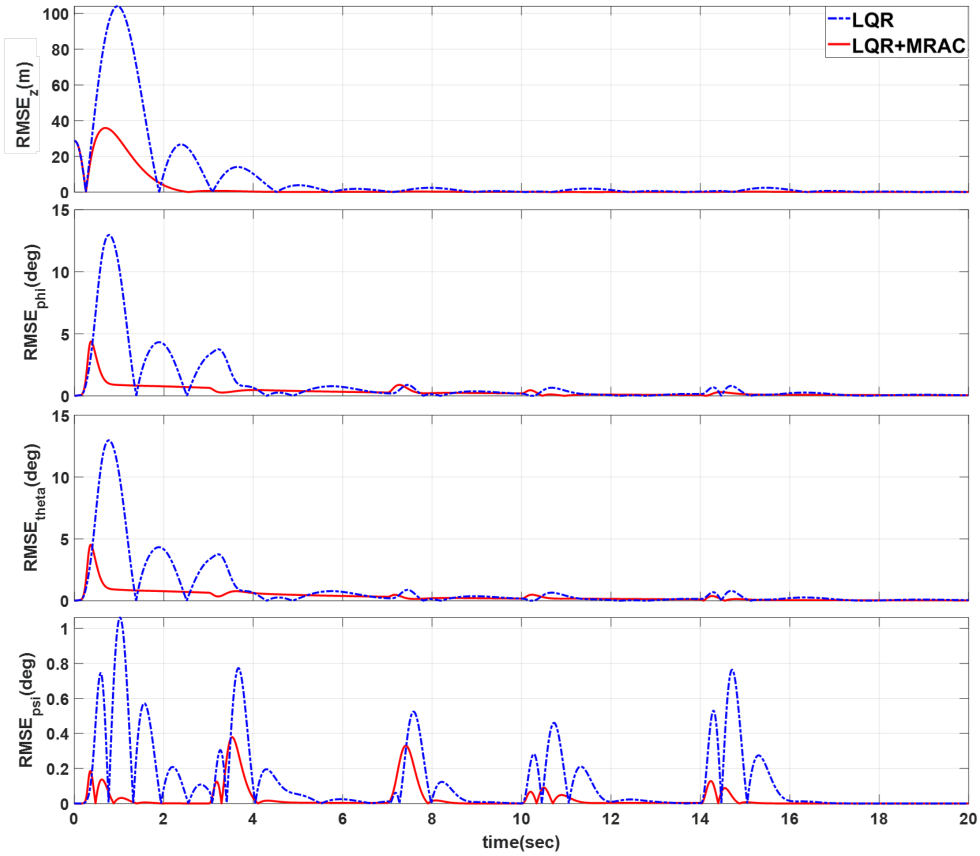

Fig. 4 Root-mean-square error of LQR and LQR+MRAC in Scenario 1

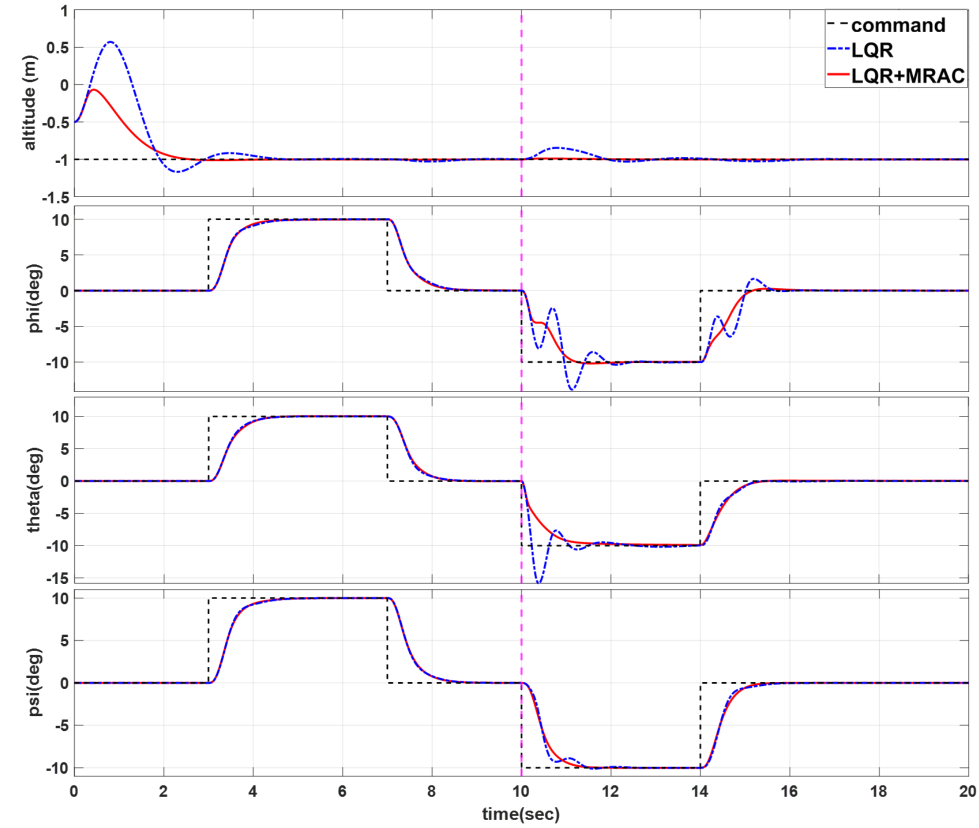

Fig. 5 Simulation result of Euler angle with LQR and LQR+MRAC in Scenario 2

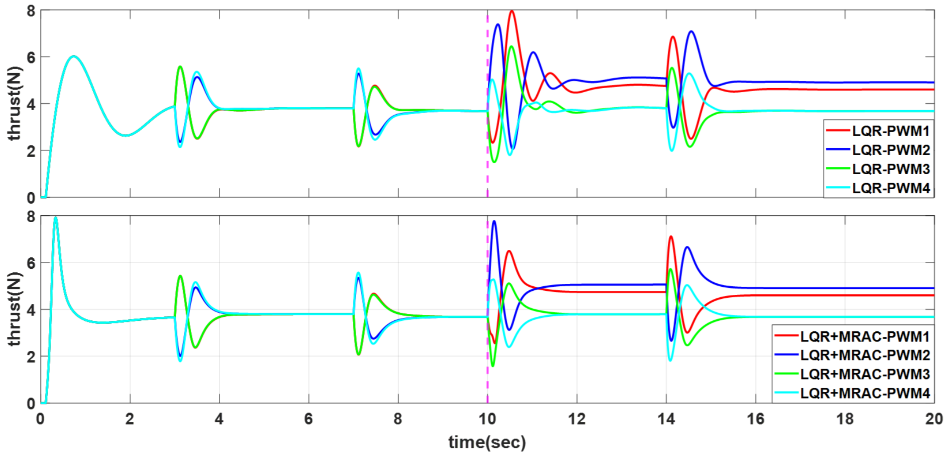

Fig. 6 Pulse-width modulation of LQR and LQR+MRAC in Scenario 2

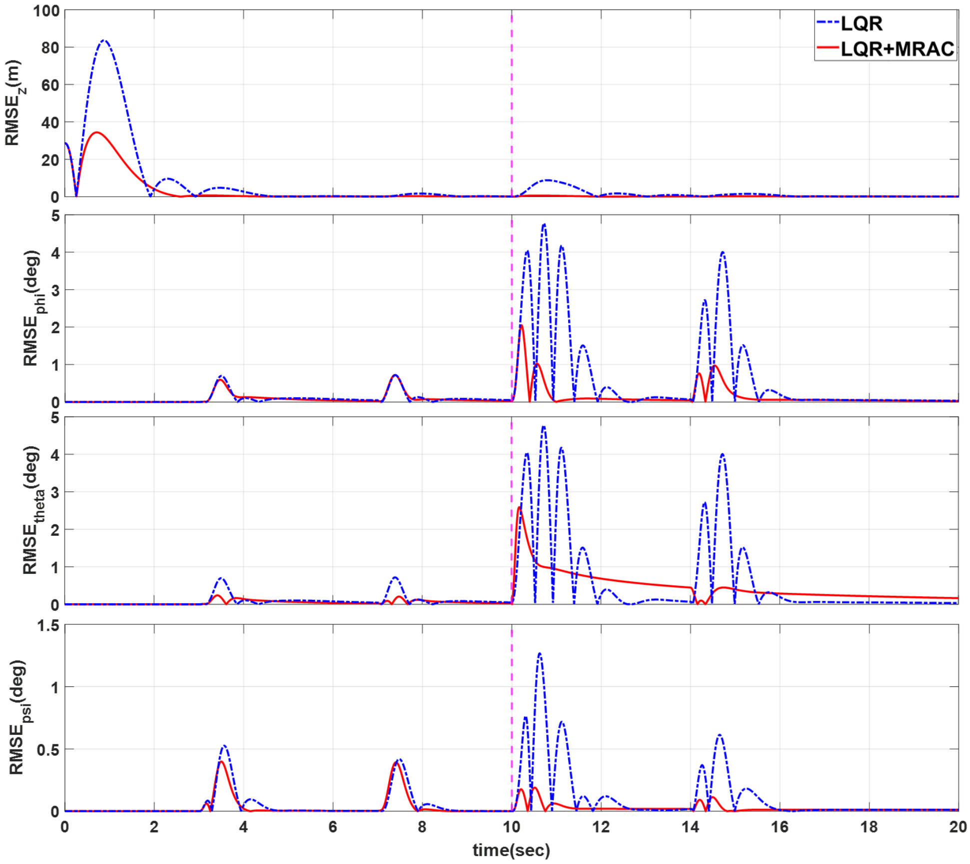

Fig. 7 Root-mean-square error of LQR and LQR+MRAC in Scenario 2

In scenario 1, the attitude and altitude performance of LQR with integral and proposed controller is presented in Figure 3. In $t = 0 \sim 4$, results of LQR with integral action show the undesired oscillation due to the mass property and aerodynamic coefficient uncertainties. After this initial time, the LQR control scheme presents the swaying behavior when step input is injected. In Figure 4, root-mean-square error of LQR and LQR+MRAC scenario 1 is shown. Also, through the root-mean-square error, we have shown the accuracy of the proposed controller numerically. As we mentioned before, the proposed controller of root-mean-square error is smaller than LQR with integral action overall simulation. In scenario 2 that fault is injected into motor1 and motor2 at $t = 10(s)$, the attitude and altitude performance of LQR with integral and proposed controller is presented in Figure 5. Before the fault is injected, LQR and the proposed controller have similar tracking performance. However, after 10 seconds, LQR controller has strong oscillation because this controller can not compensate for the loss of effectiveness effect correctly. Through Figure 6, we can understand that the LQR method continuously changes the PWM signals to follow the command accurately. However, the MRAC approach shows the smooth PWM signals even the fault is injected. In Figure 7, the root-mean-square error of LQR and LQR+MRAC in scenario 2 is shown.

Table 2 Root-mean square error of LQR and LQR+MRAC in Scenario 1

	z (m)	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$
LQR	7.72	0.93	0.93	0.12
LQR+MRAC	2.21	0.32	0.32	0.03

Table 3 Root-mean square error of LQR and LQR+MRAC in Scenario 2

	z (m)	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$
LQR	5.57	0.39	0.36	0.09
LQR+MRAC	2.20	0.12	0.26	0.03

5 Conclusion

In this paper, we derive the quadrotor UAV modelling with uncertainty from payload and motor faults. And, the closed-loop model reference adaptive controller augmented linear quadratic regulator is proposed to compensate the parametric uncertainties such as the mass, moment of inertia, center of gravity, aerodynamic coefficient, and loss of effectiveness. In the numerical simulations, the results show that the proposed controller have better performance than the LQR with integral action. Moreover, when fault is injected, the proposed controller show the passive fault-tolerant ability through the results of scenario 2.

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