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Robust Climate Optimal Aircraft Trajectory Planning Considering Uncertainty in Weather Forecast

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ABSTRACT

The air traffic management system is to deal with efficiency, safety, capacity, and environmental impacts due to the strong growth rate of air traffic over the recent years. Aviation contributes to climate change through the emission of carbon dioxides (CO_2) and aviation non-CO_2 effects, and the corresponding climate impact is anticipated to increase critically. As the non-CO_2 climate effects are highly dependent on the chemical and meteorological background conditions, there is a potential to reduce their associated climate impacts by planning climate-aware trajectories. To identify the non-CO_2 climate-sensitive regions, in this study, algorithmic climate change functions requiring some weather-related variables to quantify the climate impacts are employed. The uncertainty in the standard weather forecast can cause unreliable determination of climate-sensitive regions and aircraft dynamics and, consequently, unreliable trajectories. To this end, the ensemble prediction system is employed to characterize the uncertainty in the weather forecast. Then, robust aircraft trajectory optimization problem is formulated, capable of providing robust climate optimal trajectories taking into account uncertainty in meteorological variables. The optimization is solved for different sets of user-defined parameters, penalizing the simple operating cost and climate impact.

Keywords: Climate change; Aircraft trajectory optimization; Algorithmic climate change functions; Robust optimization.

Nomenclature

ATR	=	average temperature response over 20 years
SOC	=	simple operating cost
RF	=	radiative forcing
aCCF	=	algorithmic climate change function
NLP	=	nonlinear programming
ATO	=	aircraft trajectory optimization

1 Introduction

The emission of carbon dioxides (CO_2) and non-CO_2 effects due to aviation are contributing considerably to global warming [1]. The non-CO_2 effects include nitrogen oxides (NO_x), causing changes in ozone and methane concentrations, water vapor (H_2O), hydrocarbons (HC), carbon monoxide (CO), sulfur oxides (SO_x), and increased cloudiness due to contrail formation [1, 2]. These emissions effectuate changes in the radiative balance of the Earth's climate system. Such radiative impact has the potential to evolve the atmosphere temperature by transforming the climate system into a new equilibrium state [3].

Nowadays, aviation is responsible for only about 5% of the anthropogenic climate impact [1, 3, 4]. However, by taking into account the growth rate of the global air transport industry, the contribution is estimated to increase critically. The aviation industry experienced significant growth over the period of 1960-2018 in both activity, and climate impacts [5]. Following the recent report on aviation's growth and its corresponding anthropogenic climate impacts [5], the revenue passenger kilometers and the emission of CO_2 have been increased from 109 to 8269 billion km/year and 6.8 to 1034 Teragrams CO_2/year, respectively, in which the sharpest trend is referred to the period 2013-2018. The global air transport industry is predicted to grow at 4.4% yearly in the next 20 years [6]. It is worth mentioning that the COVID-19 pandemic has affected the expected growth rate and thus increased the uncertainty on the expected rate [7, 8]. Such strong growth in the global air transport industry outweighs the predicted annual fuel efficiency enhancement rate of 1–2% [9]. Thus, the aim of developing eco-efficient aviation becomes increasingly challenging.

The net aviation effective radiative forcing (ERF) for 2018 is estimated to be 100.9mW/m^2 (5-95% likelihood range of (55,145)) [5]. The non-CO_2 species contribute to 66% (66.6mW/m^2) of net aviation ERF with 57.4mW/m^2 and 17.5mW/m^2 caused by contrail cirrus and NO_x emission, respectively, as the main contributors, while this number is 33% (34.3mW/m^2) for CO_2 emission [5]. Mitigating the climate impact of CO_2 emission requires the development of more efficient aircraft and the use of alternative fuels or propulsion. In contrast, the non-CO_2 emissions which are responsible for approximately two-thirds of aviation ERF (i.e., $\approx$ 66%) [5, 10] vary highly with geographic location, altitude, and time of the emission [11, 12]. Taking into account such information concerning altitude and location dependencies of non-CO_2 aviation climate impacts in the aircraft path planning can potentially improve the net climate effects.

To mitigate the climate impact of non-CO_2 emissions within aircraft trajectory optimization (ATO), information on climate-sensitive regions needs to be available. Various approaches have been proposed to directly or indirectly consider the climate impacts of aviation within aircraft trajectory planning, including reducing emissions [13], avoiding persistent contrail formation areas [14], reducing RF [15] and reducing global warming potential (GWP) [11] (interested readers are referred to the survey on different ways of considering climate impacts in aircraft path planning [16]). In this study, we use the algorithmic climate change (aCCFs) functions developed within EU-project ATM4E [17, 18]. The aCCFs quantify the climate impact of ozone and methane resulting from NO_x emissions, water vapor emissions, and persistent contrail formation from some meteorological variables. These functions are computationally fast, thus, suitable to be employed by aircraft trajectory optimization techniques.

Numerous optimization approaches have been proposed in the literature to find climate-optimal trajectories (see [16] for a classification of these techniques). However, here, we focus mainly on those studies that employed aCCFs. Mainly two approaches have been considered: meta-heuristic and direct optimal control techniques. The mathematical basis of the optimization using meta-heuristic to obtain climate optimal trajectories can be found in [19]. In this study, Yamashita et al. developed the first version of air traffic simulator AirTraf, as a submodel of the European Center HAMburg general circulation model (ECHAM) and Modular Earth Submodel System (MESSy) Atmospheric Chemistry (EMAC) model. In the later studies, [3, 18], Yamashita et al. developed the second version of air traffic simulator AirTraf (AirTraf2.0) to include more routing options such as climate optimal one based on aCCFs. The other trajectory optimization technique is the direct optimal control approach. This method has been numerously employed to determine climate optimal trajectories based on different elements representing climate impacts (e.g., CCF [20], emission-based climate change functions (eCCF) [21], and PCFA for contrails [22]). As for aCCF, in [23], the optimization using trajectory optimization module (TOM) was performed considering 13,000 intra-European flights. TOM uses General Pseudospectral Optimization Package (GPOPS) through its MATLAB interface for transcription and IPOPT (interior-point NLP solver) for solving the resulting nonlinear programming (NLP). The mitigation potentials reported in the mentioned studies were promising. For instance, it was reported in [23] that a 50% reduction in climate impact could be achieved with only a 0.75% increase in fuel consumption.

In order to calculate aCCF regarding each emission, some weather-related variables, including temperature, potential vorticity, geopotential, relative humidity, and outgoing longwave radiation, are needed. These variables are obtained from standard weather forecasts, and due to many factors, including the incomplete knowledge of the state of the atmosphere, nonlinear, sometimes chaotic dynamics, they are uncertain. Such a presence of uncertainty in the calculation of aCCFs and also aircraft dynamics (due to the uncertainty in wind and temperature), if not considered, can lead to unreliable aircraft path planning. To the best of our knowledge, all the studies regarding climate-optimized aircraft trajectory planning reported in the literature (not limited to those considered aCCFs) are performed in deterministic manners. To manage and integrate the effects of meteorological uncertainties in aircraft path planning, reliable weather forecasts that are capable of representing possible deviations in weather conditions are required. To this end, probabilistic weather forecasting has been introduced to represent uncertainty in weather forecasts [24]. Among different approaches, the ensemble prediction system (EPS) is known as the most promising one, generating n individual forecasts [25]. Each forecast indicates a possible realization of weather variables, and in reality, the actual weather condition is expected to lie in the predicted dispersion of weather variables obtained using ensemble members.

This study, for the first time, addresses robust aircraft trajectory optimization considering climate impacts. The uncertainty in weather forecast is characterized using the ensemble prediction system, and the aviation's climate impacts are quantified using aCCFs. The concept of robustness that we refer to is the determination of aircraft trajectory considering the performance of all possible realizations of meteorological variables provided within the EPS weather forecast. In other words, instead of planning a trajectory based on one forecast in a deterministic manner, we aim to determine a trajectory that is optimal considering the overall performance obtained from ensemble forecasts. In this respect, from the operational point of view, the optimized trajectory is tracked as determined, then the performance in terms of variables such as fuel burn, arrival time, and climate impacts are impacted by uncertainty. This concept represents a more realistic representation of actual aircraft operations than the deterministic one. The proposed robust optimal control problem is converted to a higher-dimensional deterministic optimal control problem using the approach proposed in [26]. Then, the direct optimal control method is employed to solve the optimization problem.

The rest of the manuscript is arranged as follows. The problem statement is provided in Section 2. Section 3 presents the proposed robust optimization technique. The mitigation potential of our work in reducing aviation's climate impacts is explored in Section 4, and some concluding remarks close the paper in Section 5.

2 Problem statement

In this section, some required preliminaries to formulate and state the robust aircraft trajectory optimization considering the climate impacts are briefly presented. We first present the dynamical model of aircraft, which is considered as the dynamical constraint of the optimal control problem (Section (2.1)).

Then, the modeling of climate impacts using aCCF is presented in Section 2.2, which is included in the performance index of the robust optimal control problem formulation in Section 2.3)

2.1 Aircraft dynamical model

The trajectory of the aircraft needed to evaluate the performance of the flight in the optimization problem is to be calculated from the aircraft dynamics. As usually considered within air traffic management studies, the state of the aircraft is assumed to evolve according to the point-mass model, where the aerodynamic and propulsive performance of the aircraft is given by the BADA model [27, 28]. In this case, the equations of motion are received as follows

$$\begin{bmatrix} \dot{\phi} \\ \dot{\lambda} \\ \dot{v}_{tas} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \big(v_{tas}\cos\chi + w_y\big)\big(R_M(\phi)+h\big)^{-1} \\ \big(v_{tas}\sin\chi + w_x\big)\big((R_N(\phi)+h)\cos\phi\big)^{-1} \\ \big(\mathrm{T}(C_T) - D(C_L)\big)m^{-1} \\ -f_c(C_T) \end{bmatrix} \tag{1}$$

where λ is the longitude, ϕ is the latitude, h is the altitude, v_{tas} is the true airspeed, m is the mass, C_T is the thrust coefficient, and χ is the heading. In addition, (w_x, w_y) are the components of the wind, R_M and R_N are the Earth's ellipsoid radii of curvature in the meridian and the prime vertical respectively, T and D are the magnitudes of the thrust and drag forces, and f_c is the fuel burn rate. In line with previous arguments, by choosing directly the heading χ as a control, instead of the bank angle, we are disregarding the aircraft turn dynamics, which would require more involved computations while in principle having little to no effect on the solution accuracy. It is worth mentioning that the dynamic of aircraft relies on some meteorological variables such as components of wind and temperature. As these variables are obtained from the standard weather forecast, they are not exact. The existence of uncertainty in aircraft dynamics directly affects aircraft trajectories.

2.2 Climate impact quantified using aCCF

The climate impact of aircraft operations is modeled according to the published findings of the EU-Project ATM4E [17], which include the so-called aCCF for the non-CO_2 and CO_2 agents, measuring global climate impact using the average temperature response integrated over a 20 years (ATR20) [3]. The aCCFs quantify the climate impacts of ozone and methane resulting from NO_x emissions, water vapor emissions, and persistent contrail formation from some meteorological variables obtained from standard weather forecasts in a computationally fast manner. As the weather forecast is not exact, there exist uncertainties in the climate impacts quantified using aCCFs. In this paper, we implement the version of aCCFs reported in [3].

2.3 Problem formulation: Robust ATO considering climate impacts

The objectives of the dynamical optimization problem are interpreted as mathematical expressions and included in the objective function to be minimized (or maximized). In the context of optimal control theory, the following general form of the cost functional (or performance index) is considered for optimizing problems with uncertainty [26]:

$$J = \mathbb{E}\Big\{\mathcal{M}\big(t_0, \mathbf{x}(t_0), t_f, \mathbf{x}(t_f)\big) + \int_{t_0}^{t_f} \mathcal{L}\big(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t), \zeta\big)\mathrm{d}t\Big\} \tag{2}$$

where $\mathcal{M} : \mathbb{R} \times \mathbb{R}^{n_x} \times \mathbb{R} \times \mathbb{R}^{n_x} \rightarrow \mathbb{R}$ and $\mathcal{L} : \mathbb{R} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z}, \mathbb{R}^{n_\zeta} \rightarrow \mathbb{R}$ are the Mayer and Lagrange terms called terminal cost and cost-to-go, respectively. In Eq. (2), n_x, n_u and n_z represent the dimension of state vector $\mathbf{x}(\cdot)$, control vector $\mathbf{u}(\cdot)$ and vector of algebraic variables $\mathbf{z}(\cdot)$, respectively.

ζ denotes the vector of uncertain variables assumed to have a known probability distribution functions. The uncertain variables perturb the state, control and algebraic variables through the system dynamics as $\dot{\mathbf{x}}(t) = \mathbf{f}\big(t, \mathbf{x}(t), \mathbf{u}(t), \mathbf{z}(t), \zeta\big)$ (e.g., aircraft dynamical model in this case). The nonlinear function $\mathbf{f}(\cdot)$ is assumed to be a measurable function in ζ (see [26] for more information). Based on the nature of the optimization problem, several constraints as the type of equality and inequality path and terminal constraints may be considered.

To determine climate optimal trajectories, climate-sensitive regions need to be defined and included in the cost functional of the optimization problem. Here, we employ the version of aCCFs reported in [3] to quantify the climate impacts of each agent mathematically as a function of geographical locations, altitude, weather conditions, and time (provided in Section 2.2). In addition to the climate impacts, fuel burn and flight time are crucial factors that are usually considered as objectives to be optimized. All in all, the following performance index is defined for our purpose:

$$
\begin{aligned}
&J = \text{Simple Operational Cost (SOC)} + \text{Average Temperature Response over 20 years (ATR)} \\
&\text{SOC} = \mathbb{E}\left\{ \text{CI}\left[\text{C}_t \cdot \underbrace{[t_f - t_0]}_{\text{flight time}} + \text{C}_f \cdot \underbrace{[m(t_0) - m(t_f)]}_{\text{fuel burnt}} \right] \right\} \\
&\text{ATR} = \mathbb{E}\left\{ \underbrace{\text{EI} \int_{t_0}^{t_f} \sum_{i=1}^{5} \text{ATR}_i\big(t, \mathbf{x}(t), \mathbf{u}(t)\big) \mathrm{d}t}_{\text{ATR of CO}_2\text{ and non-CO}_2\text{ emissions}} \right\}
\end{aligned}
\tag{3}
$$

for $i \in \{\text{CH}_4, \text{Cont.}, \text{O}_3, \text{H}_2\text{O}, \text{CO}_2\}$ as

$$
\begin{aligned}
\text{ATR}_{\text{O}_3}(t, \mathbf{x}) &= 10^{-3} \times \text{aCCF}_{\text{O}_3}\big(t, \mathbf{x}\big) \cdot \dot{m}_{nox}(t) \\
\text{ATR}_{\text{CH}_4}(t, \mathbf{x}) &= 10^{-3} \times \text{aCCF}_{\text{CH}_4}\big(t, \mathbf{x}\big) \cdot \dot{m}_{nox}(t) \\
\text{ATR}_{\text{Cont.}}(t, \mathbf{x}) &= 10^{-3} \times \text{aCCF}_{\text{Cont.}}(t, \mathbf{x}) \cdot v(t) \\
\text{ATR}_{\text{H}_2\text{O}}(t, \mathbf{x}) &= \text{aCCF}_{\text{H}_2\text{O}}(t, \mathbf{x}) \cdot \dot{m}(t) \\
\text{ATR}_{\text{CO}_2}(t, \mathbf{x}) &= \text{aCCF}_{\text{CO}_2} \cdot \dot{m}(t)
\end{aligned}
\tag{4}
$$

where $\dot{m}_{nox}(t) = f_c(C_T) \cdot \text{EI}_{\text{NO}_\text{x}}$. Such a definition of cost function allows considering both cost and climate impact simultaneously as an objective to be minimized. However, there exists a trade-off between these two objectives determined by the selection of cost and environmental indices, i.e., CI and EI, respectively. The parameters C_t and C_f are used to allow considering different explanations of costs such as operational cost in USD or Euros. In this work, we assign $\text{C}_t = 0.75$[USD/s], $\text{C}_f = 0.51$[USD/kg] to express cost in USD [3].

The reason that the expected operator is used in the performance index is due to the uncertain variables considered as objectives. Uncertainties in flight time and fuel burn are related to the uncertainty in meteorological variables used in the aircraft dynamical model. In addition, the aCCFs are also uncertain due to the dependency on meteorological variables. For instance, consider the calculation of aCCFs with the ensemble forecast. In this case, for an EPS forecast containing 50 ensemble members, one can calculate 50 different aCCFs. An approach to analyzing the degree of uncertainty is to check the variability. In Section 4, for a specific case study, we will analyze the variability of ensemble members and the calculated aCCFs. In addition to the dynamical constraint and objective function, there exist some inequality and equality boundary and path constraints which will be provided in the next section.

3 Robust climate optimal trajectory optimization

Based on the presented robust optimal control problem formulation, the aim is to determine a control policy (in the form of closed-loop or open-loop) such that the cost functional Eq. (2) gets minimized and operational and physical constraints are satisfied. For each realization of uncertain parameters (i.e., $\zeta(\delta = \delta_0)$), we can find the optimal solution in a deterministic manner using well-known optimal control approaches such as the direct method. To this end, one alternative that has been proposed in the literature to cope with such dynamical optimization problems is to discretize uncertain parameters (i.e., ζ_k for $k = 1, \cdots, n$), which can be done using stochastic quadrature rule [26, 32], and then calculate the trajectories corresponded to each ζ_k, called trajectory ensemble as:

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}}_1(t) \\ \dot{\mathbf{x}}_2(t) \\ \vdots \\ \dot{\mathbf{x}}_n(t) \end{bmatrix}}_{\dot{\mathbf{x}}_a(t)} = \underbrace{\begin{bmatrix} \mathbf{f}\big(\mathbf{x}_1(t), \mathbf{u}_1(t), \mathbf{z}_1(t), \zeta_1, t\big) \\ \mathbf{f}\big(\mathbf{x}_2(t), \mathbf{u}_2(t), \mathbf{z}_2(t), \zeta_2, t\big) \\ \vdots \\ \mathbf{f}\big(\mathbf{x}_n(t), \mathbf{u}_n(t), \mathbf{z}_n(t), \zeta_n, t\big) \end{bmatrix}}_{\mathbf{f}_a\left(\mathbf{x}_a(t), \mathbf{u}_a(t), \mathbf{z}_a(t), t\right)} \tag{5}$$

where the augmented vector of control and algebraic variables are defined as $\mathbf{u}_a(t) = \Big[\mathbf{u}_1(t), \cdots, \mathbf{u}_n(t)\Big]^T$ and $\mathbf{z}_a(t) = \Big[\mathbf{z}_1(t), \cdots, \mathbf{z}_n(t)\Big]^T$. The performance index and constraints are also approximated in a similar manner (interested are referred to [26]). In this paper, we will employ a similar approach to [26] to determine robust aircraft trajectory with respect to uncertainties in the meteorological variables quantified using ensemble forecast. We use the aircraft dynamics presented in Section 2.1 with uncertain components of wind. By employing the ensemble forecast, the uncertainty is directly given in a discrete distribution fashion with equal weighting coefficients, which can be used to create the trajectory ensemble. However, there exist some issues with the direct application of this approach in aircraft trajectory optimization, and some reformulations are required and beneficial.

Within aircraft trajectory planning, it is required to obtain unique trajectories for latitude, longitude, airspeed, and altitude, satisfying predefined initial conditions. To this end, the control policies are to be obtained such that these states of aircraft dynamics follow a unique profile for all possible realizations of weather variables represented using ensemble members. Another issue is with the time as the independent variable, which will be obtained unique for all ensemble members. The uniqueness of time in all scenarios means that the aircraft's position is fixed with respect to time, considering all possible realizations of wind. In this case, the effects of uncertainty on the trajectories (e.g., uncertainty in the wind) cannot be reflected efficiently because the range of feasible solutions is limited. To cope with the mentioned issues associated with addressing uncertainties in the weather forecast, first, groundspeed v_{gs} and the course ψ are defined as additional algebraic and control variables, respectively, by taking into account the following relation

$$v_{gs}\cos(\psi) = v_{tas}\cos(\chi) + w_y, \quad v_{gs}\sin(\psi) = v_{tas}\sin(\chi) + w_x. \tag{6}$$

After these implementations, the trajectory ensemble is created with the following augmented state-space representation of the aircraft dynamical model containing uncertainty [26]:

$$\frac{\mathrm{d}}{\mathrm{d}s}\begin{bmatrix} \phi \\ \lambda \\ v_{tas} \\ m_1 \\ \vdots \\ m_n \\ t_1 \\ \vdots \\ t_n \end{bmatrix} = \begin{bmatrix} \cos(\psi)\big(R_M(\phi)+h\big)^{-1} \\ \sin(\psi)\Big(\big(R_N(\phi)+h\big)\cos(\phi)\Big)^{-1} \\ d_v \\ -f_c(C_{T,1})v_{gs,1}^{-1} \\ \vdots \\ -f_c(C_{T,n})v_{gs,n}^{-1} \\ v_{gs,1}^{-1} \\ \vdots \\ v_{gs,n}^{-1} \end{bmatrix} \tag{7}$$

with $\frac{\mathrm{d}t}{\mathrm{d}s} = v_{gs}^{-1}$, where the augmented state and control vectors are

$$\begin{aligned} \mathbf{x}_a &= \begin{bmatrix} \phi & \lambda & v & m_1 & \cdots & m_n & t_1 & \cdots & t_n \end{bmatrix}^T \\ \mathbf{u}_a &= \begin{bmatrix} d_v & \psi & \chi_1 & \cdots & \chi_n & C_{T,1} & \cdots & C_{T,n} \end{bmatrix}^T \end{aligned} \tag{8}$$

Here the untracked control variables (those that are not restricted to have the same values for all ensemble members), such as heading and thrust coefficient, are free to have different values for different scenarios. This is suitable only for real problems where the actual system has low-level controllers that can follow the determined trajectories in real time at a shorter timescale than that of the optimal control problem.

In this study, as mentioned earlier, the aim is to find an optimal aircraft trajectory in the presence of meteorological uncertainty and explore the trade-off between flight operational cost and climate impacts. To this end, we approximate the cost functional Eq. (3) with respect to the augmented dynamical system (trajectory ensemble) as

$$\begin{aligned} &J = \text{Expected SOC} + \text{Expected ATR} \\ &\quad \text{Expected SOC} : \text{CI}\left[0.75\big[\frac{1}{n}\sum_{i=1}^{n} t_i(s_f) - t_0\big] + 0.51\big[m_0 - \frac{1}{n}\sum_{i=1}^{n} m_i(s_f)\big]\right] \\ &\quad \text{Expected ATR} : \text{EI}\cdot\sum_{j=1}^{4} \text{ATR}_{\text{avg},j}(s,\mathbf{x}_a(s)) \end{aligned} \tag{9}$$

for $j \in \{NO_x, H_2O, CO_2, \text{Cont.}\}$, where

$$\begin{aligned}
\text{ATR}_{\text{avg},NO_x} &= \frac{1}{n}\sum_{i=1}^{n}\int_0^{s_f} \text{aCCF}_{i,NO_x}(s,\mathbf{x}_i(s))\text{EI}_{i,NO_x}(s,\mathbf{x}_i(s))\dot{m}_i(s)\text{d}s \\
\text{ATR}_{\text{avg},H_2O} &= \frac{1}{n}\sum_{i=1}^{n}\int_0^{s_f} \text{aCCF}_{i,H_2O}(s,\mathbf{x}_i(s))\dot{m}_i(s)\text{d}s \\
\text{ATR}_{\text{avg},CO_2} &= \frac{1}{n}\sum_{i=1}^{n}\int_0^{s_f} \text{aCCF}_{i,CO_2}\dot{m}_i(s)\text{d}s \\
\text{ATR}_{\text{avg,Cont.}} &= \frac{1}{n}\sum_{i=1}^{n}\int_0^{s_f} \text{aCCF}_{i,\text{Cont.}}(s,\mathbf{x}_i(s))\text{d}s
\end{aligned} \quad . \tag{10}$$

where $\mathbf{x}_i(\cdot)$ is the state vector of the system regarding ensemble member $i-$th. Notice that the evaluation of the performance index is now based on the distance flown (s) as the independent variable. To finalize the formulation of the optimal control problem, the boundary conditions for our proposed trajectory optimization are defined as follows

$$\begin{aligned}
&[\phi, \lambda, v_{tas}](0) = [\phi_0, \lambda_0, v_{tas,0}] \\
&[\phi, \lambda, v_{tas}](s_f) = [\phi_f, \lambda_f, v_{tas,f}] \\
&m_i(0) = m_0 \\
&t_i(0) = t_0
\end{aligned} \tag{11}$$

for $i = 1, \cdots, n$.

4 Simulation results

In this section, we apply our proposed robust trajectory optimization method to a flight that departs on May 21$^{\text{st}}$ 2018, 00:00 am UTC. In Section 4.1, the variability of meteorological situation forecasted using ensemble prediction system and its effects on the aCCFs is investigated. Then, we apply our proposed optimization to a flight from a neighborhood of Madrid to a neighborhood of Belgrade (Section 4.2).

4.1 Analysis of uncertainty in ensemble weather forecast

Within ensemble weather forecast, we are provided with n individual probable forecasts. In this study, we employ a weather forecast 3 hours in advance from ECMWF, containing 50 ensemble members. To have more reliable quantification of climate impacts in the sense of uncertainty in standard weather forecast, an approach is to calculate aCCFs for each ensemble member. Fig. (1) shows the variability of the calculated aCCFs from ensemble members. The variability is defined as the standard deviation taken from normalized variables. As can be seen, the uncertainty (or variability) on aCCF of contrails is higher than aCCF of water vapour and NO_x emission. The high uncertainty on aCCF of contrails is related to the high variability of relative humidity within the ensemble weather forecast. This is because PCFA, having high uncertainty as well, depends on meteorological variables temperature and relative humidity. The variability of meteorological variables required to calculate aCCFs and aircraft dynamical model is illustrated in Fig. (9). As can be seen, the variability of temperature is small enough compared to the variability of relative humidity. In the case of contrail's aCCF, in some regions, we have uncertainty up to 50% around mean values. According to [3, 18], which uses aCCFs to quantify climate impacts, the climate effect of contrails is higher than other species. Consequently, the robustness analysis is crucial. In the next part, we address this issue by employing our proposed robust optimization method.

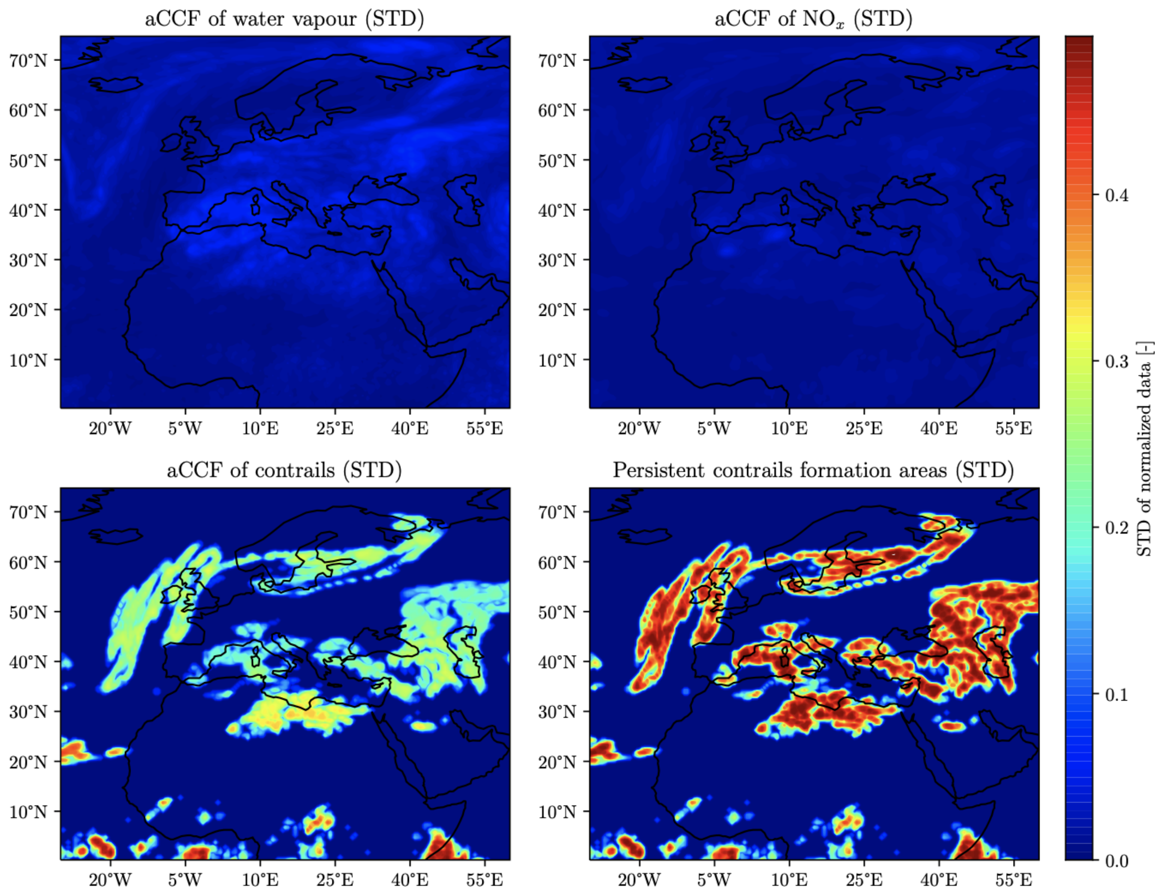

Fig. 1 Standard deviation of aCCFs taken from 50 ensemble members over European region at pressure level 250hPa (21st 2018, 00:00 am UTC)

4.2 Robust ATO considering climate impacts

In this section, the proposed optimization algorithm is applied to a flight from a neighborhood of Madrid to a neighborhood of Belgrade on May 21st 2018, 00:00 am UTC at FL340. The aircraft is an A330-341 with an initial mass of 2000 tons. As for the simulation, the initial and final true airspeeds are selected as 240 m/s and 220 m/s, respectively. The rest limiting constraints (e.g., maximum thrust coefficient and maximum Mach number) are provided within BADA4.0 specification. The direct optimal control method is employed here to solve the proposed robust aircraft trajectory optimization. The trapezoidal rule is used for the transcription of dynamical optimization to a nonlinear programming problem (NLP), which is then solved by IPOPT solver in Python.

Since the order of climate impacts and cost measured using ATR and SOC, respectively, are different, to have suitable selections for cost and environmental indices, we use the following relation to scale these parameters:

$$\mathrm{CI} = \alpha, \qquad \mathrm{EI} = (1-\alpha)\mathrm{k}; \qquad \mathrm{k} = \frac{\mathrm{SOC_{climate}} - \mathrm{SOC_{cost}}}{\mathrm{ATR_{cost}} - \mathrm{ATR_{climate}}},, \qquad 0 \leq \alpha \leq 1.0 \tag{12}$$

where for instance, $\mathrm{SOC_{climate}}$ is the SOC calculated when the optimization objective is only the climate impacts. Notice that the cost and climate impact are the mean values.

Fig. (2) shows the climate impacts associated with each species for different values of parameter α. Left-hand side figures show the climate impacts at each integration step, while the accumulated ones are depicted in the right-hand side. From Fig. (2), one can conclude:

- The climate impact of contrails is higher than the other species
- The climate impact of contrails is highly uncertain
- For smaller values of α, the climate impacts of contrail and water vapour decrease, while it increase the climate impact of NO_x and CO_2 emissions
- For smaller values of α, the range of uncertainty of contrails is squeezed

Fig. (3) depicts the net climate impacts for different values of α. The left-hand side figure shows only mean ATR values for $0.3 \leq \alpha \leq 1.0$, without depicting uncertainty ranges. It can be seen that by decreasing the value of α, the total ATR reduces. In the right-hand side figure, the net ATRs with the ranges of uncertainty are given for three values of α as used in Fig. (2). Since contrails are responsible for a great majority of total climate impacts, total ATRs are also highly uncertain. It is worth mentioning that dispersion of total ATR is decreased with smaller values of α (i.e., $\alpha = 0.3$). Fig. (4) shows the share of each species to the total climate impact for different routing options. Since the contrails have the largest contribution, the optimizer has mainly focused on avoiding contrails-sensitive regions. It is also beneficial due to the sharp behavior of contrails-sensitive areas, which are normally zero and one (if identified using PCFA). However, the other species behave smoothly (see, for instance, Fig. (8)). In some cases, this fact is beneficial for mitigation potential because contrail avoidance may be achieved with small changes in the lateral path. However, for smooth functions such as methane and ozone, it may not be that much beneficial (e.g., in Fig. (2), the climate impacts have not been considerably changed). Fig. (5) shows how the climate impact is reduced by avoiding contrail-sensitive regions.

The climate mitigation potential and its trade-off with the operational cost are depicted in Fig. (6). In the left-hand side figure, one can conclude that with an increase of 1.5% in the operational cost quantified using SOC, the climate impact is decreased by approximately 80% considering the average performance. The right-hand side plot shows the Pareto-frontier with uncertainties using absolute values (i.e., not given in percentage). For the cost-optimal routing option ($\alpha = 1.0$), the uncertainty on ATR is high, while for the climate optimal routing option ($\alpha = 0.3$), it decreases. By comparing the order of cost and climate impacts and also their deviations around mean values, one can conclude that the uncertainty in operational cost is small compared to climate impacts.

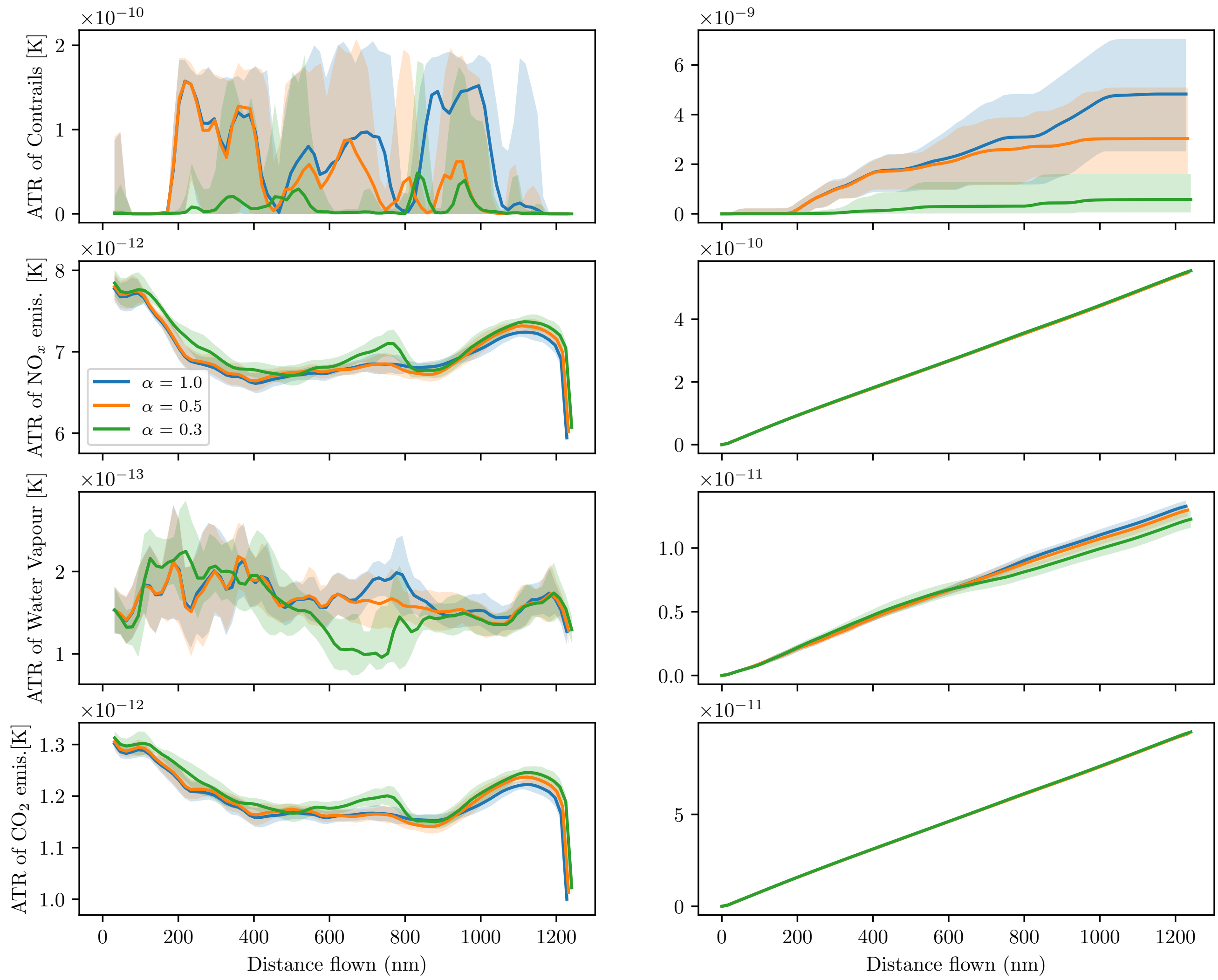

Fig. 2 The average temperature response for each species obtained from different values of α. Shaded regions show the range of uncertainties.

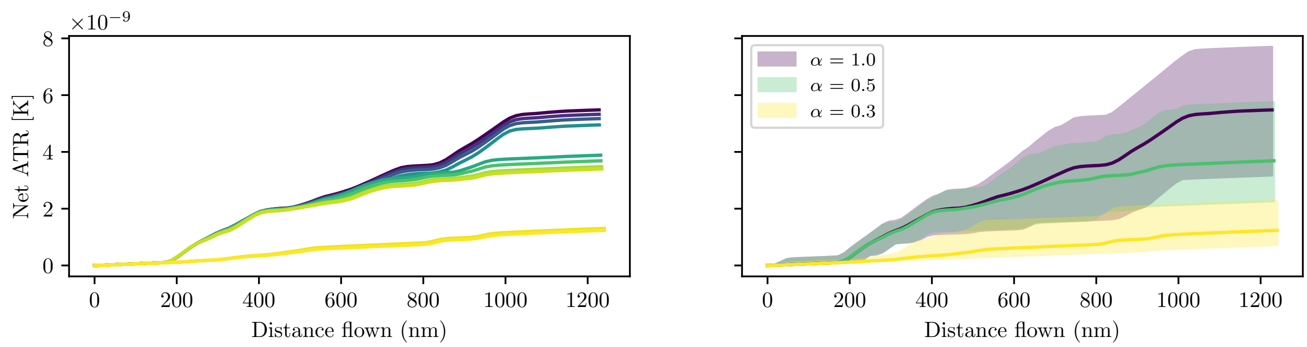

Fig. 3 The average temperature response for each species obtained from different values of α. Shaded regions show the range of uncertainties.

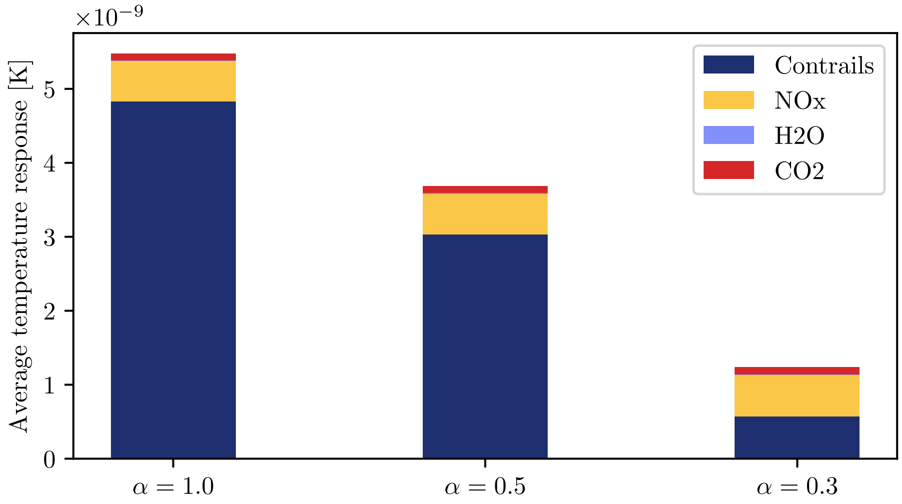

Fig. 4 The total average temperature response obtained from different values of α.

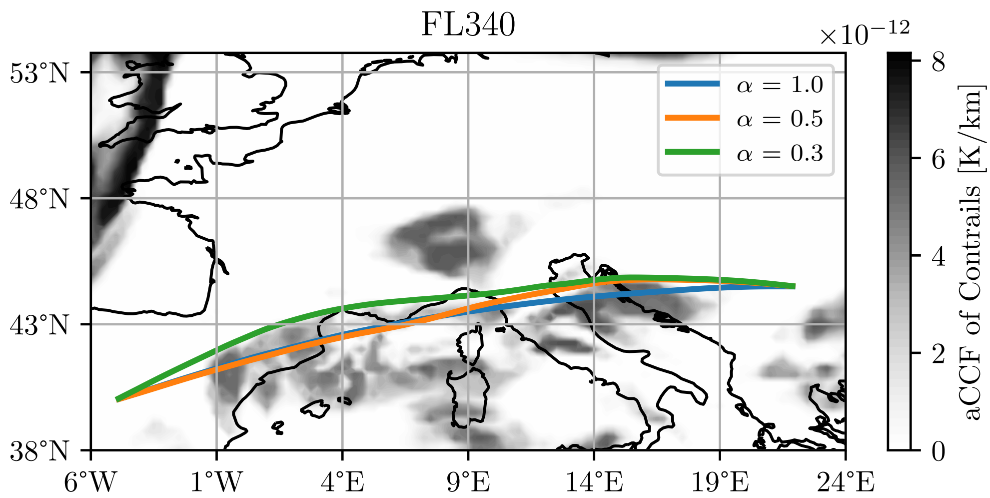

Fig. 5 Lateral paths for different values of α heat-mapped with the mean aCCF of contrails at 250hPa (FL340).

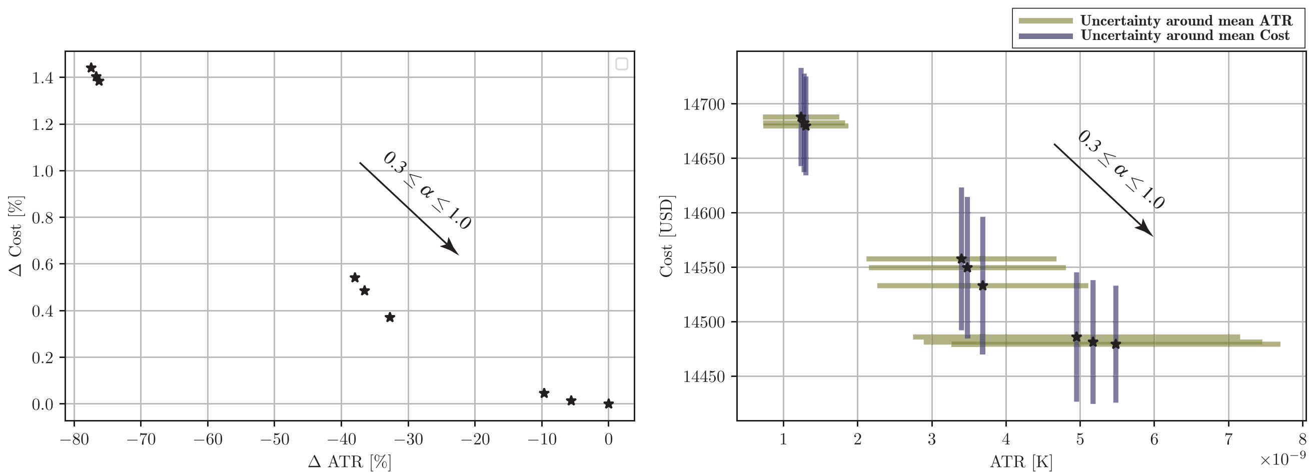

Fig. 6 Pareto-frontier for $0.3 \leq \alpha \leq 1.0$.

5 Conclusion

This paper addressed, for the first time, the robust aircraft trajectory optimization problem considering climate impacts. The climate-sensitive regions were identified using algorithmic climate change functions (aCCF) calculated from some weather-related variables. The ensemble prediction system was employed to characterize uncertainty in weather forecasts, leading to magnified uncertainties in aCCFs. It was shown that there is relatively high uncertainty in the aCCF of contrails due to the high uncertainty in relative humidity obtained from the EPS weather forecast. The robust aircraft trajectory optimization was formulated as an optimal control problem, and the direct optimal control approach was employed to solve the optimization problem. The effectiveness of the proposed methodology to deal with meteorological uncertainty in aircraft trajectory optimization was supported by simulations. A case study showed that by accepting a 1.4% increase in cost, the climate impact could be approximately decreased by 80% considering mean performance. In addition, with the climate optimal routing option, the dispersion of climate impacts was also reduced compared to the cost optimal one.

Future works:
The aircraft trajectory optimization considered in this study was performed in three dimensions (3D) (lateral path + time). As a future work, we aim to extend the results to the full 4D (lateral path + altitude + time) problem. In addition, in this study, the average value of the performance index (i.e., SOC and ATR) was considered to be minimized. Although the minimization of expected ATR reduced the dispersion of climate impacts for the considered case study, the minimization of variability (e.g., the standard deviation of performance index) in addition to the average values may provide more confident results by avoiding uncertain climate-sensitive regions.

6 Appendix

6.1 Illustration of aCCFs for the considered case study

In this section, some weather variables required to calculate aCCFs and the resulting aCCFs are depicted for a case study. As mentioned, the meteorological variables required to evaluate these aCCFs are the potential vorticity, temperature, geopotential, the solar irradiance at the top of the atmosphere, and outgoing longwave radiation, which are already available from the standard weather forecast. These variables and the calculated aCCF for March 21^{st} 2018, 00:00 am UTC over the European region at pressure level 300hPa are depicted in Figs. (7,8), respectively. To compare the contribution of agents to the total climate impact for this example, we adopt typical transatlantic values to unify the units of aCCFs based on K/Kg(fuel). These approximated values for NO_x emission and contrails are $1.3 \times 10^{-2}\mathrm{Kg(NO_2)/Kg(fuel)}$ and $0.16 \times$ Km/Kg(Fuel), respectively [33]. By using these values, we can see that the contrails and ozone are the main contributors, and the lowest effect corresponds to the water vapour. By comparing the amplitude of aCCF_{O_3} and aCCF_{CH_4}, it can be concluded that the warming impact of ozone outweighs the cooling impact of methane for this scenario.

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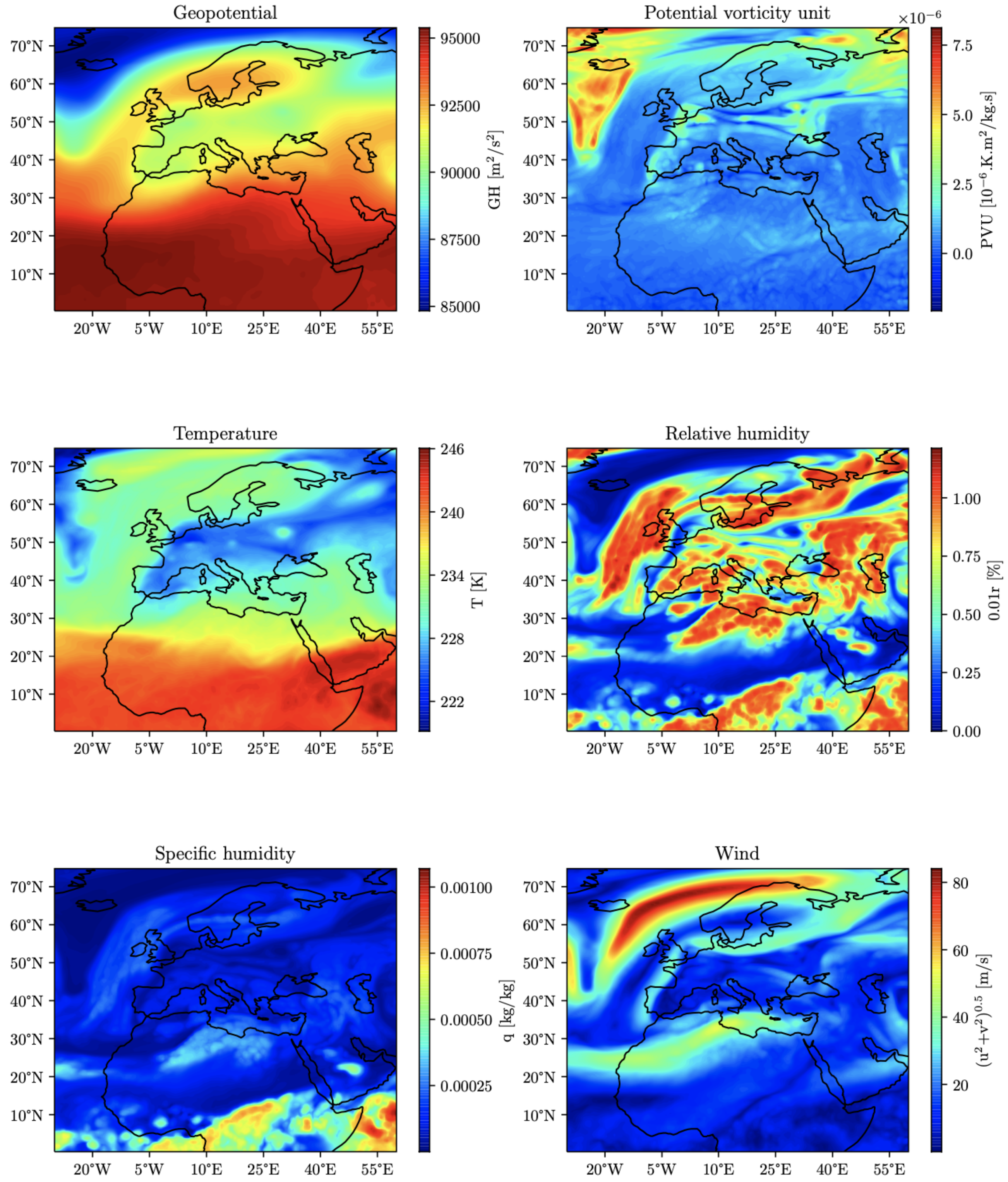

Fig. 7 Weather variables needed to calculate aCCF and aircraft dynamics on March 21st 2018, 00:00 am UTC over European region at pressure level 300hPa.

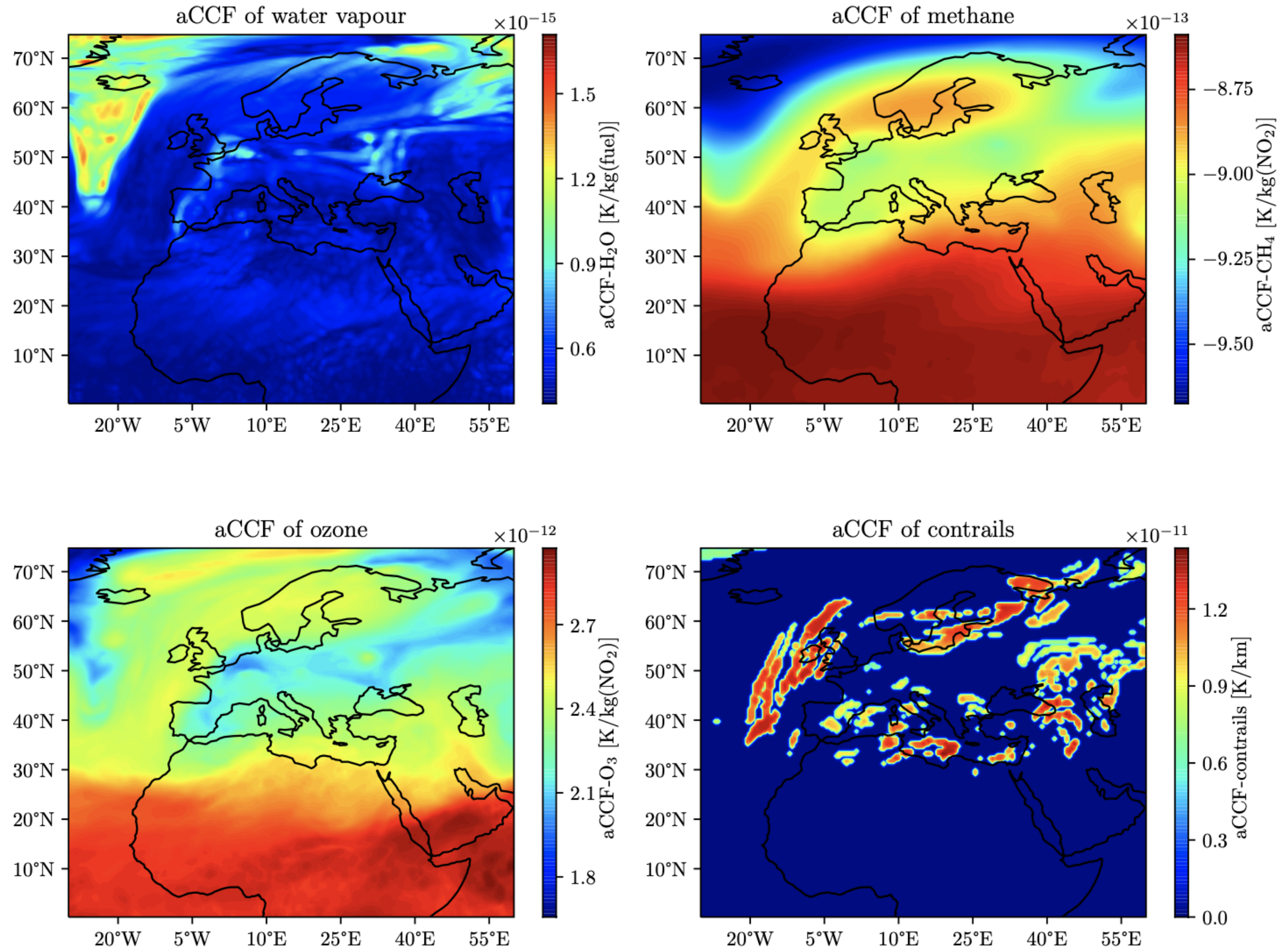

Fig. 8 Algorithmic climate change functions on March 21st 2018, 00:00 am UTC over European region at pressure level 300hPa.

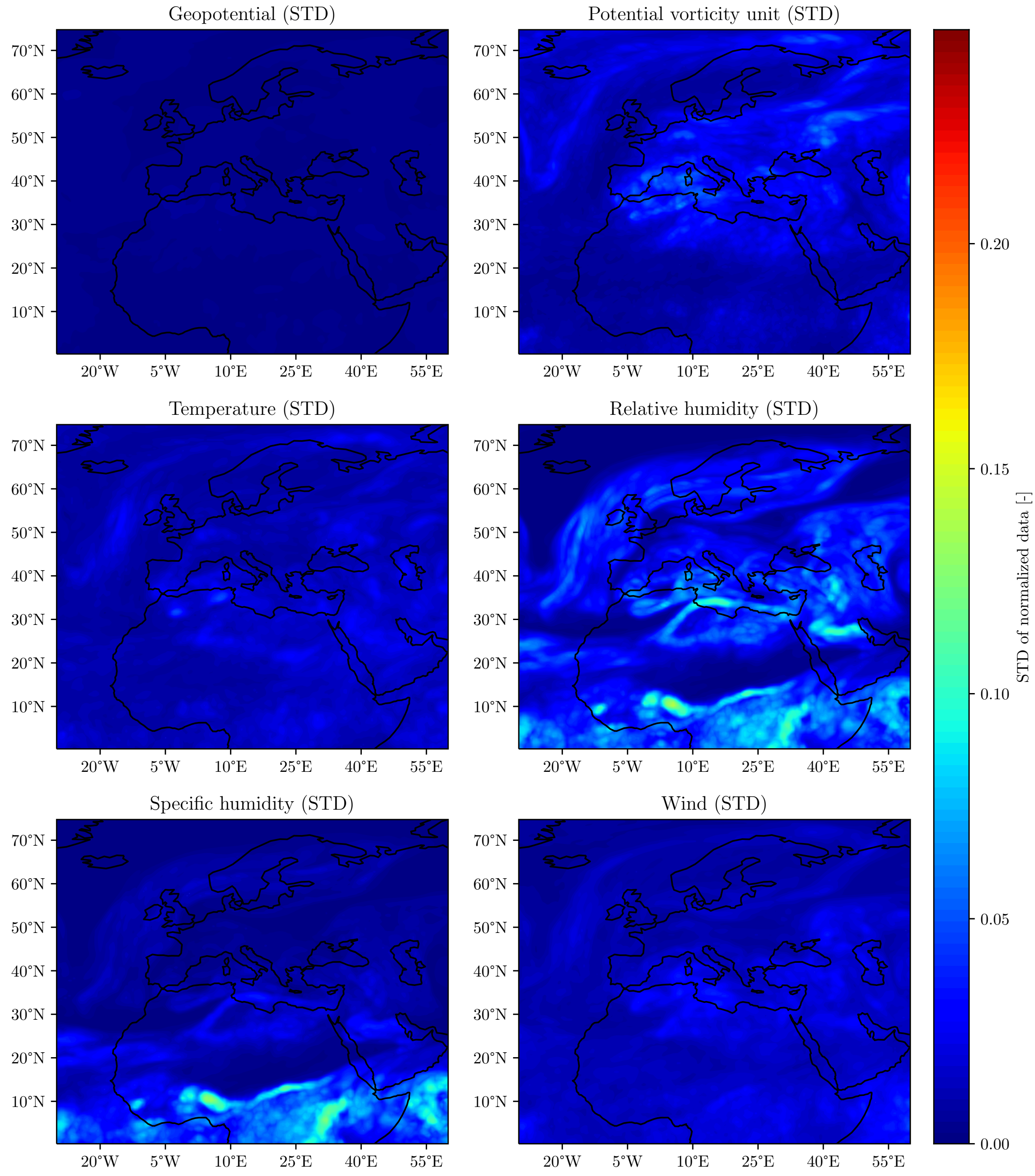

Fig. 9 Standard deviation of some meteorological variables taken from 50 ensemble members over European region at pressure level 250hPa (21st 2018, 00:00 am UTC).

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