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A Nonlinear Trigonometric Series Parameterization Approach for Smooth Trajectory Generation

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ABSTRACT

This paper introduces a parameterization method for generating trajectories with smooth control commands based on a trigonometric series, where the formulation is nonlinear with respect to the series coefficients. The proposed method aims to achieve a steadier behavior when a path constraint is temporarily active, which is often encountered with e.g., a minimum-time performance index. To this end, first, the control derivatives are parameterized instead of the control variables. Furthermore, the newly developed formulation includes a filtering function depending on the original trigonometric series. This function reduces the series value when it is within a specifiable neighborhood of zero. The performance of the proposed method is tested in two numerical benchmark problems in which a minimum-time and a minimum-effort cost function are considered, respectively.

Keywords: Trajectory Optimization; Trigonometric Series; Control Parameterization; Optimal Control

1 Introduction

Generating trajectories is an essential practice in aerospace engineering. Depending on whether the dynamics of the model are taken into consideration in the process, trajectory generation methods can be categorized differently: One class of methods considers only kinematics and constructs trajectories directly using a combination of lines, arcs, or/and clothoids, where the curvature behavior can be constrained accordingly [1–4]. Among the solutions utilizing geometric components, Dubins path determines the minimum-time trajectory for vehicles moving at a constant speed with a minimum turning radius. The theory is well established [5] and still plays an important role in trajectory generation these days [6–8]. On the other hand, accounting for the dynamics of the aircraft, trajectories are also generated via numerically solving optimal control problems [9, 10]. In this regard, an optimal trajectory is searched in a way that a performance index is minimized, and it is found effective for many constrained problems [11–14].

This paper deals with the smoothness of a trajectory that is quantified by the number of the existing continuous derivatives. The smoothness is often only guaranteed to a specific order when the trajectory is generated using the geometric components or optimal control methods with a fully discretized control grid. Modern trajectory controllers, however, utilize high-order derivatives as well [15–17]. Thus, they pose requirements on the smoothness of the trajectory. In practice, these discontinuities usually undesirably affect the performance, as most processes are continuous.

For generating smooth trajectories with infinitely differentiable controls, a trigonometric series-based parameterization method has been proposed [18], which is referred to as the original form throughout this paper. In Ref. [19], it has been applied to a flight level changes problem to provide smooth control commands as functions of the climb distance. In Ref. [20], the periodicity of controls is guaranteed by construction, through leveraging the periodicity of the trigonometric functions for the dynamic soaring application. A hierarchical structure based on the trigonometric series has been developed to enable a seamless transition connecting two steady flight conditions with strictly zero derivatives [21]. The trigonometric series parameterization is particularly suitable for a transition phase where the controls vary, but it has a drawback: The parameterized variable cannot stay temporarily constant, e.g., when a path constraint is active. Studies reveal that this issue may be insignificant when very slowly varying controls are generated, which is feasible even for a low-order series [22]. It can still be improved, as oscillations result in additional control effort and potentially compromise the objective. In this paper, a new solution based on the trigonometric series is proposed for alleviating this issue. This new formulation is nonlinear with respect to the series coefficients. It focuses on parameterizing the control derivatives because a control can be of an arbitrary value when it stays constant, while the derivative is always zero. Therefore, the proposed method aspires to further reduce the derivative to close to zero when the control is slowly varying in the original form. This is achieved by introducing a smooth filtering function that only takes effect in a neighborhood of zero. The new form still guarantees infinite differentiability of the control commands, but is expected to generate a steadier control profile when a path constraint is active.

The rest of the paper is organized as follows: Section 2 introduces the preliminaries and the main results of this paper. For evaluating the effectiveness of the proposed method, two applications are shown, the first of which is presented in Section 3, where a minimum-time problem of a vehicle moving in a horizontal plane is considered, and the second of which is described in Section 4 and is concerned with the same formulation but with a minimum-effort objective. The conclusions are drawn in Section 5.

2 Parameterization for Smooth Trajectory Generation

A trajectory optimization problem is very often of the following form:

$$\begin{aligned} \underset{\boldsymbol{u}(t),\boldsymbol{\theta}}{\text{minimize}} \quad & J\left(\boldsymbol{x}\left(t\right),\boldsymbol{u}\left(t\right),\boldsymbol{\theta},t\right) \\ \text{subject to} \quad & \dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}\left(t\right),\boldsymbol{u}\left(t\right),\boldsymbol{\theta},t\right) \\ & \boldsymbol{g}\left(\boldsymbol{x}\left(t\right),\boldsymbol{u}\left(t\right),\boldsymbol{\theta},t\right) \leq 0 \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^m$, $\boldsymbol{u} \in \mathbb{R}^n$, and $\boldsymbol{\theta} \in \mathbb{R}^p$ are the state, the control, and the parameter vector, respectively. Furthermore, $\boldsymbol{f}$ and $\boldsymbol{g}$ denote the model dynamics and all constraints of the optimization problem, respectively.

The fundamental idea of the proposed method is to reduce the change rate of the control when it is small. The time derivative of the control vector is defined as

$$\dot{\boldsymbol{u}}\left(t\right) = \boldsymbol{v}\left(t\right) \tag{2}$$

Due to the introduction of the control derivative, $\boldsymbol{v}$ becomes a new control variable and $\boldsymbol{u}$ is then a part of the state vector.

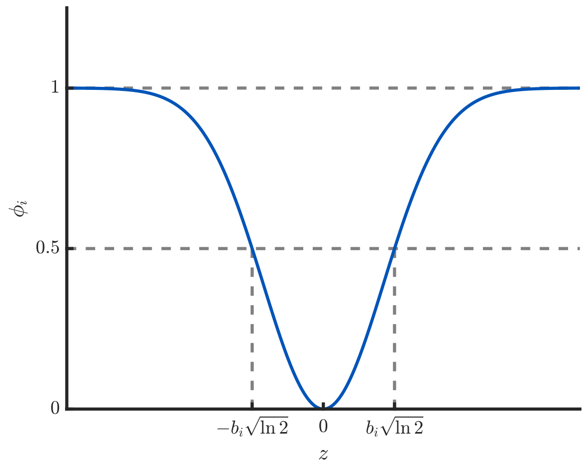

Fig. 1 Schematic representation of the filtering function ϕ

Selectively reducing the control derivative relies on introducing a filtering function, which is formulated based on the Gaussian kernel here.

$$\phi_i(z) = -e^{-(z/b_i)^2} + 1 \tag{3}$$

where the subscript i denotes the i-th control component, and b_i is a tunable parameter. The exponent can be chosen among positive even integers. This paper chooses 2 for sufficient smoothness. For demonstration purposes, the function profile is schematically shown in Fig. 1. The function value of ϕ rapidly decreases from about 1 to zero as z approaches zero in a neighborhood of zero. The value of b_i specifies this neighborhood of zero, and b_i needs to be selected individually for each control component. Moreover, in gradient-based optimization methods, it might be helpful for avoiding invalid numbers (e.g., division by zero) to implement the Gaussian-based filtering function in the following form:

$$\phi_i(z) = -e^{-(z/b_i)^2} + 1 + \varepsilon$$

where $\varepsilon > 0$ is a small constant.

If the original formulation of the trigonometric series parameterization in [18] applies, a control derivative (the symbolism $\tilde{}$ denotes a dummy variable) is expressed in the following form:

$$\begin{aligned}\tilde{v}_i(t) &= (a_0)_i + \sum_{n=1}^{N}\left((a_n)_i \cos\left(\frac{n\pi}{T_f}t\right) + (b_n)_i \sin\left(\frac{n\pi}{T_f}t\right)\right)\\ &= \boldsymbol{s}_N(t)\,\boldsymbol{c}_i, \quad i = 1, 2, \ldots, n\end{aligned} \tag{4}$$

where N is the order of the series and

$$\begin{aligned}\boldsymbol{s}_N(t) = \Bigg[&1, \cos\left(\frac{\pi}{T_f}t\right), \ldots, \cos\left(\frac{N\pi}{T_f}t\right),\\ &\sin\left(\frac{\pi}{T_f}t\right), \ldots, \sin\left(\frac{N\pi}{T_f}t\right)\Bigg]\end{aligned} \tag{5}$$

$$\boldsymbol{c}_i = [(a_0)_i, \ldots, (a_N)_i, (b_1)_i, \ldots, (b_N)_i]^{\mathrm{T}}. \tag{6}$$

Here, $\boldsymbol{c}_i$ is the coefficient vector corresponding to the i-th control variable, which needs to be determined using optimization. The trigonometric series parameterization in Eq. (4) shows that the control derivative is a linear function of $\boldsymbol{c}_i$ and infinitely differentiable with respect to time, i.e., of $\mathcal{C}^{\infty}$. As a remark, the frequencies of the trigonometric functions can be utilized to shape the time behavior of the parameterization [19], and yet this paper sticks to the original form given in [18] for the sake of simplicity.

Next, the main result of this paper is given as a modified version of Eq. (4) utilizing the filtering function as

$$v_i(t) = \phi_i\left(\boldsymbol{s}_N(t)\,\boldsymbol{c}_i\right)\boldsymbol{s}_N(t)\,\boldsymbol{c}_i = \boldsymbol{p}_{Ni}(t,\boldsymbol{c}_i) \tag{7}$$

A comparison with Eq. (4) shows that $v_i(t) = \phi_i(\tilde{v}_i(t))\,\tilde{v}_i(t)$. Then, it is intuitive that ϕ_i plays a role in reducing the absolute value of the derivative when it is close to zero. As ϕ_i is a smooth function, this expression is still of $\mathcal{C}^{\infty}$. It is to be noted that the formulation in Eq. (7) is no longer a linear function of the series coefficients as in Eq. (4) but a nonlinear function. This nonlinear nature may affect some applications or algorithms that require the linearity with respect to the series coefficients, which is featured in the original form.

Therefore, the vector of the control derivatives can be expressed by

$$\boldsymbol{v}(t) = \boldsymbol{P}_N(t,\boldsymbol{c}) \tag{8}$$

where

$$\boldsymbol{P}_N(t,\boldsymbol{c}) = \begin{bmatrix} \boldsymbol{p}_{N1}(t,\boldsymbol{c}_1) & 0 & \dots & 0 \\ 0 & \boldsymbol{p}_{N2}(t,\boldsymbol{c}_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \boldsymbol{p}_{Nm}(t,\boldsymbol{c}_m) \end{bmatrix} \tag{9}$$

$$\boldsymbol{c} = \left[(\boldsymbol{c}_1)^{\mathrm{T}}, (\boldsymbol{c}_2)^{\mathrm{T}}, \dots, (\boldsymbol{c}_m)^{\mathrm{T}}\right]^{\mathrm{T}} \tag{10}$$

Hence, by incorporating the parameterization, the new trajectory optimization problem can be formulated as

$$\begin{aligned} \underset{\boldsymbol{c},\boldsymbol{\theta}}{\text{minimize}} \quad & J(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{\theta},t) \\ \text{subject to} \quad & \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{\theta},t) \\ & \dot{\boldsymbol{u}}(t) = \boldsymbol{v}(t) \\ & \boldsymbol{v}(t) = \boldsymbol{P}_N(t,\boldsymbol{c}) \\ & \boldsymbol{g}(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{\theta},t) \leq 0 \end{aligned} \tag{11}$$

Here, it can be seen that the optimal control formulation in Eq. (11) includes two additional constraints representing the parameterization in comparison to the typical optimal control formulation in Eq. (1). Instead of determining the optimal control histories, the solution to Eq. (11) consists of the series coefficients, $\boldsymbol{c}$. Next, two applications are shown as benchmark problems assessing the effectiveness of the proposed method.

3 Application A: Minimum-time Problem

The first application is a minimum-time problem which can be found in [23, Ch. 3.3]. This application can represent any vehicle (e.g., a car or an unmanned aerial vehicle) moving in one plane.

3.1 Problem formulation

The equations of motion for the vehicle are expressed by

$$\begin{aligned} \dot{x}(t) &= V(t)\cos\chi(t)\,, \\ \dot{y}(t) &= V(t)\sin\chi(t)\,, \\ \dot{V}(t) &= \dot{V}_{\text{cmd}}(t)\,, \\ \dot{\chi}(t) &= \dot{\chi}_{\text{cmd}}(t) \end{aligned} \tag{12}$$

where x and y are the position coordinates and V and χ are the speed and course angle of the vehicle. The vehicle is controlled by the along-track acceleration and the turn rate, i.e., $\dot{V}_{\text{cmd}}$ and $\dot{\chi}_{\text{cmd}}$ on a finite horizon $\left[0, t_f\right]$, where t_f is the final time. The state and control vectors are given as

$$\boldsymbol{x} = [x, y, V, \chi]^{\mathrm{T}} \tag{13}$$

$$\boldsymbol{u} = \left[\dot{V}_{\text{cmd}}, \dot{\chi}_{\text{cmd}}\right]^{\mathrm{T}} \tag{14}$$

Each state variable is subject to boundary constraints, and control variables comply with box constraints. These are given by

$$\begin{aligned} \boldsymbol{x}(0) = \boldsymbol{x}_{\text{initial}},\ \boldsymbol{x}\left(t_f\right) &= \boldsymbol{x}_{\text{terminal}} \\ \boldsymbol{x}_{\min} \le \boldsymbol{x}(t) &\le \boldsymbol{x}_{\max} \\ \boldsymbol{u}_{\min} \le \boldsymbol{u}(t) &\le \boldsymbol{u}_{\max} \end{aligned} \tag{15}$$

Additionally, a non-sliding constraint [24] is included as

$$-\left|a_c\right|_{\max} \le \dot{\chi}_{\text{cmd}}(t) \cdot V(t) \le \left|a_c\right|_{\max} \tag{16}$$

The objective is to minimize the final time, so the cost function is considered as

$$J = t_f \tag{17}$$

3.2 Implementation

This paper presents three solutions, which are generated by the original trigonometric series parameterization in [18] (denoted by “Original”), the proposed new parameterization (denoted by “Proposed”), and a full trapezoidal discretization (denoted by “Optimal”). All three solutions are obtained using FALCON.m, an efficient optimal control tool [23]. FALCON.m enables an automatic transcription of the optimal control problem described in Section 3.1. The trigonometric series parameterizations are introduced as path constraints. Specifically for the proposed method, the model needs to be augmented by introducing control derivatives as in Eq. (11). The reason why the results generated using full trapezoidal discretization is considered “Optimal” is that it avoids the suboptimality caused by the global path constraint representing the parameterization. For consistency, the series orders are both five for the original and the proposed form, respectively.

3.3 Optimization results

The optimized results are shown in this section. The trajectories and the velocity components are shown in Fig. 2. All three of them appear to be very similar. A closer look at the state and control histories depicted in Fig. 3 reveals further details. Using control parameterization methods, the kinks in the state time histories no longer exist, indicating no discontinuity in them. Furthermore, the proposed approach does not produce undesired small oscillations in the velocity time histories as the original method does in roughly the time frame [10, 27] s. This substantiates a notable improvement of the proposed approach,

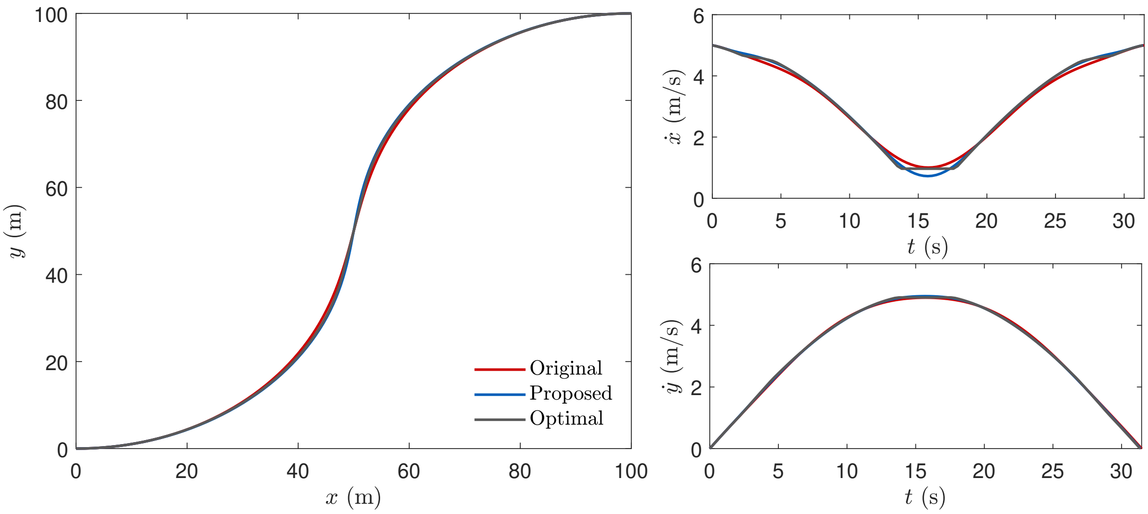

Fig. 2 Application A: Trajectories and velocity components

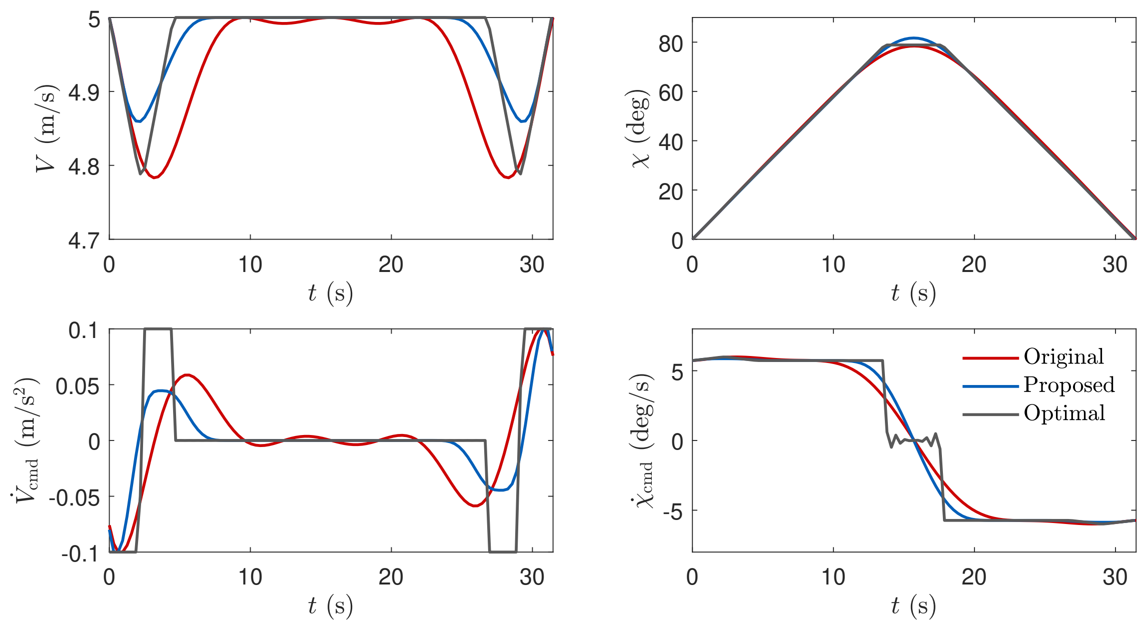

Fig. 3 Application A: State and control histories

which can also be confirmed by the control histories in the same figure. While the optimal acceleration provided by the full discretization shows a bang-bang-like phenomenon, the other two solutions are smooth. In particular, the proposed approach produces an acceleration command that stays temporarily close to zero, which correspondingly leads to a constant speed at the boundary condition. This is one of the main improvements over the original form. This phenomenon holds for the turn rate commands as well – the advantage of the parameterization methods in smoothness is thus clear.

The statistics are displayed in Table 1. In addition to the objective, i.e., the final time t_f, there are two additional quantities, $\int_0^{t_f} \dot{V}_{\rm cmd}^2\,\mathrm{d}t$ and $\int_0^{t_f} \dot{\chi}_{\rm cmd}^2\,\mathrm{d}t$ showing the efforts for speed and course angle control. Concerning the objective, while the sub-optimality of 0.34% is already minor for the original approach, it reduces to 0.10% for the proposed form. It is to be noted that $\int \dot{V}_{\rm cmd}^2\,\mathrm{d}t$ and $\int \dot{\chi}_{\rm cmd}^2\,\mathrm{d}t$ are not optimized, as the cost function solely optimizes the final time. Both parameterization methods achieve less control efforts for the two control variables in comparison to the optimal solution. Comparing with the original

Table 1 Statistics of Application A

Case	Objective [s]	$\int_0^{t_f} \dot{V}_{\text{cmd}}^2\,\mathrm{d}t$ $[\mathrm{m}^2/\mathrm{s}^3]$	$\int_0^{t_f} \dot{\chi}_{\text{cmd}}^2\,\mathrm{d}t$ $[1/\mathrm{s}]$
Original [18]	31.4608	0.0554	0.2602
Proposed	31.3862	0.0339	0.2750
Optimal	31.3535	0.0862	0.2772

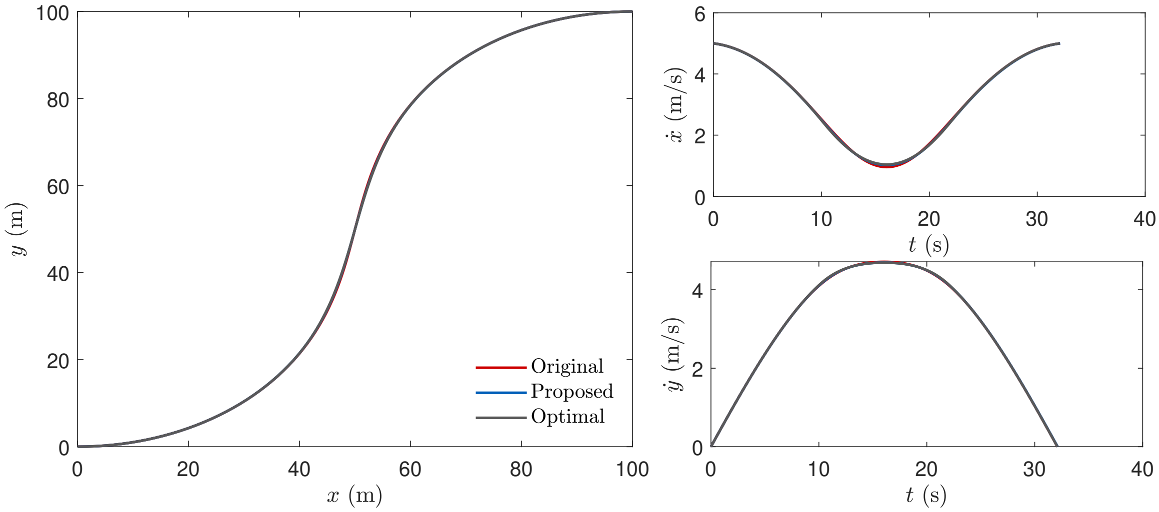

Fig. 4 Application B: Trajectories and velocity components

approach, the proposed method does not excel comprehensively in this regard, but there is an evident reduction of the control effort for the speed, whose constraint has been active for most of the time.

4 Application B: Minimum-effort problem

The second application concerns with the same problem as presented in the previous section, but it has a different objective to minimize the total control effort.

$$J = \int_0^{t_f} \dot{V}_{\text{cmd}}^2\,\mathrm{d}t + \int_0^{t_f} \dot{\chi}_{\text{cmd}}^2\,\mathrm{d}t \tag{18}$$

In general, this performance index generally does not lead to a bang-bang-like structure for the optimal solution. Therefore, this application is investigated to provide another perspective on the proposed nonlinear trigonometric series approach. Again, $N = 5$ is used for both parameterization methods.

The generated trajectories and velocity components are displayed in Fig. 4. While the trajectories shown in Fig. 2 are already similar, the trajectories in Fig. 4 can be barely distinguished, indicating very similar control profiles and performance indexes. The time histories of states and controls are shown in Fig. 5. In addition, the time histories of control derivatives are displayed in this figure. The speed and course angle time histories are quite similar for all three cases. From the time histories of $\dot{\chi}_{\text{cmd}}$, it can still be noticed that the kinks visible at around $t = 10$ s and $t = 21$ s for the optimal solution are eliminated when using parameterization methods. These kinks correspond to the steps shown in $\ddot{\chi}_{\text{cmd}}$ histories, and yet the profiles are similar in magnitude.

The corresponding statistics for the simulation results are given in Table 2. In comparison to the original form, the proposed one achieves a slightly better objective, with a minor increase in the final

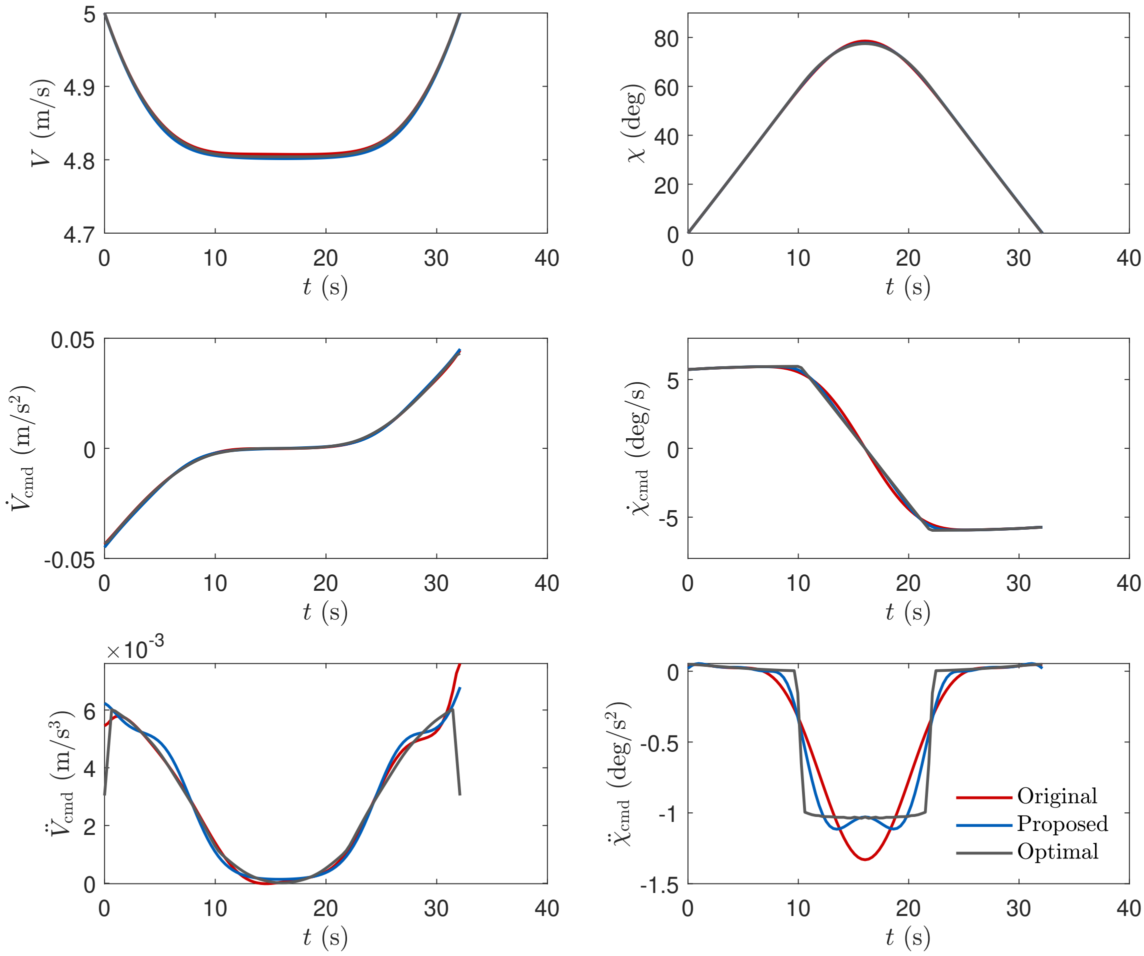

Fig. 5 Application B: State and control histories

Table 2 Statistics of Application B

Case	Objective $\left(\int_0^{t_f} \dot{V}_{\text{cmd}}^2\,\mathrm{d}t + \int_0^{t_f} \dot{\chi}_{\text{cmd}}^2\,\mathrm{d}t\right)$ [m^2/s^3]	Final time [s]
Original [18]	0.2691 (0.0103 + 0.2588)	32.1054
Proposed	0.2685 (0.0109 + 0.2575)	32.1194
Optimal	0.2680 (0.0105 + 0.2576)	32.0989

time. However, the differences across all three setups are not significant enough to draw any conclusion on the performance superiority.

The simulations in this section suggest that the proposed form still accomplishes the goal of generating smooth controls for the minimum-effort problem, but there is no fundamental difference to the original form. Given the additional nonlinearity, the proposed form might not be preferred.

5 Conclusions

Based on the trigonometric series, this paper developed a new parameterization method for generating smooth trajectories. The parameterization is nonlinear with respect to the series coefficient because of the introduction of the filtering function. The filtering function is designed to reduce the oscillations that are inevitable in the original formulation given in [18]. Performance evaluation has been done using a generic optimal control software for a simple optimal control problem with different objective functions. It exhibits clear advantages in terms of smoothness and optimality when the optimal controls have a bang-bang-like structure in a minimum-time problem, while its performance is not significantly different for a minimum-effort problem. Hence, as it has been investigated so far, although the proposed approach does not feature universally superior performance over the original formulation for all performance indexes, it can provide a notable improvement and is worth trying when path constraints are expected to be active for significant time intervals. Future works include further studies of the proposed form on other aerospace applications.

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