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PIP Selection Algorithm for Lambert Guidance Considering Engagement Geometry

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ABSTRACT

As threat of a ballistic missile that carries various devastating warheads is bigger, attention to anti-ballistic missile is grown. There are several strategies to intercept a ballistic missile, but many researches focus on intercepting the ballistic missile during the mid-course phase. Although guidance is mature field of research, intercepting a ballistic missile is still a challenging problem because of high speed of the ballistic missile. In this paper, to successfully intercept a ballistic missile, an algorithm to select predictive intercept point considering engagement geometry is proposed. To this end, a proper engagement geometry is discussed and a performance index is proposed. Then, the algorithm to find near-optimal predictive intercept point in the sense of the performance index is proposed. The algorithm calculates initial guess of the predictive intercept point by solving Lambert problem, and updates the predictive intercept point using equation of orbit. After predictive intercept point is selected, Lambert guidance law is used to get to the point. It is robust to measurement noise and needs small computational load. To demonstrate the performance of the proposed algorithm, numerical simulation is performed. As a result, the proposed algorithm shows proper performance not only from the perspective of the performance index, but also the perspective of the control input.

Keywords: Anti-ballistic missile, Lambert problem, Lambert guidance, Engagement geometry, Mid-course guidance

Nomenclature

t	=	time
$\mathbf{v_I}$	=	velocity vector of interceptor
$\mathbf{v_T}$	=	velocity vector of target ballistic missile
$\mathbf{R}(t,0)$	=	position vector of target ballistic missile measured at t
$\mathbf{V}(t,0)$	=	velocity vector of target ballistic missile measured at t
$\mathbf{R}(t,\tau)$	=	predicted position vector of target ballistic missile at t based on $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$
$\mathbf{V}(t,\tau)$	=	predicted velocity vector of target ballistic missile at t based on $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$
$\mathbf{r}$	=	position vector of interceptor launch site

$\mathbf{PIP}(t)$	=	predicted PIP position vector at t based on $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$
$\mathbf{T}$	=	thrust vector
t_f	=	the time when interceptor or target ballistic missile reaches PIP
J	=	performance index
$T(t)$	=	predicted target ballistic missile's time of flight based on $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$
$t_{go}(t)$	=	predicted time-to-go of the interceptor at t based on $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$

1 Introduction

After anti-air missiles had been developed in the 1950's, targets of anti-air missiles were expended from an aircraft to a ballistic missile as technology achieves a higher level. Nowadays, attempts such as Strategic Defense Initiative (SDI), Missile Defense (MD) were made to intercept ballistic missiles to protect against the threat of the ballistic missile which can carries multiple devastating nuclear warheads became greater. For ballistic missile interception, three strategies can be considered. The first one is intercepting the ballistic missile during the boost phase. The second one is intercepting the ballistic missile during the mid-course phase. The last one is intercepting the ballistic missile during the re-entry phase.

Intercepting the ballistic missile during the boost phase is much easier than the other two methods because the ballistic missile relatively slow. However, it requires global reconnaissance assets which are hard to achieve because rapid response is essential just after the launch of the ballistic missile. Also, even if the defender has such assets, it is politically risky to directly strike another country's missile launch site. Intercepting the ballistic missile during the re-entry is not easy either. Ballistic missiles have maximum speed during the re-entry phase of about 7km/s [1]. Also, the atmosphere gives high maneuverability to the warhead, which makes the interception problem trickier. Thus, most attempts for ballistic missile interception were focused on mid-course phase interception. Even if ballistic missiles have about 2km/s during the mid-course phase, it is much slower than the re-entry phase. Also, because there are no aerodynamic effects during the mid-course phase, it is easy to predict the trajectory of the ballistic missile and the ballistic missile has very low maneuverability compared to the ballistic missile in the re-entry phase.

A variety of mid-course guidance laws for interceptors were developed, such as ZEM guidance law [2] or Lambert guidance law [3]. However, such guidance laws have limitations in that they only consider miss distance and have not considered an engagement geometry. Considering an engagement geometry is crucial for ballistic missile interception because of two reasons: First, unlike aircraft, ballistic missiles are usually much faster than the interceptor, thus even if the seeker could lock on to the ballistic missile, it cannot physically approach the ballistic missile depending on the engagement geometry. Second, one of the significant factors for the ballistic missile interception is securing enough time for terminal guidance [4], and it highly depends on the engagement geometry. Several studies had researched mid-course guidance considering engagement geometry. Chen [5] proposed a mid-course guidance law that can control the impact angle between the target and the interceptor. However, this study considered constant gravitational field which occurs oversimplified dynamics for long-range interceptors. Na [6] showed that multistage rockets can reach Predicted Intercept Point (PIP) at the desired time with desired flight path angle but it considered an oversimplified gravitational field and it only considered a straight reference trajectory. Ann [7] suggested a novel mid-course guidance law considering engagement geometry, but it requires a pre-calculated response surface that cannot be used if the ballistic threat approaches from an unpredicted direction. Indig [8] proposes mid-course guidance law motivated by Proportional Navigation (PN) guidance law that can control the collision angle that produces near-optimal trajectory, but it requires a gain-tuning process that must be done heuristically.

In this paper, a new PIP selection algorithm is suggested for Lambert guidance law considering engagement geometry. Results of classical mechanics are used to predict the orbit of the ballistic missile. The initial guess for the PIP location is obtained from this predicted orbit. In this process, the Lambert problem is used to predict the collision angle at the initial guess of the PIP. The PIP will be updated using ballistic missile state measurement data and the initial guess of the PIP. The interceptor is guided to PIP using Lambert guidance law. The main contributions of this paper are as follows. First, proper engagement geometry is discussed and a performance index that can evaluate whether the engagement is favorable is suggested based on it. By introducing such an index, one can compare performance of algorithms from the perspective of engagement geometry much more quantitatively. Second, the proposed algorithm is near-optimal in the sense of performance index that this paper suggests. This means the suggested algorithm finds a near-optimal solution for PIP selection considering engagement geometry. Third, the proposed algorithm has feedback structure, so that it robust to uncertainty like measurement noise.

This paper is organized as follows. In Sec. 2, theoretical backgrounds will be presented. This includes preliminaries about Lambert guidance and analysis of engagement geometry in ballistic missile interception. The proposed PIP selection algorithm will be suggested in Sec. 3 and numerical simulation is performed to demonstrate the effectiveness of the proposed scheme in Sec. 4. In Sec. 5, the conclusion and future works will be presented.

2 Preliminary

2.1 Lambert Guidance

Lambert guidance is a guidance law that uses the result of the Lambert problem [9]. The Lambert problem is to find a velocity to make an orbit that passes given two points, $\mathbf{P_1}$ and $\mathbf{P_2}$, within a given time, Δt, in a given gravitational force field. Now call such a problem as a $(\mathbf{P_1}, \mathbf{P_2}, \Delta t)$ Lambert problem. It is well known that solving the Lambert problem is equivalent to solving the following simultaneous nonlinear equation with respect to y and E [10].

$$y^2 = \frac{m}{l + sin^2\frac{1}{2}E} \tag{1}$$

$$y^3 - y^2 = m\frac{2E - sin2E}{sin^3 E}. \tag{2}$$

Solving the above nonlinear equation is one of the classical research topics since Gauss, and there are many suggested algorithms to solve such an equation [10–12].

Lambert guidance is a modified form of Lambert problem. When there is a particle which can adjust thrust direction and a gravitational force field and two points, $\mathbf{P_1}$ and $\mathbf{P_2}$, are given, it finds the logic to make the particle whose initial position is $\mathbf{P_1}$ pass the point $\mathbf{P_2}$ in a given time Δt. Denote the current time as t, the interceptor's current location as $\mathbf{r}(t)$, and the interceptor's current velocity as $\mathbf{v}(t)$. Define the particle's time-to-go as $t_{go} := \Delta t - t$. Then one can solve the $(\mathbf{r}, \mathbf{R}, t_{go})$ Lambert problem. Denote the solution of the Lambert problem as $\mathbf{v_{des}}(t)$. Define $\Delta\mathbf{v}(t)$ as follows.

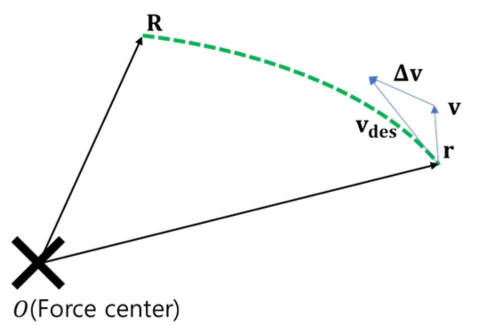
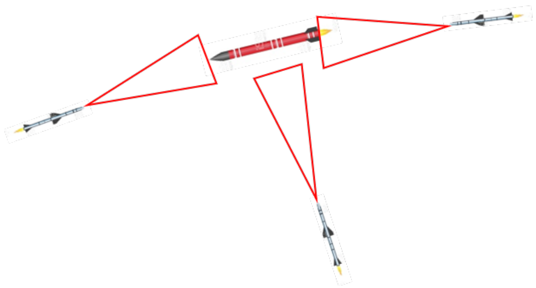
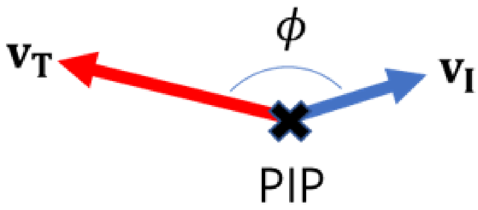


Fig. 1 Figurative representation of Lambert guidance

$$\Delta\mathbf{v}(t) = \mathbf{v_{des}}(t) - \mathbf{v}(t). \tag{3}$$



(a)



(b)

Fig. 2 (a) Possible engagement geometries (b) Geometric interception of performance index

Aligning the thrust vector parallel to $\Delta\mathbf{v}$ is the solution of the Lambert guidance law. Then, under appropriate conditions, $\mathbf{v}$ eventually converges to $\mathbf{v_{des}}$ and the interceptor reaches $\mathbf{R}$ in time t_{go}. Figurative representation of Lambert guidance is depicted in Fig. 1.

2.2 Engagement Geometry

The terminal guidance time is the key factor for successful ballistic target interception [4]. One important factor determining the terminal guidance time is the engagement geometry. The terminal guidance can be carried out only when the target is in the sight of the on-board sensor. Therefore, the terminal guidance time will be determined by the length of time that the target remains in the sight of the sensor. An interceptor is usually equipped with typical strap-down sensors for terminal guidance. One characteristic of the strap-down sensor is that it has a very narrow sight compared to gimbal-type sensors.

When an interceptor approaches the PIP, the engagement geometry can be categorized into three categories as shown in Fig. 2 (a). The first is that the interceptor approaches the target from behind. However, in most cases, the ballistic missile is much faster than the interceptor. Hence, it is physically impossible to approach the target. The left two cases are in which the interceptor is approaching the target from the side, or from the front. The latter is preferable to the former because of terminal guidance time. When the interceptor approaches the target from the side, the target is moving across the sight of the sensor. In contrast, when the interceptor approaches the target from the front, the target is moving through the direction of the sensor. Because the sight of the sensor is very narrow, when the interceptor is approaching the target from the side, it has a very short time for terminal guidance. However, when the interceptor is approaching the target from the front, it has much more time for terminal guidance compared to the side-approaching case. Thus, the front-approaching engagement geometry is the most preferable. Such an engagement geometry can be obtained by minimizing the cosine of the angle that is determined by terminal velocity vectors of the target and the interceptor as Fig. 2 (b). The performance index, J, is defined as follows.

$$J = \frac{\mathbf{v_I}^T(t_f)\mathbf{v_T}(t_f)}{||\mathbf{v_I}(t_f)||_2||\mathbf{v_T}(t_f)||_2} \tag{4}$$

where $\mathbf{v_I}(t_f)$ and $\mathbf{v_T}(t_f)$ denote the velocity of the interceptor and the target at the PIP, respectively.

3 PIP Selection Algorithm

The PIP selection algorithm is constituted by two steps. The first step is to estimate initial PIP location, $\mathbf{PIP}(\mathbf{0})$. The second step is feedback step to update the PIP location, $\mathbf{PIP}(\mathbf{t})$, using $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$.

3.1 PIP(0) Selection

In initial guess phase, the algorithm samples N points on the predicted orbit and determine $\mathbf{PIP}(0)$ by one of the sampled points that minimizes the performance index, J, defined in Eq. (4). The reference of the collision angle was determined from the trajectories of the Lambert problem. At first, time of flight of a target, $T(0)$, is calculated from position, $\mathbf{R}(0,0)$, and velocity, $\mathbf{V}(0,0)$ [13]. Then, for large enough integer N, solve $(\mathbf{R}(0,\frac{k}{N}T(0)),\mathbf{r},\frac{k}{N}T(0))$ Lambert problem for $k=1,\cdots,N$. Finally, calculate $\mathbf{PIP(0)}$ and $t_{go}(0)$. To this end, define θ_k and k^* as follows.

$$\theta_k := arccos(\frac{\mathbf{R}^T(0,\frac{k}{N}T(0))\mathbf{v_k}}{||\mathbf{R}(0,\frac{k}{N}T(0))||_2||\mathbf{v_k}||_2}) \tag{5}$$

$$k^* := \underset{k}{\operatorname{argmin}}\{cos\theta_k\} \tag{6}$$

Then, $\mathbf{PIP(0)}$ and $t_{go}(0)$ is calculated as follows.

$$\mathbf{PIP(0)} = \mathbf{R}(0,\frac{k^*}{N}T(0)) \tag{7}$$

$$t_{go}(0) = \frac{k^*}{N}T(0) \tag{8}$$

3.2 PIP$(\mathbf{t})$ Selection

In the update phase, the $\mathbf{PIP}(t)$ was determined as a point on a predicted trajectory, which is closest to the initial guess $\mathbf{PIP(0)}$. By using this simple algorithm, computational load is decreased. Time-to-go was determined as corresponding predicted time-to-go until the target ballistic missile reaches the $\mathbf{PIP(t)}$. At first, calculate target orbit, $\mathbf{R}(t,\tau)$ and $\mathbf{V}(t,\tau)$, from $\mathbf{R}(t,0)$ and $\mathbf{V}(t,0)$ [13]. After that, define τ^* as follows.

$$\tau^* = \underset{\tau}{\operatorname{argmin}}\{||\mathbf{R}(t,\tau)-\mathbf{PIP(0)}||_2\} \tag{9}$$

Then, $\mathbf{PIP(t)}$ and $t_{go}(t)$ is calculated as follows.

$$\mathbf{PIP}(t) = \mathbf{R}(t,\tau^*) \tag{10}$$

$$t_{go}(t) = \tau^* \tag{11}$$

There are two points that comments should be made for the guidance law proposed in this section. The guidance law is constructed with an assumption that the trajectory of the Lambert guidance is similar to the trajectory of the Lambert problem. It is questionable whether this assumption is valid, but it is a reasonable assumption. To arrive at a PIP in time using Lambert guidance, the velocity-to-be-gained must not be large. Hence, for a PIP that can be arrived at, it is reasonable to assume that the trajectories obtained by the guidance law and solving the Lambert problem are similar.

4 Numerical Simulation

4.1 Simulation Setting

An interceptor and a target ballistic missile is modeled as point mass in a engagement plane. Inverse-square gravitational field is considered and aerodynamic force is neglected. Dynamic equations of the interceptor is as follows.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ \frac{-\mu x}{(x+y)^{\frac{3}{2}}} + \frac{T(t)}{m(t)} cos\theta \\ \frac{-\mu y}{(x+y)^{\frac{3}{2}}} + \frac{T(t)}{m(t)} sin\theta \end{bmatrix} \tag{12}$$

A detailed model of the interceptor is given in Tab. 1. The target ballistic missile is influenced only by gravity that means the target is free-falling. At $t = 0$, the ballistic missile is assumed to have an initial position $(0, 6750)km$ and initial velocity $(2, 0)km/s$. Interceptor launch site is assumed to be at the Earth surface, with 1 degree closer to the ballistic missile's impact point. Such an environment is pictorially described in Fig. 3. The numerical simulation is conducted within MATLAB R2020b.

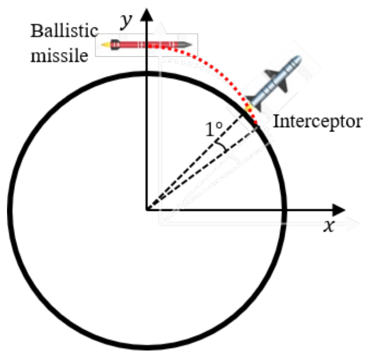


Fig. 3 Graphical description of simulation environment

Table 1 Interceptor specification

Total mass [ton]	Structure mass [kg]	I_{sp} [s]	Thrust [tonf]
4.5	500	300	13

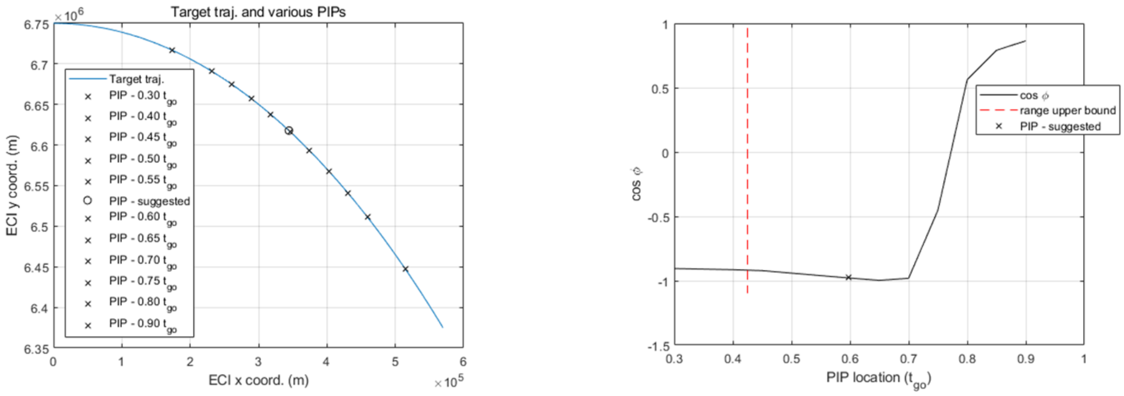
4.2 Engagement Geometry

To compare the engagement geometry with other PIPs and the suggested PIP, the performance index defined in Eq. (4) was compared. Locations of the PIPs are chosen as points on set $\{\mathbf{R}(0,t) : t \in (0, T(0))\}$. PIPs for comparing with the suggested PIP are arbitrarily chosen and all the PIPs are depicted in Fig. 4 (a). The performance indices of Lambert guidance for such PIPs represented as t_{go} are given in Fig. 4 (b). In Fig. 4 (b), the solid line represents the graph of the performance index for various PIP locations. The red line represents the boundary of t_{go} to be able to intercept the target ballistic missile. PIPs on the left side of the boundary are not reachable on time while all other PIPs are reachable and the interceptions are successful. As mentioned in Sec. 2.2, the smaller the performance index, the preferable engagement geometry is. It is shown that the proposed algorithm provides a near-optimal engagement geometry for interception. The trajectory of the Lambert guidance for some PIPs is given in Fig. 5 (a). The simulation results for this case are summarized in Tab. 2. The $24m$ of the miss distance might seem large, however, by implementing the terminal guidance after lock-on, such an error can be successfully adjusted.

Table 2 Performance index and miss distance without measurement noise

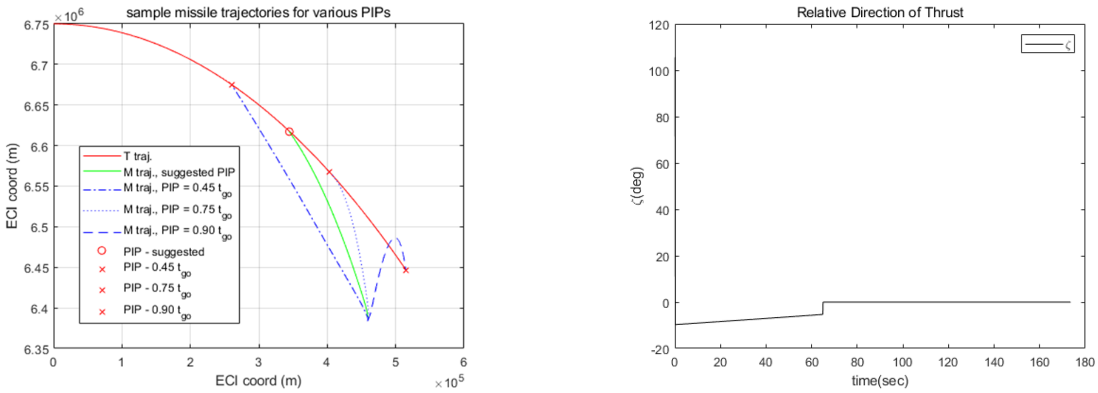
J	Miss distance [m]
0.9674	24

One another important index for evaluating guidance law is the angle between the interceptor's longitudinal body axis and the thrust vector denoted as ζ. Since the interceptor is modeled as point



(a) (b)

Fig. 4 (a) Various PIPs on the trajectory of the target ballistic missile (b) Performance index vs. PIP



(a) (b)

Fig. 5 (a) Trajectories for various PIPs (b) Time history of ζ

mass, ζ can be defined using velocity vector instead of longitudinal body axis as follows:

$$cos\zeta = \frac{\mathbf{v_I}^T\mathbf{T}}{||\mathbf{v_I}||_2||\mathbf{T}||_2} \tag{13}$$

where $\mathbf{v_I}$ and $\mathbf{T}$ denotes the interceptor's velocity vector and the thrust vector, respectively. It is desirable to keep ζ as small as possible because of two reasons. The first reason is that there is a limitation for thrust vectoring for the interceptor. The bigger the angle ζ is, the more risk of actuator saturation. The second reason is that using the thrust in thrust in lateral direction does not contribute to the speed increase of the interceptor. Therefore, it is desirable to keep ζ as small as possible. The time history of ζ for suggested algorithm is given in Fig. 5 (b). It is shown that ζ remains small.

4.3 Robustness Against Measurement Noise

In this section, numerical simulation is conducted to evaluate the robustness of the feedback scheme of the algorithm, against the measurement noise. The measurements of the ballistic missile's position and velocity are contaminated by Gaussian noise. In detail, the cartesian components of the position measurements are contaminated by Gaussian noise with variance $10m$ and the cartesian components of the velocity measurements are contaminated by Gaussian noise with variance $10m/s$. To compare the effect of feedback, the open-loop case using $\mathbf{PIP}(0)$ as PIP is given.

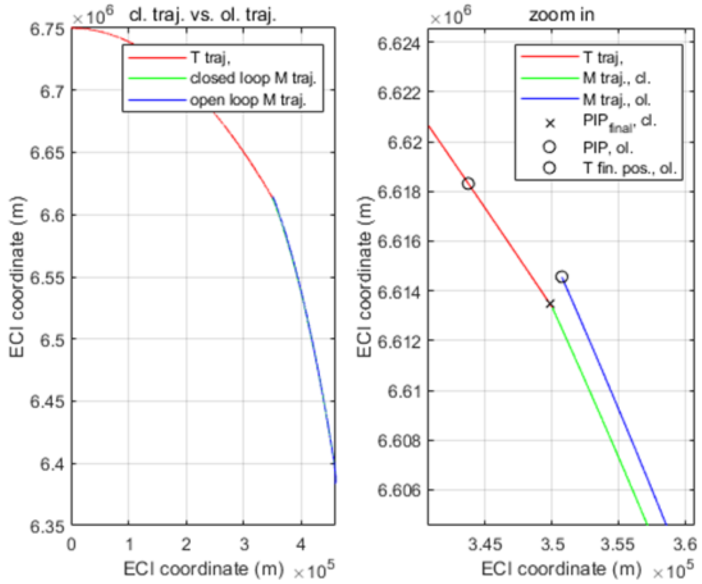


Fig. 6 Trajectories of the closed loop and open loop case

The trajectories of the closed loop case and the open loop case are given in Fig. 6. In Fig. 6, the red line represents the trajectory of the target ballistic missile, the green line represents the trajectory of the interceptor for the closed-loop case, and the blue line represents the trajectory of the interceptor for the open-loop case. The right graph is the zoomed trajectory near the PIP of the left graph. O marker of the blue line represents the PIP of the open-loop case, i.e. $\mathbf{PIP}(0)$, and the O marker of the red line is the actual position of the ballistic missile when the interceptor of the open-loop case approached its PIP. The X marker represents the PIP of the closed loop-case. As can be seen in Fig. 6, the interception was successful in the closed-loop case, but the interception was unsuccessful for the open-loop case. The performance index and miss distance is summarized in Tab. 3. Miss distance of closed loop is in a range that can be adjusted through terminal guidance, while miss distance of open loop is too large to be adjusted. The interceptor of the open-loop case had successfully arrived at its PIP but its PIP was quite far from the actual location of the ballistic missile. However, the interceptor of the closed-loop case continually adjusted its PIP and successfully intercepted the ballistic missile. From this result, one can check that the proposed algorithm's feedback scheme is useful for adjusting the measurement noise of the ballistic missile's state measurement.

Table 3 Performance index and miss distance with measurement noise

	J	Miss distance [m]
Closed loop	0.9689	57
Open loop	0.9671	2,110

5 Conclusion

In this paper, a PIP selection algorithm for Lambert guidance considering engagement geometry is proposed. To evaluate preferred engagement geometry, favorable engagement geometry is discussed and the performance index is introduced. The algorithm reduces computational load by solving the Lambert problem using equations of orbit. The performance of the proposed algorithm is demonstrated by numerical simulation which shows that the proposed algorithm provides near-optimal engagement geometry in the perspective of the performance index and is robust to measurement noise. The guidance law was designed under the assumption that the trajectories generated by the Lambert problem and the Lambert guidance are similar. There are some future works. One is to analyze the criterion of the magnitude of the velocity-to-be-gained to guarantee the trajectories generated by Lambert problem and Lambert guidance similar. Another is to reduce miss distance with more various uncertainties.

Acknowledgments

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