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Longitudinal Flight Path Control using Least Squares In-Flight Identification

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ABSTRACT

This paper describes the design of a flight path controller for longitudinal motion that ensures reference tracking with zero steady-state error for uncertain systems. Nowadays, the Total Energy Control System approach is typically used for this task. However, this controller has some drawbacks. On the one hand, the derivation of the aerodynamic speed is often used as measurable variable. The aerodynamic airspeed is usually already very noisy and by deriving the noisy signal, the quality of the measurement is further reduced. On the other hand, this approach uses integrators that, if incorrectly designed, can lead to a slow closed-loop response or even stability issues. The new approach presented in this paper uses the eigenstructure assignment to decouple altitude and speed control from each other. Steady-state accuracy is achieved by a feedforward element. To compensate inaccuracies in the plant description, the gains of the feedforward element are adjusted in flight using a recursive least squares method. The application of the method is demonstrated using a small radio-controlled aircraft.

Keywords: in-flight identification; aircraft control

Nomenclature

$\mathbf{A}, \mathbf{B}, \mathbf{C}$	=	state-space matrices (dynamic matrix, input matrix, output matrix)
$\mathbf{E}$	=	matrix for the representation of uncertainties
$\mathbf{e}$	=	error vector
$\dot{h}$	=	vertical speed
$\mathbf{I}$	=	identity matrix
$\mathbf{K}$	=	controller gain matrix
$\mathbf{K}_{FF}$	=	feedforward gain matrix
$k_{\eta q}$	=	pitch damper gain
$k_{V,\eta}$, $k_{V,TL}$, $k_{\dot{h},\eta}$ and $k_{\dot{h},TL}$	=	speed and vertical speed controller gains
q	=	pitch rate
TL	=	thrust lever
$\mathbf{u}$	=	input vector
V_{IAS}	=	indicated airspeed
$\mathbf{r}$	=	reference / demand value vector

X	=	eigenvector
x	=	state vector
y	=	vector of the measured variables
α	=	angle of attack
η	=	elevator deflection
$\boldsymbol{\Theta}$	=	estimated parameters of the feedforward element
λ	=	eigenvalue

1 Introduction

The conventional flight control system of an aircraft has a cascade structure. While the inner control loops provide stability and alleviation of atmospheric disturbances, the outer control loops ensure flight path guidance. The Total Energy Control System (TECS) approach, first introduced by A. A. Lambregts in 1983 [1], has proven to be the most effective one for controlling the aircraft in longitudinal motion [2]. It is based on the idea that total energy is composed of potential and kinetic energy. The total energy is increased by the thrust. The elevator distributes the total energy between the two individual parts. Lambregts approach is physically motivated. In terms of control theory, it corresponds to a decoupling controller. In its original form, the flight path angle and the derivative of the speed, referred to the gravity, are used as control variables. A switch allows a change in priority between speed and trajectory control. Various authors have slightly modified or extended the original concept [3, 4].

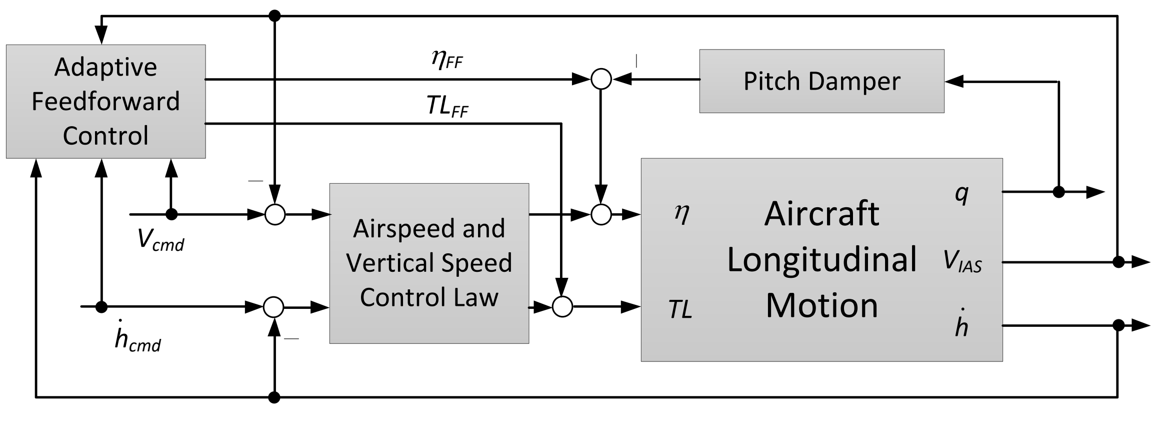


Fig. 1 Illustration of the Proposed Control Law Structure

This paper describes a new approach, which is also based on a decoupling controller. Using an eigenstructure assignment, the phugoid is divided into two aperiodic modes: the speed mode and the vertical speed mode. Speed and vertical speed are used as control variables. The control law is arranged in the forward loop, while typical state control approaches are in the feedback loop. With this approach, a dynamic decoupling of the two state variables is achieved. However, this control law alone does not enable steady-state control accuracy. Faleiro and Lambregts [5] also apply the method of eigenstructure assignment, but still use an integrator in the feedback loop. The present approach dispenses integrators in the feedback loop by using a feedforward element. This requires a precise knowledge of the plant. Since this is not always possible, the gain factors are adjusted using the method of least squares. This results in a flight path control law in longitudinal motion, which allows a decoupled control for speed and vertical speed with stationary state accuracy without using integrators in the feedback loop. Figure 1 shows the proposed control law structure. Once the gains have been determined, no further adaptation is required and thus the system reacts quickly to new reference values. In addition to advantages in flight control, the obtained factors of the feedforward element can be used for the flight performance calculation of the current and future missions. This knowledge is particularly interesting for unmanned aerial systems, for which in some cases only inaccurate flight mechanical models are available.

Adaptive control approaches for aircraft have been discussed in the literature for a long time, since flight dynamics are subject to strong parameter variation due to different configurations (e.g. flaps and landing gear) and operating parameters (e.g. mass, fuel consumption and dynamic pressure) during flight [6, 7]. Even in the case of malfunctions of actuators or structural damages, adaptive approaches can help to ensure a safe flight [8]. Many introduced adaptive control law approaches are mostly used within the control loop. However, there exist approaches in which an adaptation of the feedforward element takes place. Zeng et al. present such an approach for the suppression of aircraft structural vibrations induced by gust perturbations to increase the resilience of the flight control law in the presence of the aeroelastic/aeroservoelastic interaction [9]. For a DC motor, Brablc et al. have developed an adaptive feedforward element that enables steady-state tracking even with unknown parameters of the plant [10]. The feedforward element is designed using the recursive least squares method. The procedure presented can be considered as the initial approach for the design of the feedforward element in this paper. In contrast to the research of Brablc et al., however, a MIMO system is considered.

The application of the presented approach is demonstrated on the Unmanned Aerial System *ZOHD Nano Talon*. The *ZOHD Nano Talon* is a fixed-wing unmanned aircraft with V-tail, a wingspan of 0.86 m and a take-off mass of 0.65 kg [11]. Three aerodynamic rudders and thrust are available as input variables. Left and right aileron are actuated by a common servo motor. Left and right V-tail rudder have different servo motors and, hence, can be deflected independently. A control allocation is made in consideration of conventional control surfaces. Linear state-space models and a nonlinear simulation model with sensor and actuator dynamics are available for this aircraft in the version as Unmanned Aircraft Experimental System (UAXS). The aircraft is shown in Fig. 2.

Fig. 2 ZOHD Nano Talon

2 Eigenstructure Assignment for Decoupling of Speed and Altitude Control

As shown in Fig. 1, the inner cascade consists of the two controllers pitch damper and airspeed and vertical speed control law. Both control laws shall be designed as proportional controllers. The pitch damper has the task of increasing the damping of the short-period mode, while the airspeed and vertical speed control law is intended to decouple the flight path motion, for which the eigenstructure assignment is used. The eigenstructure assignment is a method of multi-variable control in the time domain [12, 13]. In addition to the placement of eigenvalues, it allows the specification of entries in the eigenvectors if enough control variables are available. A prerequisite for its applicability is the fulfillment of controllability according to Kálman [13]. By modifying the eigenvectors, modes can be decoupled. In aviation, the method is used for rigid-body modes, but also for flexible modes [14].

2.1 Plant Description

A nonlinear flight dynamic model is available for the reference aircraft, which has already been successfully verified in flight tests. This nonlinear model can be trimmed in different flight conditions and numerically linearized afterwards. In this paper, only longitudinal motion is considered. By applying a state-space transformation, the following first-order differential equation system

$$\underbrace{\begin{bmatrix} \delta\dot{q} \\ \delta\dot{\alpha} \\ \delta\dot{V}_{\text{IAS}} \\ \delta\ddot{h} \end{bmatrix}}_{\delta\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} M_q & M_\alpha & M_V & M_{\dot{h}} \\ Z_q & Z_\alpha & Z_V & Z_{\dot{h}} \\ X_q & X_\alpha & X_V & X_{\dot{h}} \\ Z'_q & Z'_\alpha & Z'_V & Z'_{\dot{h}} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \delta q \\ \delta\alpha \\ \delta V_{\text{IAS}} \\ \delta\dot{h} \end{bmatrix}}_{\delta\mathbf{x}} + \underbrace{\begin{bmatrix} M_\eta & M_{TL} \\ Z_\eta & Z_{TL} \\ X_\eta & X_{TL} \\ Z'_\eta & X'_{TL} \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \delta\eta \\ \delta TL \end{bmatrix}}_{\delta\mathbf{u}} \tag{1}$$

is obtained for the longitudinal motion after linearization. Instead of the flight path angle (or pitch angle) the vertical speed is chosen as state. All other state variables correspond to the classical flight mechanical state variables used in the usual literature [2]. For an airspeed of 17m s^{-1} in an altitude of 10 m with zero flight path angle, the differential equation of the linear state space system is given with

$$\begin{bmatrix} \delta\dot{q} \\ \delta\dot{\alpha} \\ \delta\dot{V}_{\text{IAS}} \\ \delta\ddot{h} \end{bmatrix} = \begin{bmatrix} -3.2 & -263.2 & 0 & 0 \\ 0.93 & -10.13 & -0.07 & 0 \\ -0.04 & -5.26 & -0.99 & -0.58 \\ 1.21 & 172.3 & 1.12 & 0 \end{bmatrix} \begin{bmatrix} \delta q \\ \delta\alpha \\ \delta V_{\text{IAS}} \\ \delta\dot{h} \end{bmatrix} + \begin{bmatrix} -118.9 & 0 \\ -0.92 & -0.03 \\ -0.35 & 18.59 \\ 15.64 & 0.55 \end{bmatrix} \begin{bmatrix} \delta\eta \\ \delta TL \end{bmatrix} \tag{2}$$

for the *ZOHD Nano Talon*. The eigenvalues of the system matrix are given by

$$\lambda_{1,2} = -6.68 \pm 15.2\,i \quad \text{and} \quad \lambda_{3,4} = -0.48 \pm 0.62\,i, \tag{3}$$

where the first eigenvalue belongs the the short-period mode and the second one to the phugoid. The damping of the short period mode is acceptable with $D = 0.4$, but can be improved. The damping of the phugoid with $D = 0.6$ is higher than for the short period mode , which is caused by the high zero-lift drag. The phugoid eigenvector is given by

$$\mathbf{X}_{3,4} = [-0.028 \pm 0.037\,i \quad 0.0004 \pm 0.0003\,i \quad -0.458 \pm 0.559\,i \quad 1]^T\,. \tag{4}$$

The eigenvector shows that the vertical speed and indicated airspeed have a significant effect on the phugoid. Both state variables are dynamically coupled in this mode.

2.2 Control Law Requirements and Design

The goal of the controller design is to increase the damping of the short-period mode and to divide the phugoid into two aperiodic motions. There, the state variables airspeed and vertical speed are decoupled from each other with respect to the eigenvectors. As control variables the pitch rate q, the indicated airspeed V_{IAS} and the vertical speed $\dot{h}$ are used. The time constant for airspeed mode shall be $T_V = 0.5$ s and for vertical speed mode $T_{\dot{h}} = 2$ s. The justification for this is that the holding of the airspeed is assigned a higher priority than the holding of the vertical speed.

In the classic eigenstructure assignment, the controller is in the feedback loop [2]. Considering the requirements, the control law

$$\underbrace{\begin{bmatrix} \delta\eta \\ \delta TL \end{bmatrix}}_{\mathbf{u}} = -\underbrace{\begin{bmatrix} k_{\eta q} & k_{V,\eta} & k_{\dot{h},\eta} \\ 0 & k_{V,TL} & k_{\dot{h},TL} \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{x} \tag{5}$$

is used within the design process. The parameter $k_{\eta q}$ is determined by using the root locus method with $k_{\eta q} = -0.125$ resulting in a damping ratio of $D = 0.707$ for the short-period mode. The four other controller parameters are determined using the eigenstructure assignment, for which the detailed procedure is described in [13, 15]. In brief, the controller gains are calculated by

$$\begin{bmatrix} k_{V,\eta} & k_{\dot{h},\eta} \\ k_{V,TL} & k_{\dot{h},TL} \end{bmatrix} = \underbrace{[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3 \quad \mathbf{r}_4]}_{\mathbf{R}} \left(\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{[\mathbf{X}_1 \quad \mathbf{X}_2 \quad \mathbf{X}_3 \quad \mathbf{X}_4]}_{\mathbf{X}} \right)^{-1}. \tag{6}$$

The vectors of the matrices $\mathbf{X}$ and $\mathbf{R}$ follow from the solution of a set of linear equation systems

$$\begin{bmatrix} \mathbf{X}_i \\ -\mathbf{r}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \lambda_i \mathbf{I} & \tilde{\mathbf{B}} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{X}}_i \end{bmatrix} \tag{7}$$

with $\mathbf{A}$ as dynamic matrix of the state-space system, λ_i as the desired eigenvalue, $\tilde{\mathbf{X}}_i$ as reduced eigenvector with respect to the states airspeed and vertical airspeed, and $\mathbf{M}$ as mapping matrix to express the full eigenvector as reduced eigenvector.

Considering Fig. 1, only the pitch damper $k_{\eta q}$ is integrated in the feedback loop. The controller for the airspeed and the vertical speed are implemented in the forward loop of the controller with

$$\begin{bmatrix} \delta\eta \\ \delta TL \end{bmatrix} = \begin{bmatrix} k_{V,\eta} & k_{\dot{h},\eta} \\ k_{V,TL} & k_{\dot{h},TL} \end{bmatrix} \begin{bmatrix} \delta V_{cmd} - \delta V_{IAS} \\ \delta \dot{h}_{cmd} - \delta \dot{h} \end{bmatrix}. \tag{8}$$

The designed controllers only provide dynamic decoupling. Static decoupling is not yet achieved. Therefore, a (static) feedforward element is used. A feedforward element is the inversion of the steady-state plant behavior. The output variables are computed so that the desired reference variables are obtained. In most cases only the stationary behavior of the plant is considered, which means $\dot{\mathbf{x}} = \mathbf{0}$. In case of longitudinal motion, this means that the eigendynamics of the short-period mode and phugoid have decayed. Assuming that the changes in the state variables are equal to 0 and that only speed and vertical speed are used as reference variables, the differential equation system of longitudinal motion is simplified to

$$\underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} M_q & M_\alpha & M_V & M_{\dot{h}} \\ Z_q & Z_\alpha & Z_V & Z_{\dot{h}} \\ X_q & X_\alpha & X_V & X_{\dot{h}} \\ Z'_q & Z'_\alpha & Z'_V & Z'_{\dot{h}} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} 0 \\ \delta\alpha_{steady} \\ \delta V_{cmd} \\ \delta \dot{h}_{cmd} \end{bmatrix}}_{\mathbf{r}} + \underbrace{\begin{bmatrix} M_\eta & M_{TL} \\ Z_\eta & Z_{TL} \\ X_\eta & X_{TL} \\ Z'_\eta & X'_{TL} \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \delta\eta \\ \delta TL \end{bmatrix}}_{\mathbf{u}}. \tag{9}$$

Table 1 Controller and Feedforward Gains for the Reference Aircraft

$k_{F,V,\eta} =$	0.019	$k_{F,\dot{h},\eta} =$	0.0003	$k_{F,V,TL} =$	0.051	$k_{F,\dot{h},TL} =$	0.031		
$k_{V,\eta} =$	-0.016	$k_{\dot{h},\eta} =$	-0.012	$k_{V,TL} =$	0.056	$k_{\dot{h},TL} =$	-0.03	$k_{\eta q} =$	-0.125

This results in

$$\mathbf{u} = \underbrace{\left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\left(-\mathbf{A}\right)}_{\hat{\mathbf{K}}_{FF}}\mathbf{r} \tag{10}$$

for the calculation of the control values with $\hat{\mathbf{K}}_{FF}$ as feedforward gain matrix with $\mathbf{r}$ as vector of the reference values. However, this approach only considers the phugoid. Changes in the angle of attack that occur when a fixed vertical speed and an aerodynamic speed are specified are not taken into account. From Eq. 9, the steady-state angle of attack α_{steady} is determined with

$$\delta\alpha_{steady} = -\frac{Z_V\,\delta V_{cmd} + Z_{\dot{h}}\,\delta\dot{h}_{cmd} + Z_\eta\,\delta\eta + Z_{TL}\,\delta TL}{Z_\alpha}. \tag{11}$$

This relationship is inserted into the equations

$$0 = X_\alpha\,\delta\alpha_{steady} + X_V\,\delta V_{cmd} + X_{\dot{h}}\,\delta\dot{h}_{cmd} + Z_\eta\,\delta\eta + Z_{TL}\,\delta TL \tag{12}$$

$$0 = Z'_\alpha\,\delta\alpha_{steady} + Z'_V\,\delta V_{cmd} + Z'_{\dot{h}}\,\delta\dot{h}_{cmd} + Z'_\eta\,\delta\eta + Z'_{TL}\,\delta TL, \tag{13}$$

from which the relationship between the control variables ($\delta\eta$ and δTL) and the reference variables (V_{cmd} and $\dot{h}_{\text{cmd}}$) can be established using the 2×2 matrix $\mathbf{K}_{FF}$. The matrix represents the feedforward element taking into account the influence of the angle of attack. Referring to Fig. 1, the relationship

$$\begin{bmatrix} \delta\eta_{FF} \\ \delta TL_{FF} \end{bmatrix} = \underbrace{\begin{bmatrix} k_{F,V,\eta} & k_{F,\dot{h},\eta} \\ k_{F,V,TL} & k_{F,\dot{h},TL} \end{bmatrix}}_{\mathbf{K}_{FF}} \begin{bmatrix} \delta V_{cmd} \\ \delta\dot{h}_{cmd} \end{bmatrix} \tag{14}$$

results, which contributes to the static decoupling of airspeed and vertical speed. Note that for this case it is assumed that the controller gains are fixed.

2.3 Design Results

After applying the methods of Sec. 2.2, the controller gain factors listed in Tab. 1 result for the introduced plant model (cf. Sec. 2.1). The closed loop has the eigenvalues

$$\lambda_{1,2} = -13,8 \pm 14\,i, \quad \lambda_3 = -2 \quad \text{and} \quad \lambda_4 = -0.5, \tag{15}$$

which fulfills the design requirement with respect to the damping of the short-period mode and the time constant for airspeed mode and vertical speed mode. The eigenvector of the airspeed mode

$$\mathbf{X}_3 = [0.02 \quad -0.008 \quad 1 \quad 0]^T \tag{16}$$

shows no participation of the vertical speed. The eigenvector of the vertical speed mode

$$\mathbf{X}_4 = [-0.03 \quad -0.004 \quad 0 \quad 1]^T \tag{17}$$

shows no participation of the airspeed. Hence, both modes are dynamically decoupled from each other.

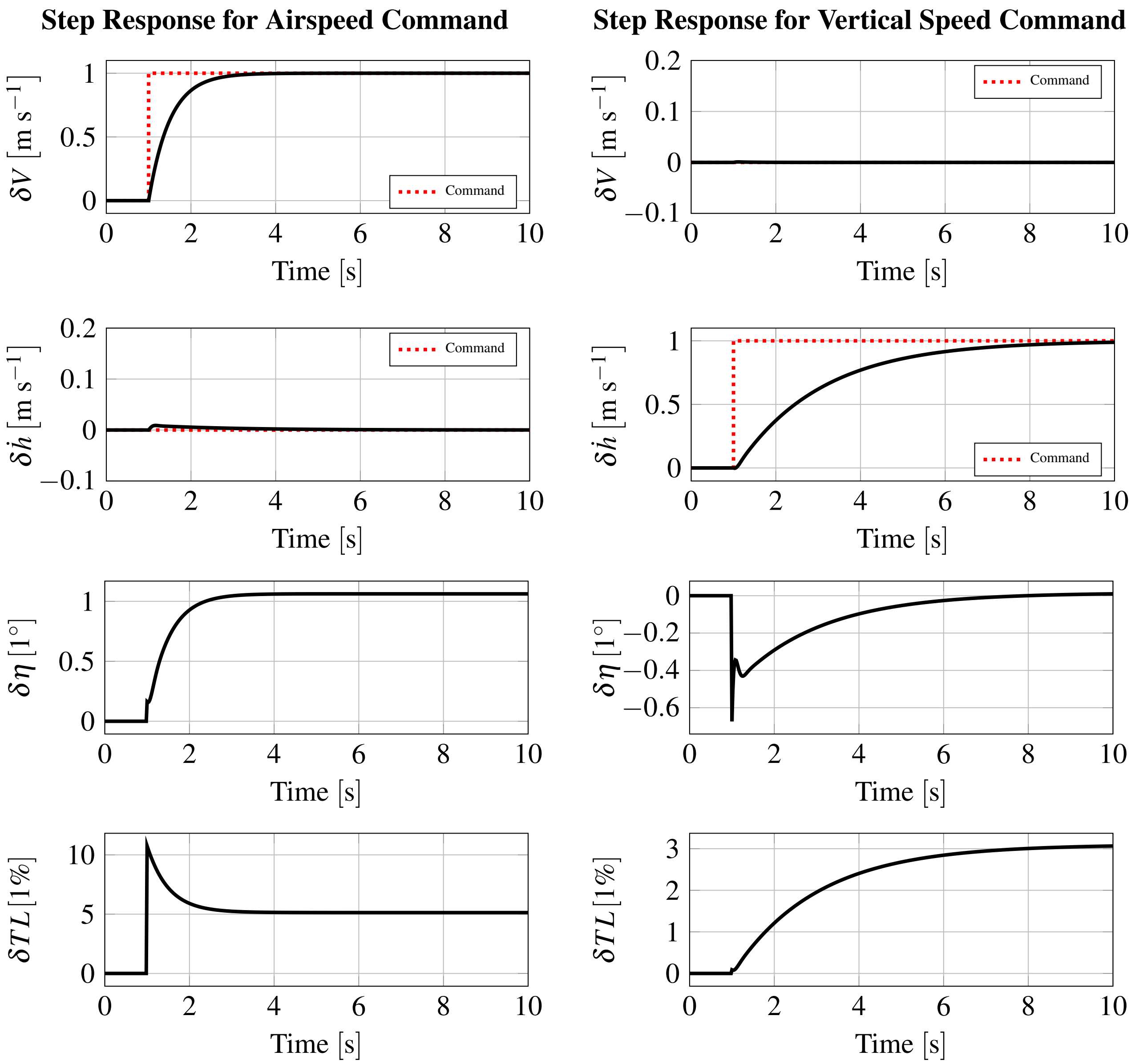


Fig. 3 Linear Closed-Loop Simulation Results

Figure 3 shows the step responses for increases in the target airspeed and the target vertical speed by 1 m s^{-1} from linear simulations. It is obvious that both state variables can be adjusted independently and that there is no significant coupling. Smaller changes in vertical speed (when airspeed command is set) and airspeed (when vertical speed command is set) result from the short period mode. Thus, the design goal is satisfactorily achieved. However, the solution is subject to limitations: i) the feedforward element is not transferable to nonlinear systems, since the trim values for thrust and elevator are not taken into account; ii) furthermore, a perfect system is assumed, where the controlled system is exactly known and no effects due to sensor noise or disturbance variables occur. Therefore, an adaptation of the feedforward element is conducted in the following section.

3 Design of a Feedforward Element Using In-Flight Identification

The previous design methods used a linear state-space model trimmed around a flight state. This means that only variations of the state variables around this flight state are considered. An equality between the linear and nonlinear state variables is valid for the pitch rate and the vertical speed. With respect to airspeed, linear and nonlinear state variables differ by the value for trimmed flight condition. Regarding the dynamic decoupling with the control law this has no influence, because the error between target airspeed and airspeed is used as input variable for the controller. However, the feedforward element for static decoupling uses the absolute target airspeed to calculate the elevator deflection and thrust. Thus, for the real-world application, a trim value for elevator η_{trim} and thrust TL_{trim} with

$$\begin{aligned} \eta_{FF} &= k_{F,V,\eta}\, V_{cmd} + k_{F,\dot{h},\eta}\, \dot{h}_{cmd} + \eta_{trim} \\ TL_{FF} &= k_{F,V,TL}\, V_{cmd} + k_{F,\dot{h},TL}\, \dot{h}_{cmd} + TL_{trim} \end{aligned} \quad . \tag{18}$$

has to be used in the feedforward element. The trim values, as well as the other parameters of the feedforward element, have to be adjusted depending on the flight condition. With the classic TECS core, this is done by the integrators. When using a feedforward element, the parameters have to be adjusted otherwise. This is realized by adjusting the parameters during the flight using the least squares method.

3.1 Polynomial Feedforward Description

The Least Squares Method estimates factors Θ_i in a polynomial function with

$$u = \Theta_0 + \Theta_1\, f_1(y) + \Theta_2\, f_2(y) + \ldots + \Theta_n\, f_n(y)\ . \tag{19}$$

The factors are real values and the functions $f_i(y)$ can be chosen arbitrarily, but depend on the default values for airspeed and vertical speed, commonly denoted by y. In preliminary investigations, quadratic and linear approaches were tested for the different functions to calculate the elevator deflection and thrust of the feedforward element, commonly denoted by u. Quadratic approaches were chosen because the dynamic pressure depends quadratically on the airspeed. Nevertheless, linear approaches for the functions $f_i(y)$ showed to be the most suitable. This leads to

$$\begin{aligned} \eta_{FF} &= \Theta_{\eta,0} + \Theta_{\eta,1}\, V_{cmd} + \Theta_{\eta,2}\, \dot{h}_{cmd} \\ TL_{FF} &= \Theta_{TL,0} + \Theta_{TL,1}\, V_{cmd} + \Theta_{TL,2}\, \dot{h}_{cmd} \end{aligned} \tag{20}$$

for the calculation of the control signals with Θ_η as factors for the elevator and Θ_{TL} for the thrust. The variables $\Theta_{\eta,0}$ and $\Theta_{TL,0}$ represent the trim values for elevator and thrust. Hence, in total, six variables have to be determined for the calculation of the feedforward element by the least squares method.

3.2 Parameter Determination Using the Least Squares Method

In general, the parameters Θ can be determined as follows: for a steady-state flight condition, the airspeed and vertical speed as well as the elevator deflection and thrust setting can be determined. Thus in Eq. 19 the value for each u and y is available and the parameters Θ can be calculated. Once the parameters are known, they can be substituted into Eq. 20 and the elevator deflection and thrust setting for a given target airspeed and vertical speed can be determined.

The determination of the parameters Θ can now be carried out in two ways: i) The nonlinear flight dynamic model is trimmed for different airspeeds and vertical speeds and the required thrust and elevator deflection are determined from the results of the trim calculation. This results in different tuples for the variables u and y with which the parameters are determined using the least squares method. ii) In the steady-state flight condition, the values for airspeed and vertical speed are measured in flight, where they correspond to an elevator deflection and thrust setting. Several different flight conditions also result in several tuples for u and y from which the parameters for Θ are determined. In this paper, the second way is chosen because a) uncertainties in the plant description are possible, b) there may be no plant description at all, or c) parameters of the plant may change in flight. By determining the parameters in flight, the aforementioned challenges can be solved. However, initial parameters can be calculated using the first way, as far as a nonlinear model is available.

To describe the method for determining the parameters in flight, the general term u is chosen for both elevator and thrust setting in the following. For each control variable, a model with

$$u_m = \hat{\Theta}_0 + \hat{\Theta}_1\, V + \hat{\Theta}_2\, \dot{h}. \tag{21}$$

is now sought which maps the functions for airspeed and vertical speed to the respective control variable. The aim of the least squares identification is now to minimize the error e_k between the expected output $u_{m,k}$ and the measured, true output u_k at discrete time k

$$e_k = u_k - u_{m,k}\,. \tag{22}$$

The measured, true output variable is obtained with

$$u_k = \Theta_0 + \Theta_1\, V_k + \Theta_2\, \dot{h}_k + n_k\,, \tag{23}$$

where n represents the measurement noise. With the assumption that in this case there are N measurement points in time, a vector for the expected measured variables and the measured variables results with

$$\underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}}_{\mathbf{u}} = \underbrace{\begin{bmatrix} 1 & V_1 & \dot{h}_1 \\ 1 & V_2 & \dot{h}_2 \\ \vdots & \vdots & \vdots \\ 1 & V_N & \dot{h}_N \end{bmatrix}}_{\boldsymbol{\Psi}} \underbrace{\begin{bmatrix} \Theta_0 \\ \Theta_1 \\ \Theta_2 \end{bmatrix}}_{\boldsymbol{\Theta}'} \quad \text{and} \quad \underbrace{\begin{bmatrix} u_{m,1} \\ u_{m,2} \\ \vdots \\ u_{m,N} \end{bmatrix}}_{\mathbf{u}_m} = \underbrace{\begin{bmatrix} 1 & V_1 & \dot{h}_1 \\ 1 & V_2 & \dot{h}_2 \\ \vdots & \vdots & \vdots \\ 1 & V_N & \dot{h}_N \end{bmatrix}}_{\boldsymbol{\Psi}} \underbrace{\begin{bmatrix} \hat{\Theta}_0 \\ \hat{\Theta}_1 \\ \hat{\Theta}_2 \end{bmatrix}}_{\hat{\boldsymbol{\Theta}}'_m}. \tag{24}$$

This leads to the sum of the squares of errors

$$I = \mathbf{e}\,\mathbf{e}^T \quad \text{with} \quad \mathbf{e} = \mathbf{u} - \mathbf{u}_m\,, \tag{25}$$

which has a minimum if the derivative from I w.r.t. $\hat{\boldsymbol{\Theta}}'$ is identical to the transposed zero vector. From this relationship, the calculation for the stationary distance parameters with

$$\hat{\boldsymbol{\Theta}}' = \left(\boldsymbol{\Psi}^T \boldsymbol{\Psi}\right)^{-1} \boldsymbol{\Psi}^T\, \mathbf{u}_m \tag{26}$$

can be derived. After determining the parameters $\hat{\Theta}_0$ to $\hat{\Theta}_3$ for each control variable, these parameters are used in the feedforward element to determine the required elevator deflection and thrust setting for each target value.

3.3 Recursive Least Square Method for Implementation in Flight Control System

The introduced method has the disadvantage that the matrix $\boldsymbol{\Psi}$ becomes larger and larger over time. This increases the computing time and can lead to runtime problems in practical implementation on a flight control system. Therefore the recursive least squares method is used. This estimation method uses only the current values for elevator, thrust, airspeed and vertical speed by using a recursive formulation. Additionally, the inversion of the matrix $\boldsymbol{\Psi}$ is not required, which saves additional computing time. The procedure is derived for example in Isermann [16], so that a detailed mathematical explanation is not needed. In each new time step $k+1$, the parameters $\hat{\mathbf{\Theta}}'$ are determined by

$$\hat{\mathbf{\Theta}}'(k+1) = \hat{\mathbf{\Theta}}'(k) + \boldsymbol{\gamma}(k)\left(u_m(k+1) - \tilde{\mathbf{\Psi}}(k+1)\hat{\mathbf{\Theta}}'(k)\right). \tag{27}$$

In this equation, $\hat{\mathbf{\Theta}}'(k+1)$ is the new estimation, $\hat{\mathbf{\Theta}}'(k)$ is the old estimation, $u_m(k+1)$ is the current value for elevator or thrust setting and $\tilde{\mathbf{\Psi}}(k+1)\hat{\mathbf{\Theta}}'(k)$ is the control variable predicted by the model. $\tilde{\mathbf{\Psi}}(k+1)$ is a line vector containing only the current values for airspeed and vertical speed. The expression in brackets indicates how much the measurement deviates from the estimation with the model. The vector $\boldsymbol{\gamma}(k)(u_m(k+1))$ corresponds to a correction. Thus, the recursive least squares method is nothing else than a Kalman filter. The correction is computed with

$$\boldsymbol{\gamma}(k) = \frac{\mathbf{P}(k)\tilde{\mathbf{\Psi}}^T(k+1)}{\tilde{\mathbf{\Psi}}(k+1)\mathbf{P}(k)\tilde{\mathbf{\Psi}}^T(k+1)+1}, \tag{28}$$

where $\mathbf{P}$ is calculated with

$$\mathbf{P}(k+1) = (\mathbf{I} - \boldsymbol{\gamma}(k)\mathbf{\Psi}(k+1))\mathbf{P}(k). \tag{29}$$

The problem with the recursive method is that initial values are needed for the parameter estimation $\hat{\mathbf{\Theta}}'_m$ and the matrix $\mathbf{P}$. For this purpose a) a priori values, b) standard values (parameter estimation corresponds to the zero vector and matrix $\mathbf{P}$ has very large entries) or c) values determined by the nonrecursive method can be used [16]. For the implementation in the control law, the last variant is chosen.

3.4 Implementation of the Recursive Least Square Method as a State Machine

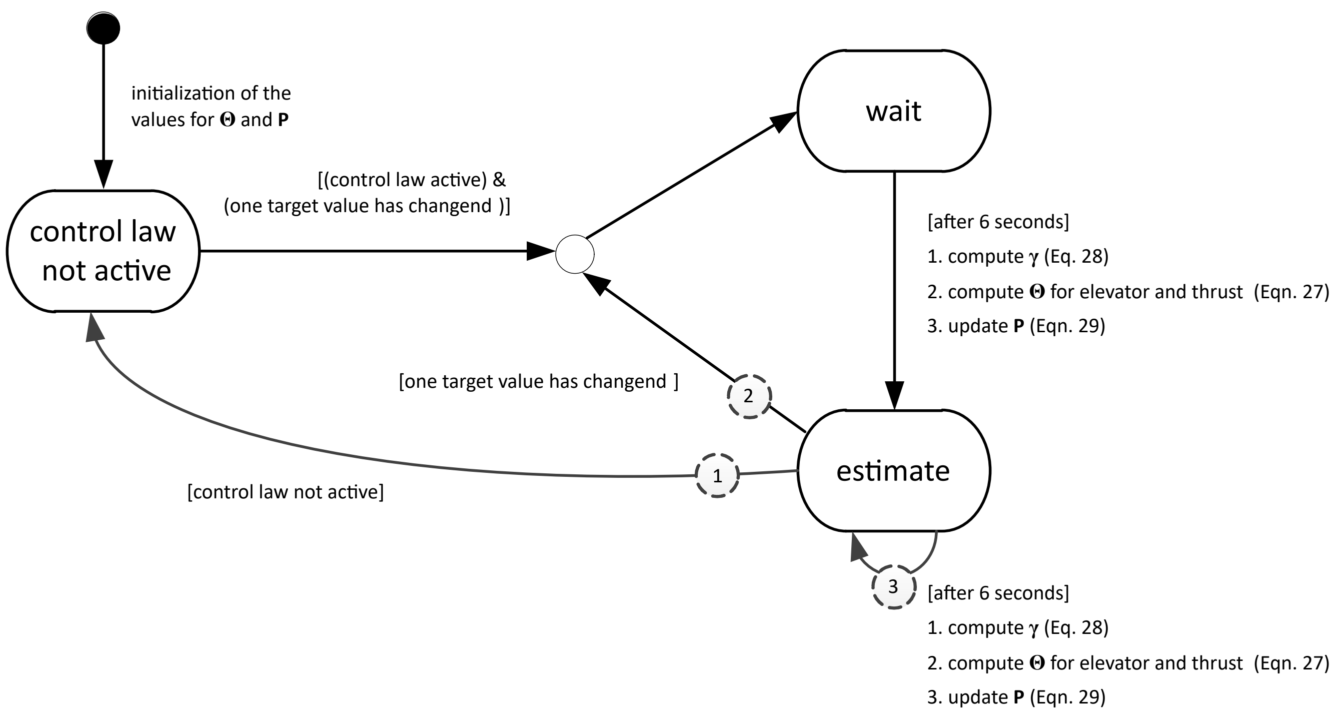
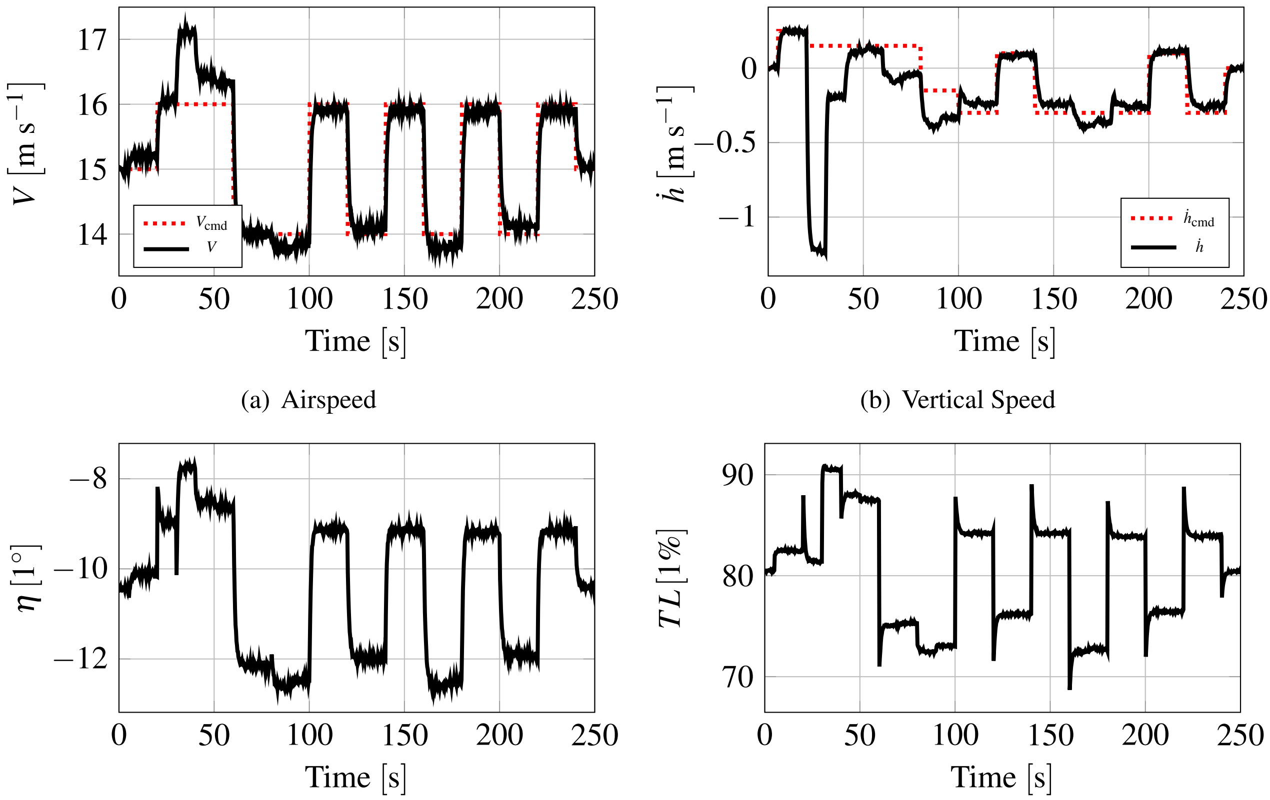


Fig. 4 Illustration of the Implementation of the Adaptive Feedforward Element as a State Machine

Figure 4 shows the state machine for calculating the feedforward factors. As soon as the control law is activated and target values for airspeed or vertical speed change, the system switches to a "wait mode". The system remains in this state for six seconds. This is to ensure that the controller can set new steady-state flight conditions. Based on the control values for elevator and thrust and the values for airspeed and vertical speed, the calculations are carried out. The new values for $\mathbf{\Theta}$ result in new control commands for elevator and thrust. This disturbs the steady flight state and the controller needs a certain time to set a new one. Once the state is set, the process starts again from the beginning.

3.5 Nonlinear Simulation Results



(a) Airspeed (b) Vertical Speed

(c) Elevator Deflection (d) Thrust Lever

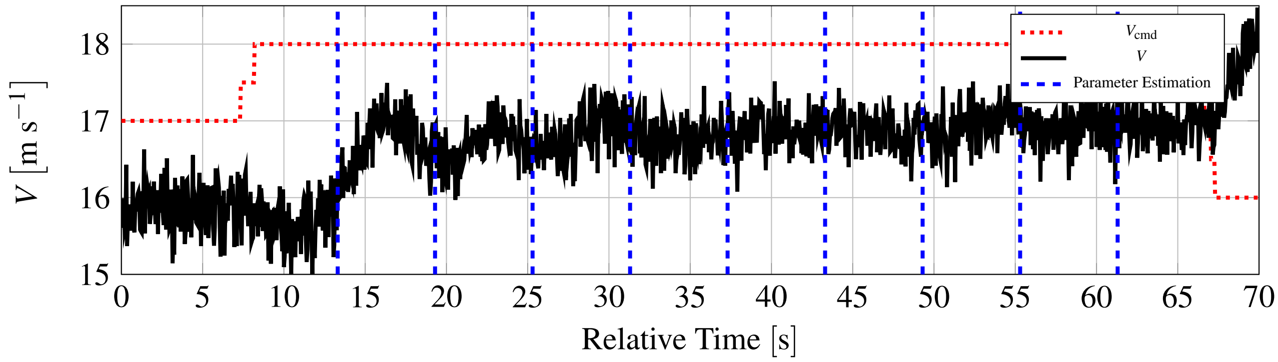
Fig. 5 Nonlinear Simulation Results for feedforward Control with Variable Gain Factors at Different Targets for Airspeed and Vertical Speed

Figure 5 shows the results of the nonlinear simulation. It can be seen that the commanded speed and vertical speed is followed very well after some time has passed. Because of noise, the feedforward parameters are continuously adjusted. The control deflections are completely within the limits. Overall, it can be seen that the presented method is robust to meet the requirements even in a nonlinear simulation, which has deviations from the linear operating point, sensor noise and unmodeled dynamics.

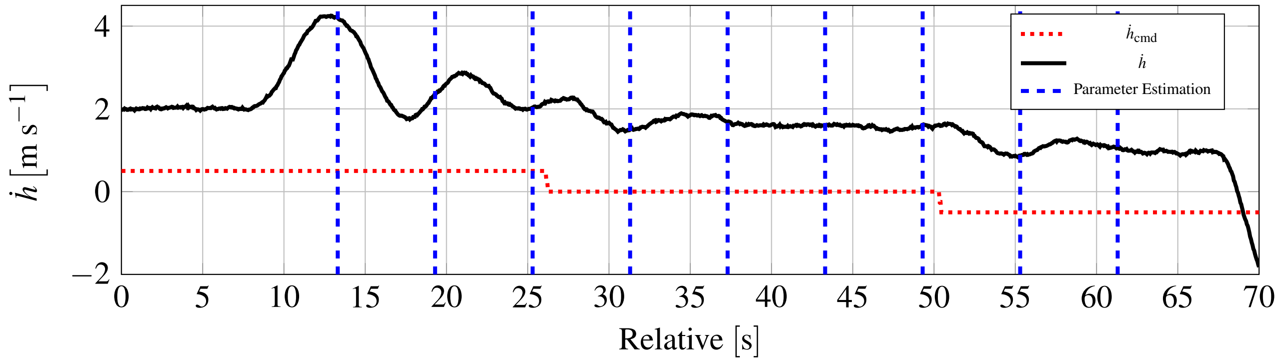
4 Flight Test Results

The flight controller of Sec. 2.2 and the adaptive feedforward, whose factors are calculated using recursive least-squares methods and the state machine of Fig. 2.2, have been integrated into the Simulink template of the UAXS[11]. Since the barometric vertical speed is not available in the Simulink template (only barometric altitude), the vertical speed is calculated using a pseudo-differentiator [2].

The aircraft was manually launched and flown to a safe altitude and trimmed out. The flight controller was then activated. For airspeed and vertical speed, the results are shown in Fig. 6 for a relative time during which the controller was activated. In addition, the timestamps at which the feedforward parameters were re-estimated are plotted. The results show that steady-state guidance accuracy is not



(a) Airspeed



(b) Vertical Speed

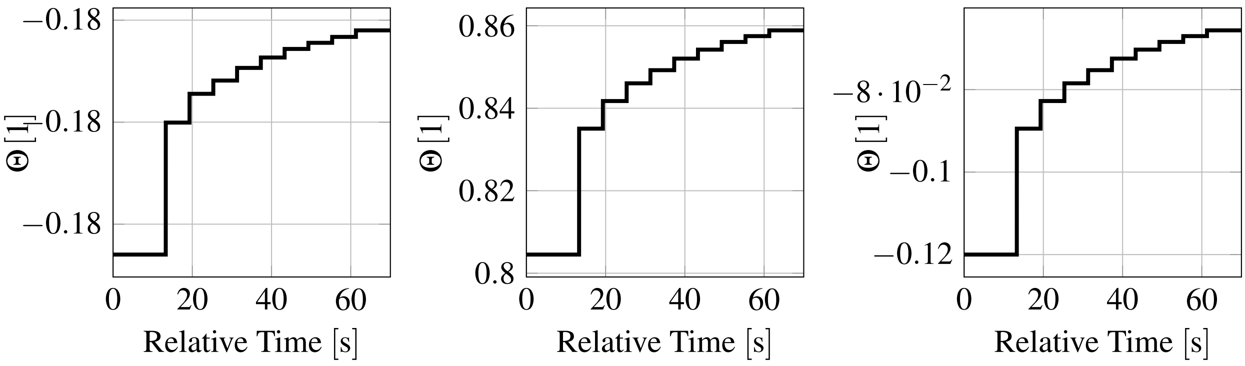
Fig. 6 Flight Test Results for Airspeed and Vertical Speed

achieved in real applications. Figure 7 shows an example of the adaptation of the feedforward elements parameters. It can be seen that an adaptation takes place, but the changes are only very small.

In the evaluation, it can be seen that the feedforward parameters are not always determined in steady-state flight conditions. The model that is determined with the recursive least squares method assumes a static behavior. Therefore, errors occur which lead to incorrect parameters of the feedforward element and thus no steady-state accuracy for airspeed and vertical speed can be achieved with the control law. The influence of estimates at non-steady flight conditions on incorrect feedforward gains can be substantiated by applying the least squares method to the flight test data. Ten steady-state timestamps were selected. The comparison shows that there are differences of 3% for steady-state thrust, for example. From a flight mechanics perspective, however, the adaptation direction of the factors - at least for the trim values - is plausible. Looking at the period between 30 and 50 seconds, it can be seen that the airspeed is too low and the vertical speed is too high. The adaptation of the trim values shows a trend for more thrust and elevator deflection. This should contribute to both an increase in airspeed and a reduction in vertical speed.

5 Conclusion

This paper presents an alternative approach to longitudinal flight path control. In contrast to the classical TECS controller, a proportional controller is used for dynamic decoupling and a feedforward for static decoupling of airspeed and vertical speed. In linear investigations this leads to success. However, the approach cannot be transferred to the nonlinear system without adjustments. For this, the trim values for thrust and elevator deflection are required but not always known exactly, especially when configuration changes occur. Therefore, the feedforward factors are adapted during flight using the recursive



(a) Parameter $\Theta_{0,\eta}$ (b) Parameter $\Theta_{0,TL}$ (c) Parameter $\Theta_{2,TL}$

Fig. 7 Illustration of Selected Feedforward Gains

least squares method. Compared to the TECS controller, the advantage is that the derivative of the airspeed is not used and integrators in the control loop are avoided. The functionality of the approach was successfully tested in nonlinear flight simulations.

However, the application in flight tests was still not successful. This can be justified by the fact that an estimation of the feedforward gains is only possible in steady-state flight conditions. During a real flight, these conditions are not always achievable due to disturbances. In future work, the concept can be extended in such a way that an adaptation of the parameters only takes place when a steady-state flight condition exists. In addition, further flight tests should be conducted to gain statistical statements about the approach.

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