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Continuous time adaptive Higher Harmonic Control

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ABSTRACT

Most helicopters experience a significant level of vibrations. Active vibration control techniques aimed at reducing helicopter vibrations have been extensively studied during the last decades. Most of the techniques developed so far make use of frequency-domain based algorithms, and thus require the computation of the Discrete Fourier Transform (DFT) of the output. Recent developments in the field have shown that time-domain adaptive methods can improve the convergence time of the widely adopted Higher Harmonic Control (HHC) algorithm, the main limitation of which is the need to update the control action after a sufficient amount of time (*i.e.*, the time required for the transient to decay following the last change in the control action). The application of such new methods is tested on a virtual helicopter and the result is compared with the one obtained with the HHC algorithm. Moreover, to improve the performance of the adaptive algorithm, continuous identification of the unknown parameters is proposed and compared with the discrete counterpart, in which the update of the control and the update of the parameters are performed synchronously.

Keywords: Higher Harmonic Control; Adaptive Control; Helicopter Vibrations

1 Introduction

Vibration is one of the major drawbacks in helicopters and the main rotor represents the principal source of it. Indeed, the main rotor transmits to the fuselage vibratory loads at the frequencies multiple of the blade passage frequency $N\Omega$, where N is the number of blades and Ω is the rotor angular velocity. Therefore, active vibration control aims to attenuate disturbances with known frequency, where the main goal is the first harmonic $N\Omega$. The problem has been extensively studied and several approaches have been proposed [1], [2]: Higher Harmonic Control (HHC), Individual Blade Control (IBC), and Active Control of Structural Response (ACSR). The main difference between these approaches relies on the actuation system; both HHC and IBC aim at vibration reduction on the rotor, HHC uses the non-rotating swashplate at higher harmonics of the rotor rotational speed while IBC controls each blade individually in the rotating frame using actuated pitch links. On the other hand, ACSR controls the vibrations in the fuselage. Regardless of the approach, the most adopted control algorithm is the T-matrix algorithm. The T-matrix algorithm is a frequency-domain method, based on a quasi-steady representation of the plant. Since the disturbance has a known frequency, the dynamic model of the helicopter can be reduced to a linear relationship in the frequency domain between the outputs and the inputs at that frequency. This relationship is represented by the T-matrix, which is an accurate description of the system, but only at steady state. For this reason, in a digital implementation of an active vibration control scheme, the time interval between the updates of the control action must be long enough to allow the transients to decay. The T-matrix algorithm has been extensively studied, in [3] the convergence and the robustness of the algorithm were analysed considering only the steady state model, in [4] a continuous time analysis of the HHC was presented for the first time, while in [5] a discrete time stability analysis was carried out using a coupled rotor-fuselage model. The T-matrix cannot be computed reliably from first-principle models and is usually estimated from data; the estimate can be computed both offline or online. Therefore, researchers have investigated both robust and adaptive control approaches, in [6] the control problem was formulated in a robust control framework and an H_∞ synthesis was used to design fully parametrized gain matrices; on the other hand, adaptive techniques based on recursive least squares methods have been widely used to estimate the T-matrix online, in [7] Johnson presented a survey of different adaptive implementations. Recently, a different adaptive approach was presented in [8]. Namely, Kamaldar and Hoagg moved the quasi-steady model from the frequency domain to the time domain and proposed an adaptive algorithm in which the adaptive law is updated minimizing an instantaneous cost function. This approach avoids the computation of the Discrete Fourier Transform (DFT) of the outputs needed to extract the harmonic information and reduces the convergence time. Indeed, they showed that the update of the control action can be accelerated with respect to the standard HHC algorithm. In this paper, the adaptive algorithm used in [8] is investigated and a modification of the algorithm is proposed to further improve the convergence time. The paper is organised as follows: in Section 2 the HHC algorithm is presented, while in Section 3 the adaptive HHC algorithm used in [8] is illustrated and a continuous identification is proposed to improve the performance of the algorithm. Finally, in Section 4 the performance is assessed through numerical simulations. In particular, a two-mass structure example is used to compare the performance of the adaptive HHC with the proposed one, then the revisited algorithm is applied to a virtual helicopter and compared with the standard HHC.

2 The Higher Harmonic Control Algorithm

The HHC algorithm has been developed to reduce the vibratory loads transmitted by the main rotor of the helicopter to the fuselage. It takes its name from its early implementations, in which the swashplate was controlled at higher harmonics of the rotor rotational speed to modify the airload distributions and thus attenuate the vibrations transmitted to the structure. In general, the algorithm is used to attenuate disturbances with known frequency and is formulated in the frequency domain. The update of the control input is performed at discrete times $t(k) = k\tau$, where τ is approximately the time required for the plant to reach the steady state. Let $u \in \mathbb{R}^m$ be the vector of control inputs and $y \in \mathbb{R}^p$ the vector of measured outputs; then the general HHC system is based on a quasi-steady model relating the response of the plant to the harmonics of the control inputs at the disturbance frequency

$$y_\omega(k) = T u_\omega(k) + d, \tag{1}$$

where $u_\omega(k) \in \mathbb{R}^{2m}$ is the vector that contains the cosine and sine harmonics of the control input, $d \in \mathbb{R}^{2p}$ represents the vibration affecting the system (assumed constant) and $y_\omega(k) \in \mathbb{R}^{2p}$ is the vector containing the harmonics of the response. $T \in \mathbb{R}^{2p\times 2m}$ is a constant coefficient matrix. Assuming that the dynamics relating u to y is linear time-invariant, then T is related to the frequency response matrix $G(j\omega)$ by the following equation

$$T = \begin{bmatrix} Re\, G(j\bar{\omega}) & Im\; G(j\bar{\omega}) \\ -Im\, G(j\bar{\omega}) & Re\, G(j\bar{\omega}) \end{bmatrix}, \tag{2}$$

where $\bar{\omega}$ is the frequency of the disturbance we aim to eliminate. The general HHC algorithm aims to derive the control harmonics that minimize the effect of d on $y_\omega(k)$, the control signal u is then obtained from $u_\omega(k)$ through a modulation of its components. In [9] Shaw and Albion proposed the following control law:

$$u_\omega(k+1) = u_\omega(k) - T^\dagger y_\omega(k), \tag{3}$$

where $T^\dagger \in \mathbb{R}^{2m\times 2p}$ denotes the Moore-Penrose pseudoinverse of the T-matrix, which guarantees deadbeat rejection of the disturbance in one discrete-time step.

3 The Adaptive Higher Harmonic Control Algorithm

Online identification techniques can be adopted in case T is unknown. In this section, the adaptive HHC proposed in [8] is briefly presented and, then, a modification of that algorithm is proposed. The following adaptive HHC algorithm is formulated in the time domain, thus the determination of the harmonics of y is not required. Let $y_{hss}(t, u_\omega) \in \mathbb{R}^p$ be the steady state response of the closed-loop system, $u \in \mathbb{R}^m$ be the vector of control inputs, $u_\omega \in \mathbb{R}^{2m}$ be the vector that contains the cosine and sine harmonics of the control input and $\phi(t) \in \mathbb{R}^{2p\times p}$ be defined as follows:

$$\phi(t) = \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix} \otimes I_{p\times p} \tag{4}$$

where $\otimes$ is the Kronecker product. Equation (1) can be expressed in time-domain as shown in equation (5), where the subscript $*$ is used to characterize the true values of the parameters,

$$y_{hss}(t, u_\omega) = \phi^T(t)(T_* u_\omega + d_*), \tag{5}$$

and $T_* \in \mathbb{R}^{2p\times 2m}$ represents the linear relationship between the input and the output that holds in the frequency domain and $d_* \in \mathbb{R}^{2p}$ contains the harmonics of the disturbance. It can be noted that the main difference between equation (1) and equation (5) is represented by the term $\phi(t)$.

The control law

$$u_\omega = -T_*^\dagger d_* \tag{6}$$

is obtained by minimizing the average power of y_{hss}:

$$\frac{2\pi}{\omega}\int_0^{\omega/2\pi} |y_{hss}(t,u_\omega)|^2 = \frac{1}{2}|T_* u_\omega + d_*|^2, \tag{7}$$

where T_* and d_* are computed iteratively using a gradient descent algorithm, therefore $u_\omega(k)$ is computed at each time step using equation (6), based on the current estimate of the adaptive law. The control signal is obtained modulating the components of u_ω. More precisely, let $\tau > 0$ be the update time of the control action. Then for each $k \in \mathbb{N}$ and for all $t \in [k\tau, (k+1)\tau)$, the control update equation is given by:

$$u(t) = \left(\begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix} \otimes I_{m\times m}\right) u_\omega(k). \tag{8}$$

The following adaptive law is denoted D-AHC (discrete-adaptive harmonic control). For all $j \in \mathbb{Z}^+$, define $\Lambda_j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \otimes I_j \in \mathbb{R}^{2j\times 2j}$ and consider $\mathscr{T} = \{T \in \mathbb{R}^{2p\times 2m} : T = \Lambda_{\mathrm{p}}^T T \Lambda_{\mathrm{m}}\}$ which is the set of $2p \times 2m$ matrices that have the block structure of equation (2). For each $k \in \mathbb{N}$, the sampled data are defined as:

$$y(k) = y(k\tau) \in \mathbb{R}^p \tag{9}$$

$$\phi(k) = \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix} \otimes I_p \in \mathbb{R}^{2p\times p} \tag{10}$$

and consider the cost function:

$$\begin{aligned} J(k,\hat{d},\hat{T}) =& \frac{1}{2}\left|y(k+1) - \phi^T(k+1)\left(\hat{T}u_\omega(k) + \hat{d}\right)\right|^2 + \\ & \frac{1}{2}\left|y(k+1) - \phi^T(k+1)\left(\Lambda_{\mathrm{p}}^T \hat{T}\Lambda_{\mathrm{m}} u_\omega(k) + \hat{d}\right)\right|^2, \end{aligned} \tag{11}$$

where the second term constrains the estimate $\hat{T}$ to be contained in the $\mathscr{T}$ set, since for $\hat{T} \in \mathscr{T}$ the two terms are equal and then $J(k)$ can be interpreted as a measure of how well $\phi^T(k+1)\left(\hat{T}u(k) + \hat{d}\right)$ approximates the measurement $y(k+1)$, which itself is an approximation of y_{hss}. If the steady state assumption is satisfied and assuming no measurement errors are present, then $J(k)$ is minimized by $\hat{d} = d_*$ and $\hat{T} = T_*$, which leads to $J(k) = 0$.

To obtain the update equations the gradient descent method is used, and the derivatives of the functional are evaluated at $\hat{d} = d(k)$ and $\hat{T} = T(k)$

$$d(k+1) = d(k) + 2\gamma_d \eta(k)\phi(k+1)[y(k+1) - \phi^T(k+1)(T(k)u_\omega(k) + d(k))] \tag{12}$$

$$T(k+1) = T(k) + \gamma_T \eta(k)\{\phi(k+1)[y(k+1) - \phi^T(k+1)(T(k)u_\omega(k) + d(k))]u_\omega^T(k) + \Lambda_p \phi(k+1)[y(k+1) - \phi^T(k+1)(T(k)u_\omega(k) + d(k))]u_\omega^T(k){\Lambda_m}^T\}, \quad (13)$$

where $\eta(k) = \frac{1}{1+u_\omega^T(k)u_\omega(k)}$ is a normalization term. The stability of the algorithm and the convergence of y to zero are provided assuming steady state conditions (*i.e.*, $y(k+1) = y_{hss}(t_{k+1}, u_\omega(k))$) in [8]. This implies that the control update time must be long enough to allow the transients to decay. In [8], the authors have shown through simulation results that the update time can be drastically reduced by adjusting the update gains of the adaptive law, without destabilizing the plant and therefore better results in terms of convergence time can be obtained. However, this procedure relies on a fine tuning of the gains, which may result in instability.

The proposed modification aims to obtain faster convergence time of the parameters and consequently of the response, while keeping the update of the control action at a slow rate. The basic idea is to decouple the sampling rate used for the parameters identification from the one used to update the control action. Indeed, it was noted that by performing the identification at a higher sampling rate than the control update, the performance is improved without the need to reduce too much the update time. Taking this approach to the limit, a continuous identification of the parameters can be performed. Therefore, the adaptive law is now described by equations (14) and (15). In this way, larger adaptive rates can be used since the update is not immediately reflected on the control action. The following equations describe the update laws of the proposed continuous time adaptive HHC denoted as C-AHC :

$$\dot{d} = 2\gamma_d \eta(k)\phi(t)[y(t) - \phi^T(t)(T(t)u_\omega(k) + d(t))] \quad (14)$$

$$\dot{T} = \gamma_T \eta(k)\{\phi(t)[y(t) - \phi^T(t)(T(t)u_\omega(k) + d(t))]u_\omega^T(k) + \Lambda_p \phi(t))[y(t) - \phi^T(t)(T(t)u_\omega(k) + d(t))]u_\omega^T(k){\Lambda_m}^T\}, \quad (15)$$

where $u_\omega(k) = T^\dagger(t = k\tau)d(t = k\tau)$ is given by sampling the identified parameters at every control update. The adaptive law described in equations (14) and (15) is denoted as C-AHC to distinguish it from the D-AHC. Moreover, a projection operator can be used to enforce a bound on the adaptive parameters [10]. The modified algorithm is expected to lead to a faster convergence of the parameter estimates, since it performs more iterations within the same amount of time. However, the stability proof of the feedback system under the continuous adaptive law (C-AHC) that does not rely on the steady state assumption is still an open problem.

4 Numerical Simulations

In this section the algorithm is tested on two applications: the first one is a two mass-structure, which is used to compare the D-AHC with the C-AHC; the second one is a virtual helicopter where the C-AHC is compared with the HHC algorithm.

4.1 Two Mass Structure Example

In this subsection the proposed algorithm is compared with the one proposed in [8], applying them to the system shown in Fig. 1, where ψ_1 and ψ_2 are the control forces and d_1 and d_2 are the disturbance forces. The numerical values used are the following: $m_1 = 2$(kg), $m_2 = 1$(kg), $c_1 = 60$(kg/s), $c_2 = 50$(kg/s), $c_3 = 40$(kg/s), $k_1 = 300$(N/m), $k_2 = 200$(N/m), and $k_3 = 400$(N/m). Let $y = \xi_1$, $u = \psi_1$,

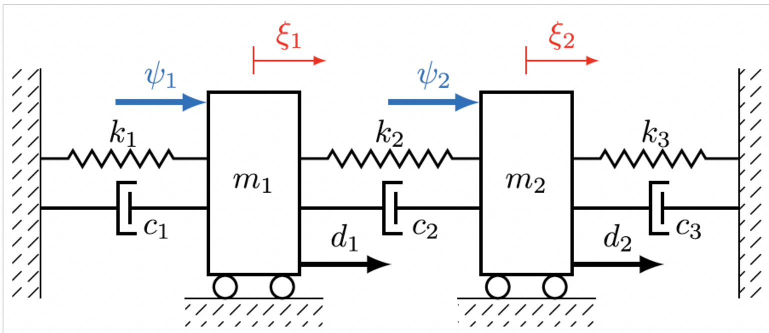

Fig. 1 Two-mass structure, [8]

$\psi_2 = 0$, $d_1 = 0$ and $d_2(t) = 10\cos(\omega_1 t) + 10\sin(\omega_1 t)$, where $\omega_1 = 9\pi$ (rad/s). Let $G_{yu}(j\omega)$ be the frequency response function from the control u to y, then the initial condition for the estimate of the T-matrix is obtained using equation (2) and considering $G_0 = 5\exp(j\frac{5\pi}{12})G_{yu}(j\omega_1)$. The update time of the control action is 0.25(s) and the control system is turned on after 1(s). The update gains for the D-AHC are $\gamma_d = \gamma_h = 2$ which in [8] were shown to give the best convergence time, while for the C-AHC are $\gamma_d = \gamma_h = 15$. As anticipated for the C-AHC the choice of the gains is easier and larger values can be picked, since the identification algorithm has time to adapt before the control action is updated. In Fig. 2 the closed-loop responses obtained with the C-AHC and the D-AHC algorithms are compared, Fig. 2(a) shows the response using the C-AHC while Fig. 2(b) shows the response using the D-AHC. In addition, Fig. 3 compares the control action of the two algorithms. It can be seen from Fig. 2 that the convergence of the proposed method, the C-AHC, is remarkably faster compared to the D-AHC. Furthermore, to compare the performance of the algorithms, the Root Mean Squares (RMS) and the settling time of the responses, denoted t_s, are shown in Table 1. The RMS is defined as $x_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}|x_i|^2}$ and t_s is the time (computed starting from $t = 1(s)$) required by the closed-loop system response to reach and stay within a range of 2% of the steady-state value produced by the disturbance. From Tab. 1 it can be seen that the RMS of the C-AHC response is 18% lower than the one obtained with the D-AHC.

Table 1 Performance comparison between C-AHC and D-AHC

Algorithm	$y_{RMS}(mm)$	$u_{RMS}(N)$	$t_s(s)$
C-AHC (proposed)	$1.1x10^{-3}$	5.3	1.74
D-AHC ([8])	$1.3x10^{-3}$	5.7	3.29

4.2 Virtual Helicopter Example

The framework of this example is Active Control of Structural Response (ACSR). ACSR is based on the superposition of the primary uncontrolled vibration response and the controlled secondary vibration response, controlled in such a way to minimize the vibrations transmitted from the main rotor at key locations of the fuselage. These locations are selected to account for the global vibration behavior of the structure. The helicopter model is built based on data representative of a generic, medium weight (6 t) helicopter with a conventional articulated 5 blade main rotor and tail rotor configuration. The model has been realized using MASST, Modern Aeroservoelastic State Space Tools, a MATLAB tool developed at Politecnico di Milano for the aeromechanical and aeroservoelastic analysis of fixed and rotary wing aircraft [11]. The model is built from subcomponents, each component is assembled in an overall model using the Craig Bampton's Component Mode Synthesis approach. The airframe elastic model was generated in NASTRAN while both the main rotor and the tail rotor aeroelastic models are

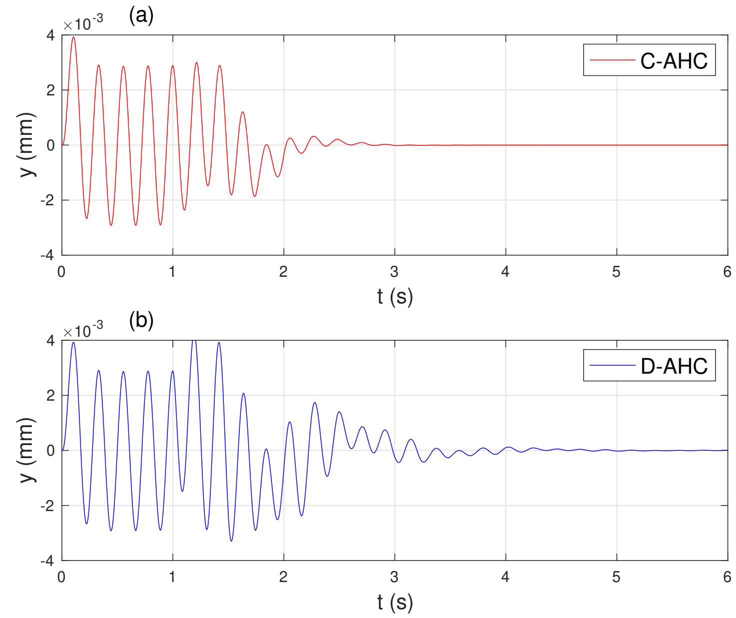

Fig. 2 Closed-loop time response of the two mass structure example: a) the C-AHC is implemented with an update time of $0.25(\mathrm{s})$ and $\gamma_d = \gamma_h = 15$; b) the D-AHC is implemented with an update time of $0.25(\mathrm{s})$ and $\gamma_d = \gamma_h = 2$.

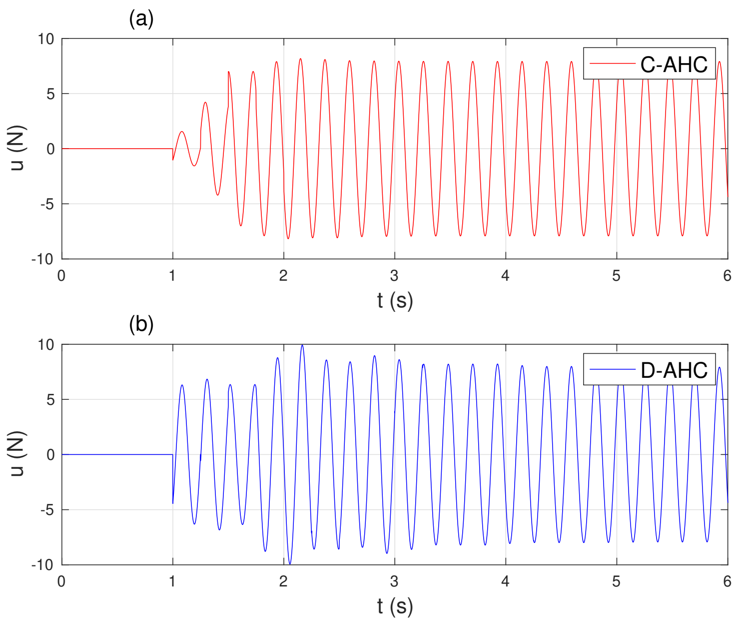

Fig. 3 Control signal of the two mass structure example: a) the C-AHC is implemented with an update time of $0.25(\mathrm{s})$ and $\gamma_d = \gamma_h = 15$; b) the D-AHC is implemented with an update time of $0.25(\mathrm{s})$ and $\gamma_d = \gamma_h = 2$.

obtained in CAMRAD/JA. The model is formulated as a linear system in state-space form, given by the following equations:

$$\dot{x} = Ax + Bu \tag{16}$$
$$y = Cx + Du, \tag{17}$$

where $y \in \mathbb{R}^{10}$ is the vector containing the accelerations measured by the sensors placed on the airframe critical points; the vector $u \in \mathbb{R}^{7}$ accounts for the three external forces and for the four actuator forces acting on the gearbox struts. In order to fairly compare the HHC algorithm and the C-AHC the T-matrix is considered known. Since the HHC attenuates the vibrations in one step in case the T-matrix is known, the difference in terms of convergence time between the invariant HHC and the adaptive one is not expected to be significant if the same control update time is chosen. However, the C-AHC can be implemented with a shorter update time, resulting in a faster convergence.

The objective is to attenuate the vibration at the blade passage frequency of $N\Omega = 25$ Hz, the simulation lasts 20(s), the disturbances are the three forces components acting on the main rotor hub. Because of the linearity of the system, the amplitude of the disturbance can be chosen arbitrarily; therefore the response obtained is not a real representation of the vibration level experienced by the helicopter, but it is more than sufficient to compare the performance of the control algorithms. Thus, the disturbance is initially set to 1 in each direction and then reduced after 11(s) by 50%, 30% and 20% in the longitudinal, lateral and vertical direction, respectively, to show how smaller update times can improve the response when abrupt changes in the disturbance occur. The control system is turned on after 2(s). The HHC algorithm is implemented with an update time of 0.5(s), as a smaller update time leads to instability, while the C-AHC is implemented with an update time of 0.1(s) and $\gamma_d = 8$. The C-AHC algorithm implemented is a simplified version of the one described by equations (14) - (15), since the T-matrix is known and only the identification of the disturbance is considered. The update equations used are the following:

$$\dot{\bar{d}} = \gamma_d \phi(t)[y(t) - \phi^T(t)(Tu_\omega(t = k\tau) + d(t))], \tag{18}$$
$$\dot{d} = proj(d, \dot{\bar{d}}, f) = \begin{cases} \dot{\bar{d}} - \frac{\nabla f(d) \nabla f(d)^T}{\|\nabla f(d)\|^2} \dot{\bar{d}} f(d), & if \;\; f(d) > 0 \;\; \wedge \;\; \dot{\bar{d}}^T f(d) > 0 \\ \dot{\bar{d}}, & otherwise \end{cases} \tag{19}$$

where a projection operator is used following [12] to bound the estimate of the disturbances and $f(d) = \frac{(1+\varepsilon)\|d\| - d_{max}^2}{\varepsilon d_{max}^2}$.

Fig. 4 and Fig. 5 show the response scaled with respect to the corresponding steady state value without the control action. In Fig. 4 the time response of one of the outputs for both the controllers HHC, Fig. 4(a), and $C-AHC$, Fig. 4 (b) are shown. It can be seen that the transient is slightly improved in the C-AHC case; the same consideration can be done for the other outputs that are not shown here. Indeed, Fig. 5 shows the norm of the amplitudes of the vibrations as a measure of the global vibration behaviour; it can be seen that the peak due to the change in the disturbance obtained with the C-AHC is 40% lower than the one obtained with the HHC. The reason is that the C-AHC algorithm can be implemented with a smaller update time, thus, it is able to react earlier to the change in the disturbances.

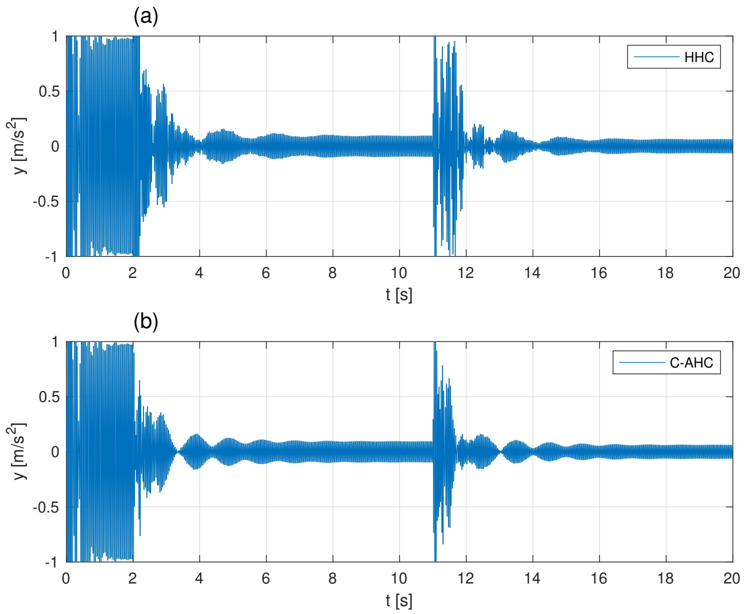

Fig. 4 Closed-loop scaled response of a selected output location of the virtual helicopter: a) HHC response using an update time of the control action of 0.5(s); b) C-AHC response using an update time of the control action of 0.1(s).

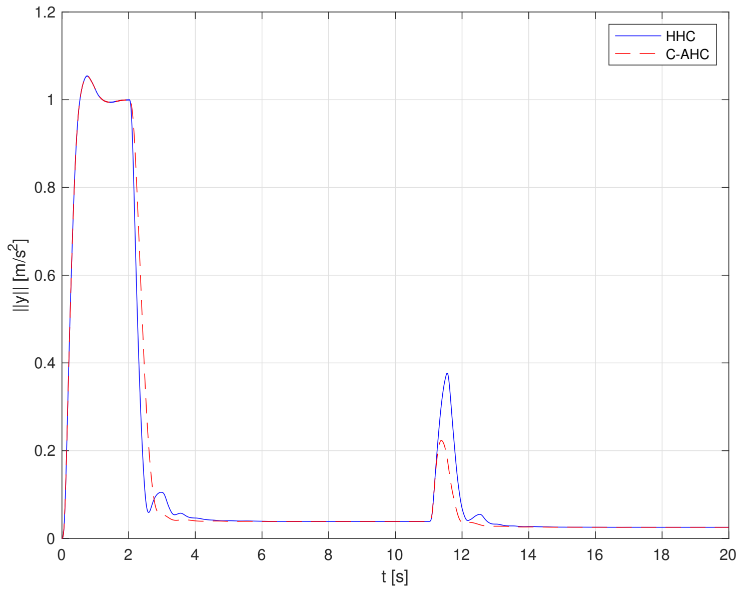

Fig. 5 Scaled norm of the amplitudes of the closed-loop response of the virtual helicopter.

5 Conclusion

In this paper the problem of active vibration control in helicopters is considered and a modification of an existing adaptive control algorithm for harmonic disturbance suppression is proposed. In particular, the T-matrix and the disturbance are identified in a continuous manner and the control action is updated in discrete time. The performance of the algorithm is assessed through numerical simulations, and the advantages of the presented technique with respect to the state-of-the-art are highlighted.

References

[1] Ch. Kessler. Active rotor control for helicopters: motivation and survey on higher harmonic control. *CEAS Aeronautical Journal*, 1(3), July 2011. DOI: 10.1007/s13272-011-0005-9.

[2] P. Friedmann and T. Millott, A. Vibration reduction in rotorcraft using active control: a comparison of various approaches. *Journal of Guidance, Control, and Dynamics*, 18(4), July 1995. DOI: 10.2514/3.21445.

[3] D. Patt, L. Li, J. Chandrasekar, D. Bernstein, and P. Friedmann. The HHC algorithm for helicopter vibration reduction revisited. *Journal of Guidance, Control, and Dynamics*, 28(5), September 2005. DOI: 10.2514/1.9345.

[4] R. Hall, S. and M. Wereley, N. Performance of higher harmonic control algorithms for helicopter vibration reduction. *Journal of Guidance, Control, and Dynamics*, 16(4), July 1992. DOI: 10.2514/3.21085.

[5] M. Lovera, P. Colaneri, C. Malpica, and R. Celi. Discrete-Time, Closed-Loop Aeromechanical Stability Analysis of Helicopters with Higher Harmonic Control. *Journal of Guidance, Control, and Dynamics*, 30(5):1249–1260, 2007. DOI: 10.2514/1.13874.

[6] M. Lovera and R. Mura. Baseline vibration attenuation in helicopters: Robust MIMO-HHC. In *IFAC Proceedings Volumes*, volume 47, pages 8855–8860, 2014. DOI: 10.3182/20140824-6-ZA-1003.02740.

[7] W. Johnson. Self-tuning regulators for multicyclic control of helicopter vibration. Technical report, NASA, 1996.

[8] M. Kamaldar and B. Hoagg, J. Adaptive harmonic control for rejection of sinusoidal disturbances acting on an unknown system. *IEEE Transactions on Control System Technology*, 28(2), March 2020. DOI: 10.1109/TCST.2018.2873283.

[9] J. Shaw and N. Albion. Active control of the helicopter rotor for vibration reduction. *Journal of the American Helicopter Society*, 26(3), July 1981. DOI: 10.4050/JAHS.26.32.

[10] P. loannou and B. Fidan. *Adaptive Control Tutorial*. SIAM, 2006.

[11] A. Tamer, V. Muscarello, P. Masarati, and G. Quaranta. Evaluation of vibration reduction devices for helicopter ride quality improvement. *Aerospace Science and Technology*, 95, 2019. DOI: https://doi.org/10.1016/j.ast.2019.105456.

[12] E. Lavretsky and T. Gibson. Projection operator in adaptive systems. *arXiv preprint arXiv:1112.4232*, 2011. DOI: 10.48550/arXiv.1112.4232.