CEAS-GNC-2022-048

CEAS EuroGNC 2022

"Conference on Guidance, Navigation and Control"

3-5 May 2022 @ Technische Universität Berlin, Germany

Destination and Time-Series Inference of Moving Objects Using Conditionally Markov Sequences

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ABSTRACT

This study aims to design a destination and time-series inference algorithm for tracking moving targets. The destination of the vehicle is considered as the drone's intent, and the inference is accommodated in the Bayesian framework. The destination-driven dynamic model is described by a Conditionally Markov (CM) model. The CM model is utilised in the intent-driven trajectory estimation based on a multiple model adaptive estimator (MMAE), where the bank of Kalman filters produce state estimates and the inferred destination. The proposed inference algorithm is applied to a moving target tracking scenario, and performance is evaluated via the numerical simulation.

Keywords: Intent Inference; Target Tracking; Multiple Model Adaptive Estimation; Bayesian Inference

1 Introduction

Destination inference has potential benefits for better understanding object's behaviour, enhancing recognition of the pattern, thus supporting a proper decision making. The destination imposes the object's intent and an expected future behaviour, which could provide more accurate state estimations. In general, the destination inference has been interpreted as selection of the most probable intent within a finite set of the destination candidates that are priorly collected. Then, the intent inference problem has been investigated using probabilistic approach based on estimation theory [1–4], Bayesian inference framework [5–7], pattern matching [8, 9], and machine learning techniques [10, 11]. In addition, the intent inference has been extended to many applications such as surveillance [9], air-traffic control[2, 3, 10] human-machine interaction [4, 11, 12], and intelligent interactive displays[5, 6, 13].

For imposing destination information into time-series inference, destination-driven dynamic modeling is necessary. Unlike the Markov process (MP), the destination-driven dynamic model generally comprises of destination component in addition to the initial distribution and evolution law, which allows for more accurate tracking by the effect of the expected future information into the current estimates. However, the destination-driven dynamic model requires a more general stochastic process that exhibits non-causal properties. The destination implies future position of the object, hence the destination information imposes the non-causal property of the stochastic process. Several approaches in the stochastic process have been found to address the non-causal property of the stochastic sequence such as Reciprocal process (RP) [14–17], and Markov Bridge [6], and Conditionally Markov (CM) [15, 18–21].

There are several studies for intent inference and state estimation. In [1, 2], hybrid state estimators using an interactive multiple model (IMM) filtering were proposed. State estimation using Markov process models for finite flight modes were designed in parallel, and destination inference along with state estimation was performed by computing the likelihood between the target's velocity vector and the destination. However, these studies focused more on the flight mode inference for state estimation, and the destination information was not directly incorporated into the inference. Refs. [5, 6] incorporated the destination information into the dynamic model for Bayesian intent inference. Ref. [5] used an Ornstein-Uhlenbeck (OU) process to build the state transition probability, and the destination inference was performed by sequentially calculating the likelihoods via MMAE. Ref [6] derived RP model as the state transition probability using the Markov Bridge. The MMAE based filters captured the influence of the endpoint on the dynamic model and showed the effective performances in human-machine interaction. However, this approach requires the full information of the terminal distribution including a final time and velocity at which the target will reach the destination. In practice, some waypoints are partially informative, and thus a supplementary approach to adapt this issue is needed. In addition, the Markov-bridging model in [6] assumes that the initial distribution is independent of the terminal distribution, it could be inconsistent with the Markov nature in that it dictates the transition density $p(x_N|x_0)$. To handle a more generic model that includes such a correlation, it is necessary to describe the dynamic model with a general and constructive framework.

In this regard, CM can suitably address the destination and time-series inference in a constructive and generalised manner. The CM sequence comprises three components: initial distribution, terminal distribution, and transition probability. The principle of CM is that the state evolution can be described as the state transition probability conditioned on the current state and end point. Refs. [15, 20] showed that CM is a more general sequence than MP and RP. It was also shown that a Markov-induced dynamic model properly describes the destination-oriented motion [19], and a Kalman filter can be implemented in state estimation based on the CM model [21]. The studies on CM have been recently extended to a series of waypoints tracking target estimation [20], whereas intent inference based on CM process have not been conducted yet.

Motivated by the works [19–21], we present a destination and time-series inference algorithm using CM sequences. Specifically, this study addresses following considerations for the inference algorithm.

- CM-based intent inference that could capture more general destination-oriented motion than existing approaches based on MP and RP.
- Estimation of un-predetermined terminal velocity and time which gives cost-effective estimates and alleviates the dependency on the accuracy from prior distribution.

For intent inference, a destination is regarded as intent of the object, and Bayesian inference enables to estimate the intended destination by choosing the maximum posterior probability density among a finite candidates. The destination-driven state transition density is formulated via CM model for each candidate destination. In this way, the state transition model via CM process properly describes more general destination-driven target motion in the time-series inference compared to the Markov bridging model in Ref. [6]. For imposing arrival time and terminal velocity required in the CM model, this study includes an estimation of the terminal values before time-series inference step, thereby easing the dependency on the prior distribution for state estimation. In return, the proposed intent inference gives a reliable destination and state estimates.

The rest of the paper is organised as follows. Section 2 briefly introduces a preliminary result of CM process along with MP and RP. Based on the mathematical background, the intent inference algorithm is proposed in Section 3. To demonstrate the effectiveness of the proposed algorithm, a numerical study is performed. The results are discussed in Section 4. Lastly, Section 5 summarises concluding remarks with future research.

2 Mathematical Background

This section presents a preliminary and mathematical basis that describes destination-driven stochastic sequences. There are many ways to generate sample path of a stochastic sequence depending on different consideration of the stochastic process. Definitions of the stochastic processes such as MP, RP, and CM are briefly introduced, and related results that investigate the inclusive relation between those processes are presented. Using the definitions, we present the dynamical modeling of the stochastic sequence in terms of the Markov-induced CM_L model derived in [19].

2.1 Definitions and preliminaries

The following notations are used throughout this study.

$$[x_k]_i^j = \{x_i, x_{i+1}, \cdots, x_j\}, \quad [x_k] = [x_k]_0^N$$

There are many stochastic processes to describe a stochastic sequence. In this section, we briefly review some stochastic processes with important properties. First, the Markov process (MP) can be equivalently defined as

Lemma 1. $[x_k]$ is Markov iif $F(\xi_i|[x_k]_0^j) = F(\xi_i|x_j)\ \forall j < i, \forall \xi_i \in \mathbb{R}^d$, where $F(\cdot|\cdot)$ denotes the conditional cumulative distribution function.

In MP, the probability density function (PDF) satisfies $p(x_k|[x_i]_0^{k-1}) = p(x_k|x_{k-1})$. Then, the stochastic sequence can be described by the MP as

$$p([x_k]) = p(x_0, x_1, \cdots x_N) = p(x_N|x_{N-1}) \cdots p(x_2|x_1)p(x_1|x_0)p(x_0) \tag{1}$$

The above sequence complies with the *causual* property. For a destination-driven dynamics, the terminal state influences the transition of the current state, which makes the resultant sequences non causal. For imposing such a non-causalty, the following definitions of RP, and CM are introduced as

Lemma 2. $[x_k]$ is RP iif $F(\xi_i|[x_k]_0^j, [x_k]_l^N) = F(\xi_i|x_j, x_l)\ \forall j < i < l \in [0, N], \forall \xi_k \in \mathbb{R}^d$.

Lemma 3. $[x_k]$ is $[k_1, k_2] - CM_c$, $c \in \{k_1, k_2\}$ iif $F(\xi_i|[x_k]_{k_1}^j, x_c) = F(\xi_i|x_j, x_c)\ \forall j < i \in [0, N], \forall \xi_i \in \mathbb{R}^d$.

Remark 1. We use the following notation $(k_1 < k_2)$

$$[k_1, k_2] - CM_c = \begin{cases} [k_1, k_2] - CM_F & \text{if } c = k_1 \\ [k_1, k_2] - CM_L & \text{if } c = k_2 \end{cases}$$

When the CM interval is the whole time interval, $[0, N] - CM_c$ sequence is called CM_c.

Note that the RP can be characterised as the three-point distribution where the conditional PDF depends on its adjacent points. On the other hand, CM describes the stochastic sequence partitioned on the end point and the previous point. Hence, a PDF in CM_L satisfies $p(x_k|[x_i]_0^{k-1}, x_N) = p(x_k|x_{k-1}, x_N)$. Upon the relation, the CM_L sequence can be generated by the following steps: i) generate the endpoint states from density $p(x_0, x_N)$, ii) generate other states from $p(x_k|x_{k-1}, x_N)$

$$\begin{aligned} p([x_k]) &= p([x_k]_0^{N-1}|x_N)p(x_N) \\ &= p([x_k]_1^{N-1}|x_0, x_N)p(x_0|x_N)p(x_N) = \left(\prod_{i=1}^{N-1} p(x_i|x_{i-1}, x_N)\right) p(x_0|x_N)p(x_N) \end{aligned} \tag{2}$$

Several studies investigated inclusive properties between MP, RP, and CM [14, 15, 19]. In this study, we summarise properties of the stochastic processes in the CM_L viewpoint. From those studies, suppose the sequence $[x_k]$ is a zero-mean nonsingular Gaussian (ZMNG). The dynamic model based on (2) can expressed as

Theorem 1. A ZMNG $[x_k]$ is CM_L iff it satisfies

$$x_k = G_{k,k-1}x_{k-1} + G_{k,N}x_N + e_k \tag{3}$$

and boundary condition

$$x_N = e_N, \quad x_0 = G_{0,N}x_N + e_0 \tag{4}$$

where $e_k \sim \mathcal{N}(0, Q_k)$ and $(G_{k,k-1}, G_{k,N})$ represent state transition matrices.

Based on (3) and (4), following theorems addresses how the CM_L model become a reciprocal (Markov) process as

Theorem 2. A ZMNG $[x_k]$ is reciprocal iff it satisfies (3) with (4) and

$$G_k^{-1}G_{k,c} = G'_{k+1,k}G_{k+1}^{-1}G_{k+1,c} \tag{5}$$

Theorem 3. A ZMNG $[x_k]$ is Markov iff it satisfies (3)-(5) and

$$G_0^{-1}G_{0,N} = G'_{1,0}G_1^{-1}G_{1,N} \tag{6}$$

The above theorems indicate that CM_L is more generalised process than RP and MP, i.e., $MP \subset RP \subset CM$.

2.2 Markov-induced Reciprocal CM_L Model

Consider a (nearly) constant velocity model in 2-dimensional space $x_k = [p_x, p_y, v_x, v_y]'_k$ from the origin $x_0 \sim \mathcal{N}(\mu_0, C_0)$ to $x_N \sim \mathcal{N}(\mu_N, C_N)$ with the evolution law

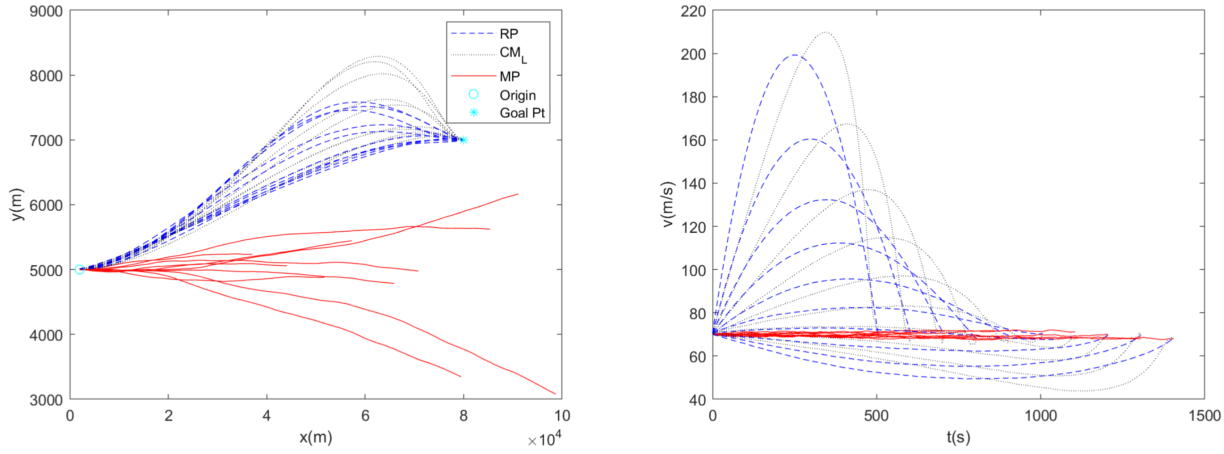
$$x_k = Fx_{k-1} + w_{k-1} \tag{7}$$

where $w_{k-1} \sim \mathcal{N}(0, Q)$, and state trasition matrix F is given by

$$F = \begin{bmatrix} I & \Delta t I \\ \mathbf{0} & I \end{bmatrix}, Q = q\begin{bmatrix} \frac{\Delta t^3}{3} I & \frac{\Delta t^2}{2} I \\ \frac{\Delta t^2}{2} I & \Delta t I \end{bmatrix} \tag{8}$$

where I is the identical 2 by 2 matrix, and $\mathbf{0}$ is the 2 by 2 zero matrix. The covariance matrix considering the cross-correlation between the origin and destination are given by $C = \begin{bmatrix} C_0 & C_{0,N} \\ C_{N,0} & C_N \end{bmatrix}$. Then, the dynamic model starting from x_0 to x_N with evolution law (7) can be re-expressed in terms of a Markov-induced CM_L model [19] as

$$\begin{aligned} x_k &= G_{k,k-1}x_{k-1} + G_{k,N}x_N + e_k \\ x_0 &= \mu_0 + G_{0,N}(x_N - \mu_N) + e_0 \\ x_N &= \mu_N + e_N \end{aligned} \tag{9}$$



(a) Trajectories (b) time histories of velocity in x direction

Fig. 1 Sample paths using different stochastic sequences (black: CM, blue: RP (Markov-induced CM_L), red: MP)

where the matrices are calculated to represent the evolution law (7) in terms of CM_L as

$$\begin{aligned}
&G_{0,N} = C_{0,N}C_N^{-1},\ G_N = C_N,\ G_0 = C_0 - C_{0,N}C_N^{-1}C'_{0,N} \\
&G_{k,k-1} = (I - G_{k,N}F^{N-k})F \\
&G_{k,N} = G_k(F^{N-k})'C_{N|k}^{-1} \\
&G_k = \left(Q^{-1} + (F^{N-k})'C_{N|k}^{-1}F^{N-k}\right)^{-1} \\
&C_{N|k} = \sum_{n=k}^{N-1} F^{N-n-1}Q(F^{N-n-1})'
\end{aligned} \tag{10}$$

Figure 1 shows the sample paths generated by different stochastic sequences. Red coloured paths are generated by MP from an initial point using evolution law (6), and CM sequences are generated using (3) and (4). Blue-dotted lines represent Markov-induced reciprocal CML sequences which are generated using (7)-(10). As shown in Fig. 1, MP sequences do not reach the designated goal point. On the other hand, RP and CM sequences incorporate the terminal information and hence reach the goal point. Note that the terminal incident N must be specified to describe the CM_L model, the sample paths are generated with various terminal time N. Trajectories driven by CM and Markov-induced CM are formed from the origin to designated destination, wheares the MP does not properly generate the sample paths to the destination. Shown in Fig. 1-(b), speed variation can be observed in CM and RP sequences in order to satisfy the boundary conditions. This result indicates that a proper choice of terminal incident N is necessary to estimate a CV behaivour during entire tracking period.

3 Bayesian Inference for Destination and Time-Series Estimation

This section presents how we design an intent and time-series inference for moving objects. First, it is assumed that a set of destinations $g \in \mathcal{G} = \{1, 2, \cdots d\}$ is given, where d is the number of destination candidates and $\mathcal{D} = \{D_1, D_2, \cdots, D_d\}$, $D_i \in \mathcal{R}^4$ consists of terminal position and velocity at the destination. For the inference problem, the following assumptions are used

Assumption 1. The true intent g_i is invariant.

Assumption 2. The true intent g at any time has a mode space S that is time invariant and identical to the time-invariant finite model set $\mathcal{G}$ used.

Using the above Assumptions, the intent inference problem is interpreted as to choose a maximum posterior probability $\rho_k^{(i)} = p(g|Z_k)$ among set of destinations, for the observed data $Z_k = [z_1, z_2, \cdots, z_k]^T$

$$\hat{g} = \arg\max_{g\in\mathcal{G}} \rho_k^{(i)} \tag{11}$$

In the meantime, state estimation is regarded as a means of time-series inference.

$$\hat{x}_k = \sum_i \rho_k^{(i)} \hat{x}_k^{(i)} \tag{12}$$

The MMAE based inference algorithm is designed to obtain the posterior probability $\rho_k^{(i)}$ and the state estimates $\hat{x}_k^{(i)}$ for each destinations g_i. We construct multiple models of CM_L for each destination among the destinations, and the MMAE produces state estimates for time-series inference. Aformentioned in Sec. II.B, the CM_L model requires terminal conditions that may not be specified in prior. This study present how to deal with the undetermined condition by instantaneous estimation of the values.

3.1 Handling terminal conditions

This section presents how we handle undetermined terminal condition in the inference problem. In Ref. [6], $[v_x, v_y]$ is assumed to be known along with the terminal position for each destination. The disperency of the velocity at the destination would contribute to degradation of the intent inference. Usually, terminal t_f and N should also be known or estimated by a prior distribution. Ref. [6] adopted quadrature rule to marginal out t_f on the conditional PDF, which requires parallel calculation of multiple KF for each quadrature points $t_f^j, j = 1, \cdots N_p$.

To relieve this limitation, terminal constraints are calculated, and these values are accommodated to determine the end-point distributions for each destination. Given destination $d_i = [d_i(1), d_i(2)]$, the relative distance r and the line-of-sight angle λ can be calculated as

$$r^{(i)} = \sqrt{(d_i(1) - p_{x,k})^2 + (d_i(2) - p_{y,k})^2}, \quad \lambda^{(i)} \quad = \tan^{-1}\left(\frac{d_i(2) - p_{y,k}}{d_i(1) - p_{x,k}}\right) \tag{13}$$

When the moving target has a constant speed, the desired motion of the CV model is to move toward the destination by aiming its heading to the destination, which coincides with the LOS. In this respect, the terminal velocity $[\hat{v}_x, \hat{v}_y]$ and ideal time-to-go t_{go} can be expressed in terms of the initial speed V_0, and instantaneous geometry with destination as

$$\hat{v}_x = V_0 \cos\lambda^{(i)}, \quad \hat{v}_y = V_0 \sin\lambda^{(i)} \tag{14}$$

$$\hat{t}_{go} = \frac{r^{(i)}}{V_0} \tag{15}$$

Considering uncertainty, the probability distribution of the terminal velocity can be assumed as

$$(v_x, v_y) \sim \mathcal{N}((\hat{v}_x, \hat{v}_y), \Sigma_{v,f}) \tag{16}$$

For the ideal time-to-go, the mathematical expression indicates the shortest time-to-go given the current position. It can be justified in that the desired motion of the target is described by a constant velocity model directly heading towards the destination. By the definition $\hat{t}_f = \hat{t} + t_{go}$, the terminal incident N_f

can be obtained as

$$N_f = round(\hat{t}_f/\Delta t) = k + round\left(\frac{r}{V_0 \Delta t}\right) \tag{17}$$

where Δt is the time step for discretization. It would be desirable to consider the uncertainty in the calculated value by assuming a probability distribution as

$$t_{go} \sim \mathcal{N}(\hat{t}_{go}, \sigma_1^2) \tag{18}$$

where σ_1 and $\Sigma_{v,f}$ are covariances.

3.2 Filtering for State Estimation

State-augmentation based filtering is adopted for the destination-driven state estimation [21]. An augmented state variable $y_k = [x_k, x_N]^T$ is introduced. Then, the evolution law can also be expressed as the ZMNG form.

$$y_k = G^y_{k,k-1} y_{k-1} + e^y_{k-1} \tag{19}$$

where

$$G^y_{k,k-1} = \begin{bmatrix} G_{k,k-1} & G_{k,N} \\ \mathbf{0}_4 & I_4 \end{bmatrix}, G^y_k = \begin{bmatrix} G_{k+1} & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{0}_4 \end{bmatrix} \tag{20}$$

where I_4 and $\mathbf{0}_4$ denote the 4-by-4 identity and zero matrcies, repectively. Since the evolution law is described in a linear form, the state predictive density and its priori estimates (for $i = 1, \cdots, D$) are obtained as

$$\begin{aligned} p(y^{(i)}_k | Z_{k-1}) &= \mathcal{N}(y^{(i)}_k; \hat{y}^{(i)}_{k|k-1}, \Sigma^{ss(i)}_{k|k-1}) \\ \hat{y}^{(i)}_{k|k-1} &= G^{y,(i)}_{k,k-1} \hat{y}^{(i)}_{k-1|k-1}, \\ \Sigma^{ss,(i)}_{k|k-1} &= G^{y,(i)}_{k,k-1} \Sigma^{(i)}_{k-1} (G^{y,(i)}_{k,k-1})^T + G^y_k \end{aligned} \tag{21}$$

On the other hand, the observation $z_k = [p_x, p_y]^T$ can be expressed in the state space form $z_k = H_k y_k + v_k$ where

$$H_k = \begin{bmatrix} I & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{22}$$

Then, observation predictive density and its estimates can be described as

$$\begin{aligned} p(z_k | Z_{k-1}, g_i) &= \int_{\mathbb{R}^n} p(z_k | y^{(i)}_k) p(y^{(i)}_k | Z_{k-1}) dy^{(i)}_k \sim \mathcal{N}(\hat{z}^{(i)}_{k|k-1}, \Sigma^{mm,(i)}_{k|k-1}) \\ \hat{z}^{(i)}_{k|k-1} &= H_k \hat{y}^{(i)}_{k|k-1}, \\ \Sigma^{mm,(i)}_{k|k-1} &= H_k \Sigma^{(i)}_{k|k-1} H^T_k + V_k \end{aligned} \tag{23}$$

The state posteriori for each mode i can be calculated as

$$\begin{aligned} K^{(i)}_k &= \Sigma^{(i)}_{k|k-1} H^T_k (\Sigma^{mm,(i)}_{k|k-1})^{-1} \\ \hat{y}^{(i)}_{k|k} &= \hat{y}^{(i)}_{k|k-1} + K^{(i)}_k (z_k - \hat{z}^{(i)}_{k|k-1}), \\ \Sigma^{(i)}_k &= \Sigma^{(i)}_{k|k-1} - K^{(i)}_k \Sigma^{mm,(i)}_{k|k-1} (G^{(i)}_k)^T \end{aligned} \tag{24}$$

Finally, the state estimates and its covariance matrix can be obtained as

$$\hat{x}_k = \sum_i \mu^{(i)}_k \hat{x}^{(i)}_k = \sum_i \rho^{(i)}_k \begin{bmatrix} I_4 & \mathbf{0}_4 \end{bmatrix} \hat{y}^{(i)}_{k|k} \tag{25}$$

where $\rho_k^{(i)}$ is the mode probability determined in the next section.

3.3 Intent Inference

Main principle of the intent inference is to calculate the mode probabilities based on Bayesian Inference

$$\rho_k^{(i)} \triangleq p(g_i|Z_k) \propto p(Z_k|g_i)p(g_i) \tag{26}$$

where $p(Z_k|g_i)$ is the likelihood and $p(g_i)$ is the prior distribution of the destination. Without knowledge of any prior information, it is natural to set the prior function as the uniform distribution $p(g_i) = 1/d$. When the uniform distribution is chosen as the prior, the MAP can be equivalent to the maximum likelilhood (ML) problem. Prediction error decomposition (PED) gives the likelihood expressed as the recursive form.

$$p(Z_k|g_i) = p(z_k|Z_{k-1}, g_i)p(Z_{k-1}|g_i) \tag{27}$$

Using Eq. (26), the likelihood can be recursively calculated. Upon the updated likelihood, the posterior probability is normalised to evaluate the mode probability.

$$\rho_k^{(i)} = \frac{p(Z_k|g_i)p(g_i)}{\sum_{g_j \in \mathbb{G}} p(Z_k|g_j)p(g_j)} \tag{28}$$

The overall flow of the destination and time-series inference using MMAE are summarised in Algorithm 1.

Algorithm 1 MMAE for Intent Inference (Markov-induced Reciprocal CML model)

procedure MAP($\hat{g}$,$\hat{x}$)
  Input: $p(g_i)$ (priors), d_i (intended goal point) $i = 1, \cdots, D$
  Initialise $\hat{x}_0^{(i)}$,$P_{0|0}^{(i)}$
  for each observation at time instants $k = 1, \cdots, N$ **do**
    for all Intent candidate $i = 1, \cdots, D$ **do** ▹ Multiple Model Inference
      Calculate Terminal conditions t_f, N_f
      Calculate State Matrices $G_{k,k-1}^{y,(i)}, G_k^{y,(i)}$
      Calculate $\hat{y}_{k|k-1}^{(i)}$ and $\Sigma_{k|k-1}^{(i)}$ ▹ One-step State prediction
      Calculate $\hat{z}_{k|k-1}^{(i)}$ and $\Sigma_{k|k-1}^{mm,(i)}$ ▹ Measurement Update
      Calculate Likelihood $l_k^{(i)} = p(z_k|Z_{k-1}, g_i)$ and $L_k^{(i)} = p(Z_k|g_i)$ ▹ Likelihood
      Compute unnormalised posterior probability $\hat{p}(g_i|Z_k)$
      Calculate $\hat{x}_{k|k}^{(i)}$ and $P_{k|k}^{(i)}$ ▹ State update for the next observation
    end for
    Calculate normalised posterior probability $\rho_k^{(i)} = \frac{\hat{p}(g_i|Z_k)}{\sum_j \hat{p}(g_j|Z_k)}$
  end for
  return $\hat{g} = \arg\max_{g_i \in \mathcal{G}} \rho_k^{(i)}$
  return $\hat{y}_k = \sum_i \rho_k^{(i)} \hat{y}_{k|k}^{(i)}$
end procedure

4 Numerical Simulation

Numerical simulation is carried out to validate the proposed inference algorithm. A moving target tracking scenario is considered, where the target is moving from an origin $(0, 0)$ with constant speed

$V_0 = 10m/s$ towards a destination in the following set

$$\mathcal{D} = \{[-40,150],[-40,250],[50,350],[100,150]\}$$

Target motion is described as

$$\begin{aligned}\dot{r} &= -V_0\cos(\lambda-\gamma)\\ \dot{\lambda} &= \frac{V_0}{r}\sin(\lambda-\gamma)\\ \dot{\gamma} &= \frac{a_{cmd}}{V_0}\end{aligned} \tag{29}$$

where r and λ represent the relative distance and LOS angle from the target position to a destination D specified, respectively. γ is the flight path angle of the target, and a_{cmd} is the lateral acceleration. Two cases of inference scenario are conducted as:

- Case i: $D = [-40,250]$, $a_{cmd} = 3V\dot{\lambda} + \mathcal{U}(0,2)\sin((r/r_0)^3)$.
- Case ii: $D = [50,350]$, $a_{cmd} = 3V\dot{\lambda} + \mathcal{U}(0,2)\sin((r/r_0)^3)$.

where $\mathcal{U}$ represents the uniform random distribution. To track the target, the inference algorithm is executed. Simulation parameters are selected as below for the proposed inference algorithm.

$$\Delta t = 0.1, C_0 = C_N = diag\{1,1,0.1,0.1\}, C_{0N} = diag\{0.5,0.5,0.01,0.01\}$$

$$Q = \begin{bmatrix}0.0003 & 0 & 0.0050 & 0\\ 0 & 0.0003 & 0 & 0.0050\\ 0.0050 & 0 & 0.1000 & 0\\ 0 & 0.0050 & 0 & 0.1000\end{bmatrix}, V_n = \begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}$$

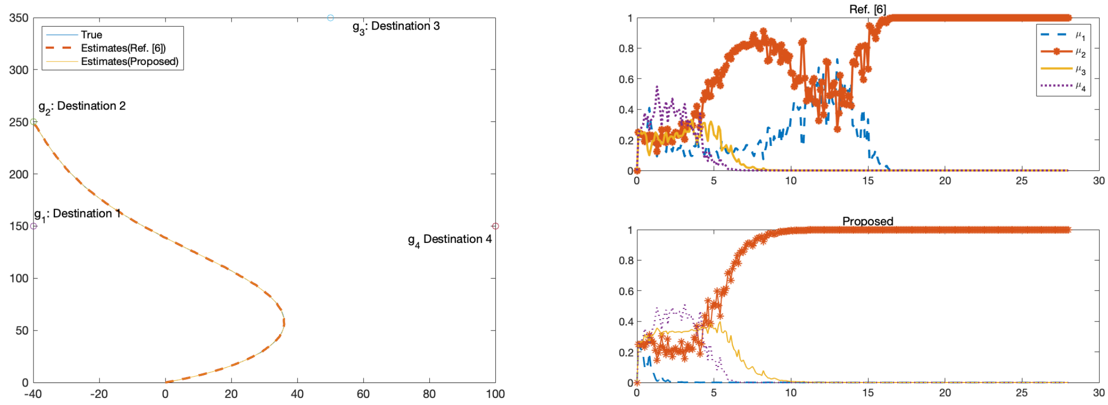
The performance of the proposed algorithm is compared with existing method in Ref. [6]. The simulation parameters are identically set, and the prior distribution of the final time is chosen as $T \sim \mathcal{U}(15,35)$. Two performance measures are taken into account to compare the performances statistically

$$RMS = \sum_{i=0}^{N}\sqrt{(p_x-\hat{p}_x)^2+(p_y-\hat{p}_y)^2} \tag{30}$$

$$SR = \sum_{i=1}^{N}\frac{S(i)}{N}, \qquad S(i) = \begin{cases}1 & \text{if } \hat{g} = g\\ 0 & \text{otherwise}\end{cases} \tag{31}$$

Figure 2 shows the simulation result for case 1. The target initially tends to move toward destination 4 and gradually turns the heading counter-clockwise to reach destination 2 as shown in Fig. 2-(a). The trajectories using both inference algorithms show almost similar responses to the actual one, which indicates the time-series inference is satisfactory for both approaches. In the destination inference performance, Ref. [6] cannot clearly determine the destination around 13 seconds, At the instance, the target closes to both destinations 1 and 2, and these probabilities remain similar until the target is much close to the true destination. On the other hand, the proposed inference algorithm increases ρ_4 at the beginning but quickly captures the correct inference probability ρ_2. This may attribute the effect that the posterior probability for mode 1 is calculated to be very small by the terminal conditions calculated in the beginning phase. Consequently, the proposed intent algorithm captures the true destination while easily distinguishing actual intention (mode 2) from others.

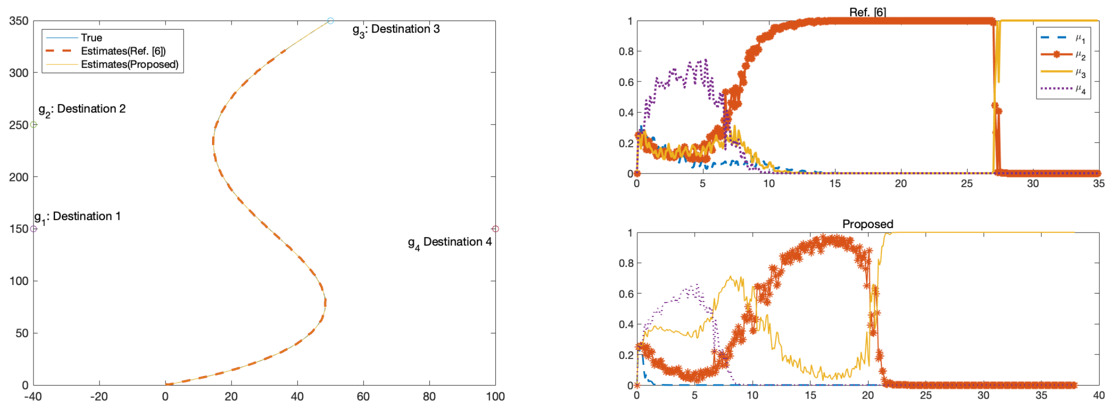
In the second case, the guidance command allows for a sinusoidal pattern of the motion. And the curved trajectory toward destination 3 is made as shown in Fig. 3-(a). Since the distance from the origin to the destination is the farthest, the sinusoidal random manoeuvre leads to an actual terminal time of



(a) Trajectory (Intent = g_2) (b) Inference Probability $\mu_k^{(i)}$ (Intent = g_2)

Fig. 2 Simulation Result (Case 1)

around 37 seconds, which is outside the prior distribution in Ref. [6]. As a result, the inference algorithm [6] no longer produces state estimates after 35 seconds and fails the time-series inference. On the other hand, the proposed algorithm produces state estimates consistently. The terminal conditions (v_x, v_y, t_f) are recursively updated in the CM_L model, and it compensates for unmatched terminal time. As shown in Fig. 3-(b) the intent inference probability increases as the state estimates get close to destinations 4 and 2, but set consistent after 20 seconds. Table 1 summarises the statistical comparison results in both cases. The inference using the proposed algorithm outperforms the one in Ref. [6] in terms of both root-mean-square error (RMS) and successful rate. It can be concluded that the inference performance using the proposed algorithm is still effective with the change of destination and terminal conditions.



(a) Trajectory (Intent = g_3) (b) Inference Probability $\mu_k^{(i)}$ (Intent = g_3)

Fig. 3 Simulation Result (Case 2)

Table 1 Simulation Result

	RMS		Successful rate	
	Ref. [6]	Proposed	Ref. [6]	Proposed
Case 1	0.1083	0.1050	0.8429	0.8571
Case 2	NaN	0.1060	0.2032	0.5567

5 Conclusion

In this study, we addressed a destination and time-series inference problem for moving target tracking. The multiple model adaptive estimator was adopted for the inference algorithm. The constructed conditionally Markov model suitably represents the destination-driven time series, and it could be incorporated in intent inference under the Bayesian framework. Because the formulated conditionally Markov model allows to describe more general stochastic motion, the inference algorithm can be further extended to other applications such as guided target tracking and inference group behaviours. The proposed approach effectively alleviated the dependency of the prior distribution setting by repeatedly updating the terminal conditions upon the relative geometry.

For future studies, the intent inference problem can be further investigated under the more complex condition where the destination property changes. e.g., moving or change of destination. An interactive multiple model estimation can be incorporated into the inference algorithm to address such problems. Also, a data-driven approach and learning-based inference can be designed to capture a more general destination-driven dynamic motion of the target for intent inference, and the intent inference can also be extended to meta-level intent inference.

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