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Formation Control of a Guided Munition Swarm with Potential Function Method

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ABSTRACT

Munition swarm is favorable for many reasons when it is compared to a single munition attack. Coordinated simultaneous attack can provide flexible attack opportunities and inflict more severe damage. In addition to that, it is harder to defend an area when the number of attackers is increased. Formation control of these air to ground munitions offers a coherent arm flight. Various methods are applied for this purpose to different systems such as UAV's, missiles, fighters and quadrotors. Potential function method can be useful to provide cooperative flight. The method creates virtual potential energy for different positions for agents of a swarm. These potential functions are used to create position commands for agents. Agents move toward the positions with less potential energy. Position commands are realized using position tracker. PD controller is designed for position tracker which creates acceleration commands. Full state feedback acceleration autopilots are designed to apply these commands. A 6-DoF simulation is created with MATLAB/Simulink to investigate the results of the proposed method. Finally, simulation results are shown in figures and discussed.

Keywords: cooperative guidance; formation control; potential function method; guided munition swarm

Nomenclature

X, Y, Z	=	Aerodynamic forces at body axis
L, M, N	=	Aerodynamic moments at body axis
u, v, w	=	Velocity components at body axis
x, y, z	=	Translational position components at inertial reference frame
$\hat{C}^{(n,m)}$	=	DCM matrix from m reference frame to n reference frame
ϕ,θ,ψ	=	Euler angles for roll, pitch and yaw

p,q,r	=	Body angular velocities
α	=	Angle of attack
β	=	Side slip angle
l_{ref}	=	Reference length of airframe
S_{ref}	=	Reference area of airframe
I_{ab}	=	The moment of inertia along defined axis by ab
F_i	=	Inertial reference frame
F_b	=	Body fixed reference frame
$\vec{V}_{b/i}^{(b)}$	=	Velocity of F_b with respect to F_i resolved in F_b
$\bar{r}$	=	Matrix Form of $\vec{r}$
$\bar{r}$	=	Matrix Form of $\vec{r}$
$\vec{\omega}_{b/i}{}^{(b)}$	=	Angular Velocity of F_b with Respect to F_i Resolved in F_b
$\tilde{r}$	=	Skew Symmetric Matrix Form of $\vec{r}$

1 Introduction

The usage of data transfer between vehicles is an extremely popular concept these days. This concept is applied to systems such as unmanned air vehicles, quadrotors, unmanned ground vehicles and unmanned sea vehicles. In addition to this, connection between distinct vehicle types are also another vital concept. Possible advantages of communication between members on the missile guidance concept are a uniform, safe, and collaborative flight and active collision avoidance while approaching the target.

Various algorithms and control strategies are employed to manage the formation of quadrotor swarms, as well as to guide and position drone systems and UAVs [1,2]. Similar techniques involve connecting multiple vehicles to accomplish complex tasks and using swarm intelligence control mechanisms to direct the movement of ground robots and underwater vehicles. An obstacle avoidance potential field is utilized to instruct the formation of an unmanned ground vehicle swarm [3], while autonomous underwater vehicle swarms are utilized to operate under environmental disturbances [4]. These advancements in technology enable multiple vehicles to work together and achieve more challenging missions.

Formation control is a crucial research area in the field of unmanned systems, since it enables multiple vehicles to collaborate towards a common objective such as target tracking, reconnaissance, or surveillance. It improves operational efficiency, reduces mission time, and enhances overall operations effectiveness. The development of formation control methods for UAVs, quadrotors, munitions, and missiles has made significant progress in recent years. These methods range from centralized to decentralized control, from model predictive control to potential field-based control, leader follower concept to virtual leader concept [5]. Each of these methods has its own advantages and disadvantages thus are suitable for different scenarios and applications.

For aerial attacks, guidance methods such as triangle intercept guidance, proportional navigation guidance, and command to line of sight guidance have been employed [6]. Navigation correction algorithms are also applied using Kalman filters with datalink range measurements [7,8]. Multi-munition attacks are preferred to increase the chances of overcoming anti-missile systems [9-11]. Furthermore, munition swarms are cost-effective when they contain seeker-less munitions along with fully equipped munitions. Attacking with multiple munitions simultaneously means more targets for air defense systems to block, which makes munition swarms an attractive option for attacking essential targets.

Examples of multiple vehicle formation control algorithms for systems such as UAV's, missiles, quadrotors and ground vehicles are presented as it is explained before. Artificial potential functions are also applied to swarm models to create desired formations. In general, these studies focus on 2-Dimensional models, particle assumptions or vehicles moving on a specified plane [12-15]. Artificial potential function method is applied to 6-DOF model of munitions with aerodynamic data and gravity in this study. The method is applied to munitions which do not have any control on speed. The study offers uniformity and active collision avoidance for a munition swarm with only pitch and yaw acceleration commands. The algorithm offers uniform and safe arm flight while all the swarm members fly through the target.

The outline of the paper is as follows: In section 2, swarm concept and engagement scenario are explained. Next, Section 2.1 is reserved to explain the model of a single munition containing physical and aerodynamic properties. Then, Section 2.2 and 2.3 explain how the formation control algorithms are applied using the potential function method and position controller. The simulation outputs of MATLAB/Simulink program are shown and discussed in Section 2.4. The conclusion is presented in Section 3.

2 Munition Swarm Concept

Multiple air to ground munitions without any propulsion system are released to hit a stationary target located on the ground. All members can communicate with each other and share their position information within a specified range. Members of the swarm are called "agents" and are expected to fly uniformly without any collision. Each agent must be able to move according to the others dynamically to avoid any irregularity or collision. Engagement geometry for an example scenario is shown below. In Fig. 1, λ is the missile-target line of sight angle, ε is the look angle which is the angle between missile body frame x-axis and line-of-sight, and γ is the flight path angle.

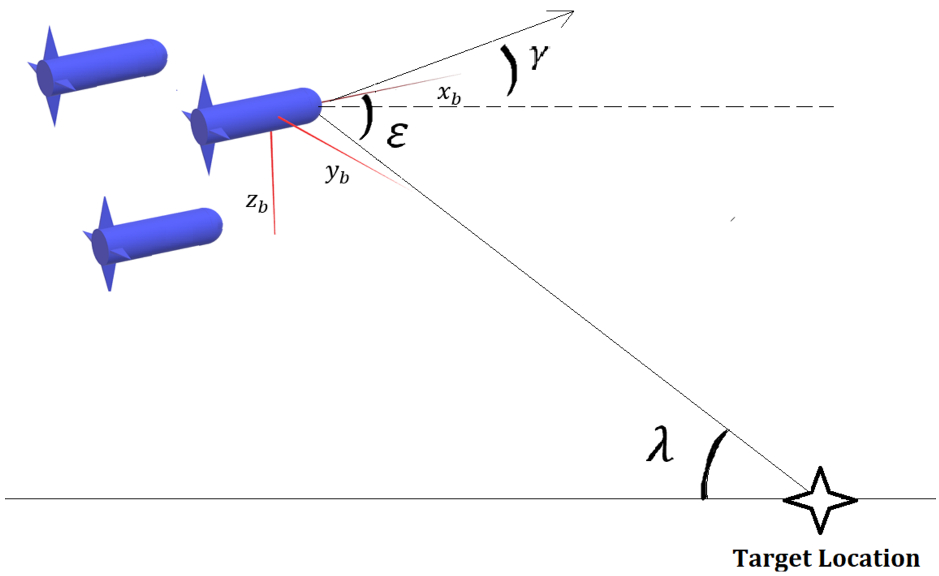

Fig. 1 Engagement Geometry

The munition swarm consists of different types of agents. Some members of the swarm are assumed to have the precise target position information, which are named as “informed agents”. They can be thought as fully equipped munitions. Other agents are named as “naïve agents”. These agents do not have any seeker system and they can only interact with other agents using RF data link. Main guidance commands are generated by the informed agents. The other members of the swarm align themselves according to the informed agents using formation control algorithms.

In this study, agents are considered as drop munitions. Munitions are commanded to make skid-to-turn maneuvers. They are controlled with only aerodynamic control fins and guided to fly with zero roll angle. Since they don’t have any control authority on changing their speed, only pitch and yaw acceleration commands are used to guide the munitions.

Informed agents are commanded with pursuit guidance method towards the target position directly. On the other hand, naïve agents provide uniform and safe arm flight according to positions of neighbor agents. Position tracker and potential function method are applied together to ensure this purpose. The desired horizontal position through the target is calculated using the potential function method. A position tracker is implemented to reach the desired position. On the vertical plane, naïve agents directly follow the position of primary informed agent using pursuit guidance.

All munitions are modeled as they have the same physical and aerodynamic properties. According to these properties 6 degrees of freedom motion equations are developed.

2.1 Single Munition Model and Simulation

The physical properties and aerodynamic derivatives of the considered munitions are shown in Table 1.

Table 1 Physical Properties and Aerodynamic Derivatives

m=25 kg	$C_{Y_\delta} = -14.32$	$C_{Y_\beta} = -22.91$
$I_{xx} = 0.08\ kgm^2$	$C_{Z_\delta} = -14.32$	$C_{Z_\alpha} = -22.91$
$I_{yy} = I_{zz} = 0.98\ kgm^2$	$C_{L_\delta} = -1.72$	$C_{L_p} = -14$
$I_{xy} = I_{xz} = I_{yz} = 0$	$C_{M_\delta} = -28.65$	$C_{M_\alpha} = -11.46$
$S_{ref} = 0.0177m^2$	$C_{N_\delta} = 28.65$	$C_{N_\beta} = 11.46$
$l_{ref} = 0.15$m	$C_{X_0} = -1.2$	$C_{Z_q} = C_{Y_r} = 0$

Aerodynamic forces and moments are modeled as functions of angle of attack, sideslip angle, angular rates and control surface deflection angles. Single munition aerodynamic data is created with Missile DATCOM program for this study. Missile DATCOM is a semi-empirical datasheet component build-up method to predict missile aerodynamic coefficients and stability characteristics. Aerodynamic coefficients are linearized around the equilibrium point to create linear force and moment equations. Aerodynamic derivatives are considered as they are not affected from Mach number of the body.

Using dynamic pressure, referance area, referance length and velocity; related aerodynamic coeffients are found to create aerodynamic forces and moments.

Generic form of the static force equation can be written as

$$F_i = QS_{ref}\, C_{F_i}, \qquad i = \alpha, \beta, \delta \tag{1}$$

And the static moment equation is calculated as

$$M_i = QS_{ref}l_{ref}C_{M_i}, \qquad i = \alpha, \beta, \delta \tag{2}$$

The dynamic moment is also calculated by

$$M_i = \frac{QS_{ref}l_{ref}^2 C_{M_i}}{2V}, \qquad i = p, q, r \tag{3}$$

The aerodynamic forces can be expressed as

$$X = X_0 \tag{4}$$

$$Y = Y_\beta \beta + Y_\delta \delta_r + Y_r r \tag{5}$$

$$Z = Z_\alpha \alpha + Z_\delta \delta_e + Z_q q \tag{6}$$

Similarly, the aerodynamic moments are given by

$$L = L_p p + L_\delta \delta_a \tag{7}$$

$$M = M_\alpha \alpha + M_\delta \delta_e + M_q q \tag{8}$$

$$N = N_\beta \beta + N_\delta \delta_r + N_r r \tag{9}$$

Simulation of a single munition is created according to Newton-Euler's 6-DOF equations of motion expressed in vehicle body frame as

$$\sum \bar{F}^{(b)} = m\bar{a}_{b/i}^{(b)} = m\left(\dot{\bar{V}}_{b/i}^{(b)} + \tilde{\omega}_{b/i}{}^{(b)}\bar{V}_{b/i}^{(b)}\right) \tag{10}$$

$$\sum \bar{M}^{(b)} = \left(\dot{\bar{H}}^{(b)} + \tilde{\omega}_{b/i}{}^{(b)}\bar{H}^{(b)}\right) \tag{11}$$

$\tilde{\omega}_{b/i}$ stands for skew symmetric matrix form of $\vec{\omega}_{b/i}$ and $\bar{H}^{(b)}$ is the column matrix form of angular momentum.

2.2 Potential Function Method

The horizontal position of an agent is determined using the potential function method while all agents are going toward the target. The potential function method is preferred because the method works online and determines new position command for all agents each time step and it can prevent collision and separation of members. Interactions of agents are limited within a defined range and defined number of neighbor agents. Only the closest neighbors can affect the position of an agent. Effective neighbors are found with these logics. Final command is created with superposing all the influence of these effective neighbors. n_{en_i} is defined as the total number of effective neighbors for each agent.

The potential function between any two agents is written in equation (12). The discussion below follows the development in [12] with application to swarm of munitions. The constants a, b and c are all positive constants that define the shape of the potential function. y_i defines the horizontal position of i^{th} neighbor agent and y_{agent} is the horizontal position of the agent of concern [15].

$$J_{agent_i} = \sum_{i=1}^{n_{en_i}} \left[\frac{a}{2}\left|y_i - y_{agent}\right|^2 + \frac{bc}{2} e^{\left(-\frac{\left|y_i - y_{agent}\right|^2}{c}\right)} \right] \tag{12}$$

An example potential versus distance plot with constants a=0.25, b=50, and c=40 is shown in Fig. 2.

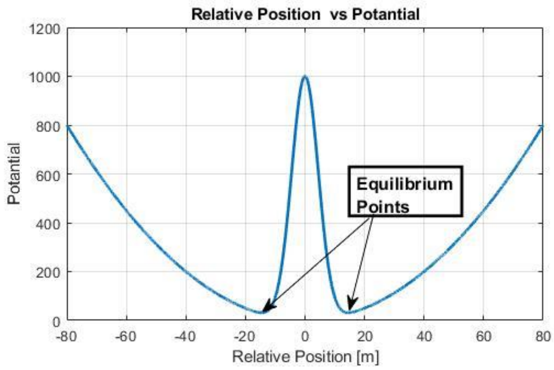

Fig. 2. Relative Position vs Distance Plot

The effect of one agent on another is calculated with nonlinear virtual potential functions created before. Attraction and repulsion forces are superposed according to the position difference between agents. There are two equilibrium positions (d_{eq}) where attraction and repulsion forces cancel each other. In example, for distances larger than the equilibrium position, attraction forces are stronger, and for distances smaller than the equilibrium position, repulsion forces are stronger for a positive side.

Instead of using the method directly, force and relative position graph is needed to be reshaped while application. Effect distance at the horizontal axis is named as d_{eff} and the limitation of attraction force distance is named as d_{af}. For distances larger than d_{eff}, agents do not have any force effect on each other. If the distance is larger than d_{af} it applies force as it is at the d_{af}. These limitations are added to prevent the attraction forces to become so dominant and cause collision of agents. d_{eff} is also added in order to add the effect of datalink range and make the simulation more realistic.

The force on an agent is calculated using relative position d_i as shown in equation (13).

$$F_{pot_i} = -d_i \left(a - be^{\left(-\frac{|d_i|^2}{c} \right)} \right), \qquad where\ d_i = y_i - y_{agent} \tag{13}$$

An example force versus relative position plot with constants a=0.25, b=50, c=40 and $d_{eff} = 60$, $d_{af} = 30,\ d_{eq} = 14.56$ is shown in Fig. 3.

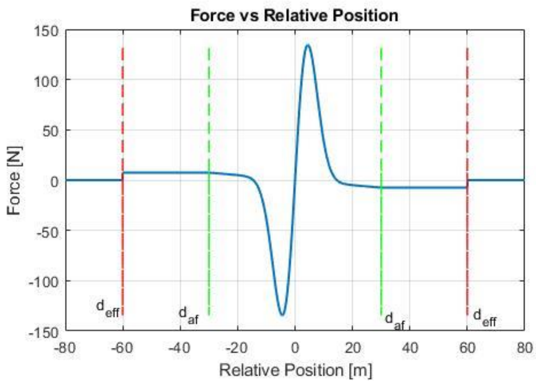

Fig. 3. Force vs Relative Position Plot

The resultant force on each agent is calculated by superposing the forces created by effective neighbors of the agent of concern. Total number of effective neighbors is shown with n_{en_i}. In order to find an equilibrium position of each agent, the resultant force on the agent is integrated with time starting from the initial horizontal position of the agent. This application finds an appropriate position for each agent at each time step even when any agent could not follow the desired position due to any disturbance or aerodynamical effect. All agents will align their positions accordingly and the formation will be arranged so that the swarm can fly with a more regular configuration. Since this method is used only in horizontal plane, horizontal positions are used for these calculations. Desired horizontal position ($y_{desired_{k+1}}{}_i$) at the next step is calculated using the total force and the desired horizontal position at the current step ($y_{desired_k}{}_i$).

$$F_{totalpot_i} = \sum_{i=1}^{n_{en_i}} F_{pot_i} \tag{14}$$

$$y_{desired_{k+1}}{}_i = F_{totalpot_i} dt + y_{desired_k}{}_i \tag{15}$$

As can be easily seen, the motion of the agents defined in (13) is along the negative gradient of the potential function in (12). It is possible to analyze the stability of the system using the sum of the potential functions in (12) for all agents as Lyapunov function candidate and utilizing the Lasalle's invariance principle [12]. In fact, it is possible to show that, as long as the neighborhood topology is connected, the munitions swarm will converge to a constant relative configuration. The final configuration depends on the selected parameters of the potential in (12) and the neighborhood structure.

2.3 Position Controller

Position controller is designed to apply the desired horizontal position which is the output of the potential function method.

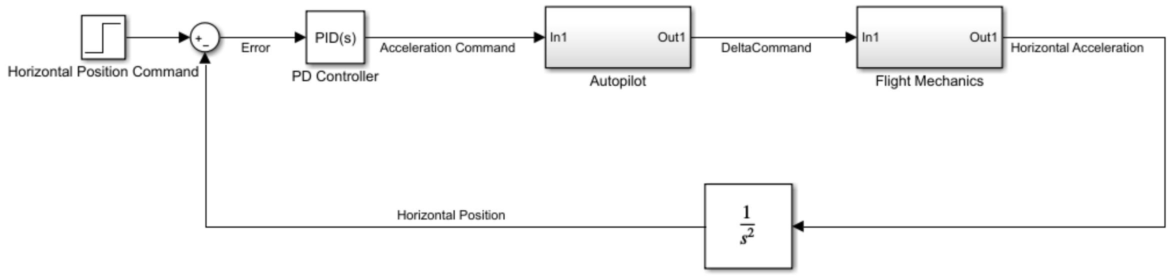

Fig. 4. Position Tracking Loop

In Fig. 4, the Simulink diagram of the position tracking loop is presented. The loop contains autopilot, flight mechanics and a controller. Horizontal position is reached with double integrating horizontal acceleration. A PD controller is chosen for position controller.

$$a_{y_{command_i}} = K_P\left(y_{desired_k}{}_i - y_i\right) + K_D \frac{d\left(y_{desired_k}{}_i - y_i\right)}{dt} \tag{16}$$

Acceleration commands are realized with acceleration autopilots for both pitch and yaw plane. Since this is a symmetric munition, autopilots for pitch and yaw are the same. State feedback design with acceleration error integral as a last state is applied. Fig. 5 and Fig. 6 show the autopilot structure and the step response, respectively.

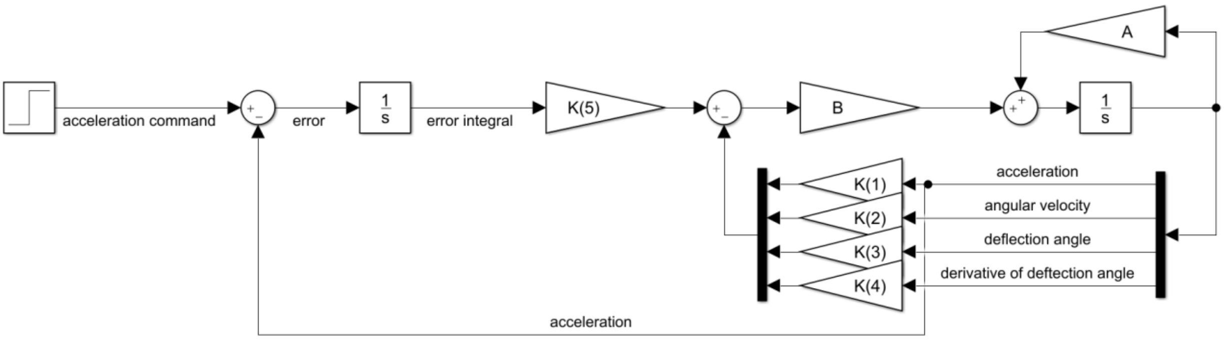

Fig. 5. Autopilot Scheme

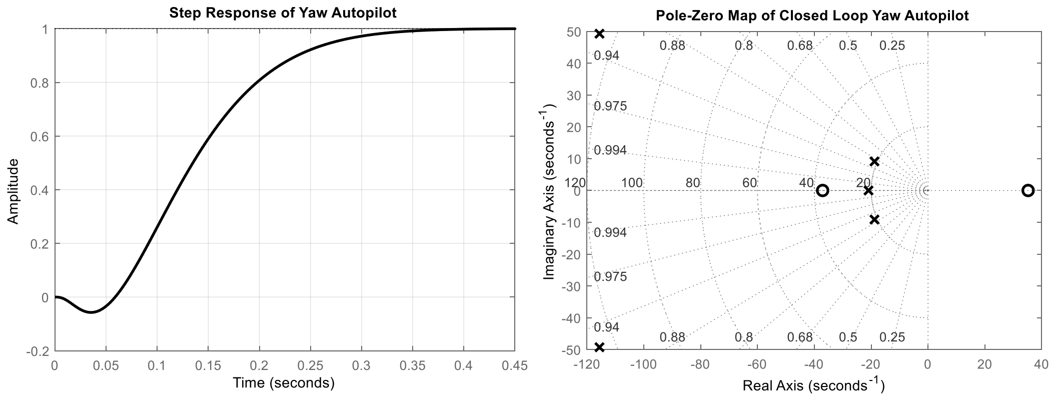

Fig. 6. Step Response and pole zero location of Acceleration Autopilots

As it is seen from Fig. 6., the closed loop dynamics of acceleration autopilot is much faster when it is compared to the outer-loop which is the position tracker. Therefore, the autopilot and system dynamics together are assumed to be ideal while designing the position controller.

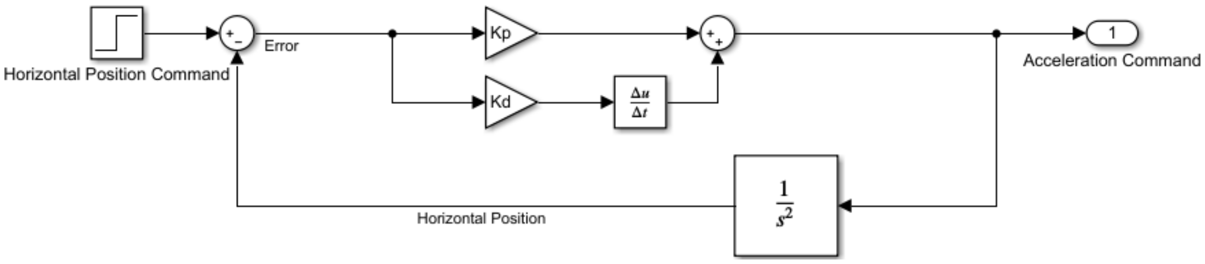

Fig. 7. Position Controller Scheme

Nonlinear simulation results show that this is a proper assumption and position trackers performance is very close to the linear design shown in Fig. 7.

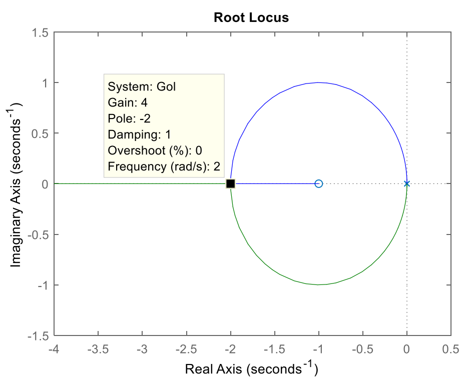

Fig. 8. Position Controller Root Locus

While designing a PD controller, for simplicity the zero of the controller is placed at -1 at first. Poles are located such that the final design will be critically damped. As a result of this both poles are placed at -2, and both gains are calculated as 4. Although the poles are placed on the real axis, the output has an overshoot because of a zero which is closer to the imaginary axis than the closed loop poles.

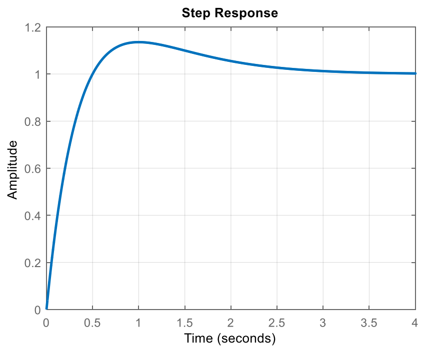

Fig. 9. Position Controller Step Response

Step response of the position controller can be seen in Fig. 9. Settling time, rise time around a second is considered as sufficient when the dynamics of the flight is considered. The maximum overshoot is not preferred more than %15 to not create any disorder for position controller.

2.4 Results and Discussion

The outputs of the formation control algorithm related with the motion of the munitions will be investigated in this section. The method needs at least one informed and one naïve agent to be applied. Guidance commands differ for informed and naïve agents. For informed agents, guidance commands are created to reach the target directly by changing its velocity vector toward the target position.

A Scenario with 1 informed agent and 8 naïve agents is simulated as an example. Target is located at 4000 meters away on the ground at the heading direction. The informed munition is released from 3000 meters altitude and uninformed agents are also released around that altitude to show the effect of altitude difference. The Informed agent is positioned at the center of the horizontal axis and naïve agents are placed to left and right of informed agent. The distance between agents is 10 meters for all. The informed agent is released 0.1 seconds before the naïve agents. The potential function constants are a=0.025, b=15, c=5 and boundary parameters are $d_{eff} = 60$, and $d_{af} = 20$.

Applied guidance commands and vertical trajectory of the informed agent can be seen in Fig. 10-11.

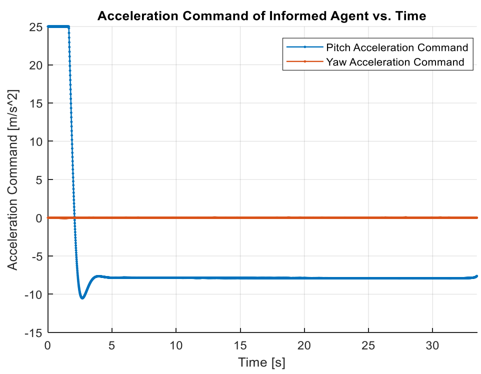

Fig. 10. Acceleration Commands of Informed Agent

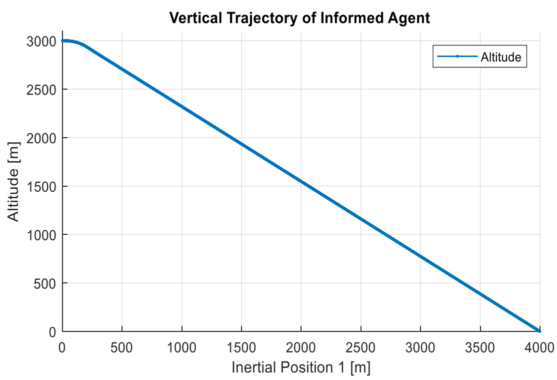

Fig. 11. Vertical Trajectory of Informed Agent

Since the target is in the heading direction of the agent, its acceleration command is around 0 at the yaw axis, but because of the large altitude difference between the target and release point of the munition, it has relatively large acceleration commands at the pitch axis of the body frame. The acceleration commands shown in Fig.12-13 are applied for the 2nd agent of the swarm, which is one of the naïve agents.

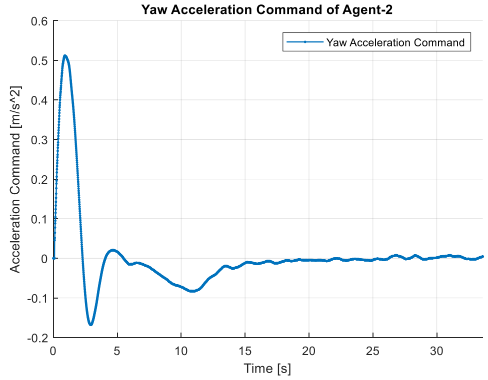

Fig. 12. Yaw acceleration Command of Agent-2

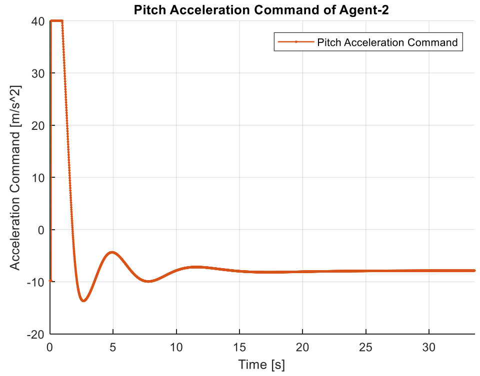

Fig. 13. Pitch Acceleration Command of Agent-2

Naïve agents follow the trajectory commanded by the potential function method at the horizontal plane. Low energy points are commanded with this method. The position controller generates the commands at yaw plane of the trajectory. Acceleration commands at the pitch axis are applied to follow the position of the informed agent. Therefore, similar trajectories are occurred at vertical axis even when altitude difference exists between different agents. In order to stay in the design regime of the linear acceleration autopilots and due to maneuver capabilities of the munition, acceleration commands for informed agents are limited at 25 m/s^2 and for naïve agents the limit is 40 m/s^2. Acceleration command limit is higher for naïve agents to ensure they can follow and align themselves according to informed agent. Horizontal and vertical trajectories of all agents is shown in Fig. 14-15.

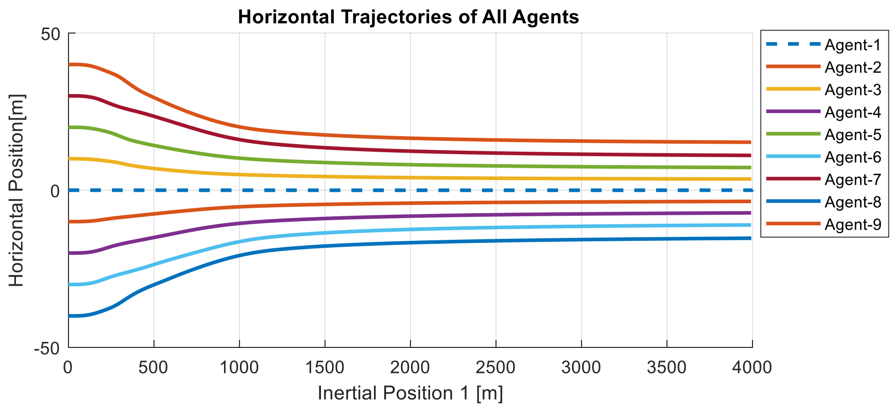

Fig. 14. Horizontal Trajectories of 9 Agents Flight

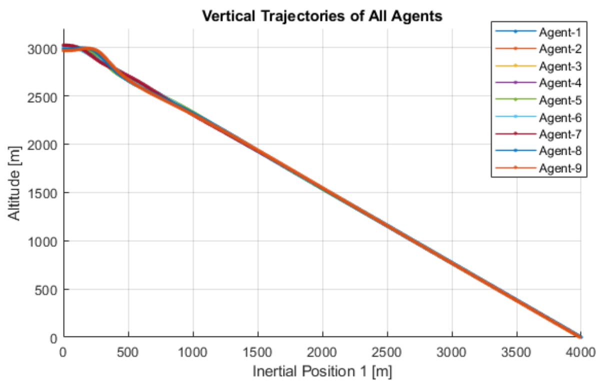

Fig. 15. Vertical Trajectories of 9 Agents Flight

Horizontal trajectory examples for different various number of agents are plotted in Figs 16-18. For each case number of naïve agents is increased. Different runs are compiled to get each plot.

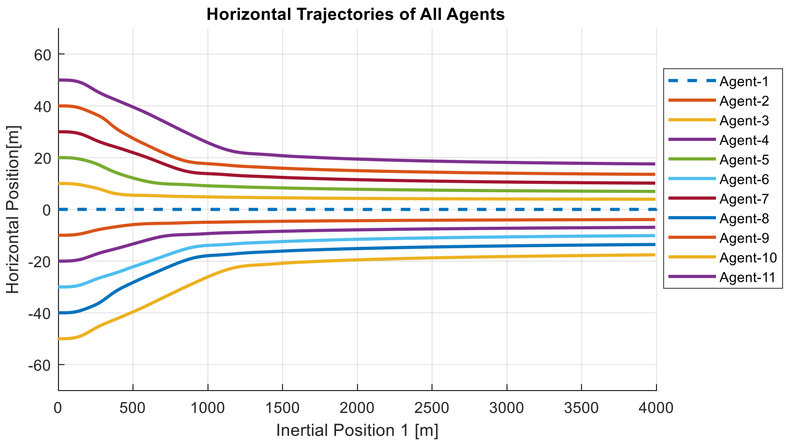

Fig. 16. Horizontal Trajectories of 11 Agents Flight

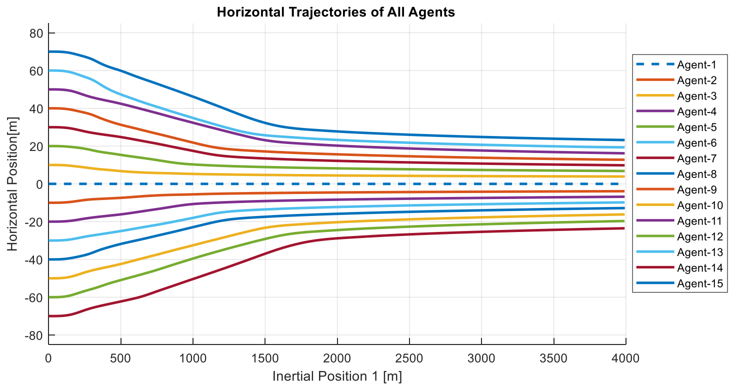

Fig. 17. Horizontal Trajectories of 15 Agents Flight

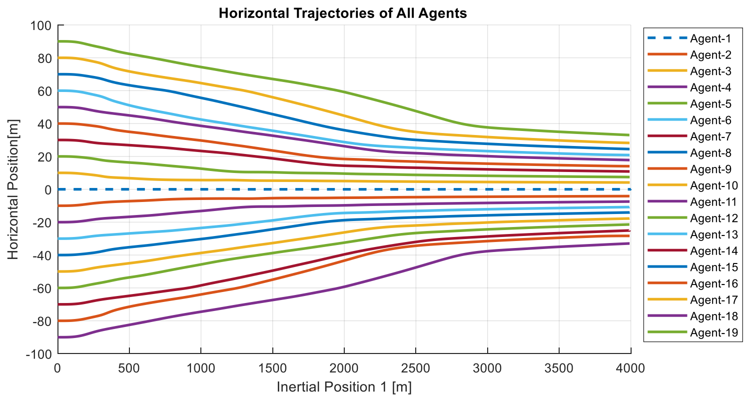

Fig. 18. Horizontal Trajectories of 19 Agents Flight

It is seen that for each case, the agents tend to come closer and fly with much more uniform state. Attraction and repulsion forces come into balance and horizontal position for each agent converge at some point. When the agent number increases, it takes more time to reach the steady state position. This is because of the increased number of interactions.

Potential function method can also provide formation transformation property to the swarm. Formation of the swarm might be changed during flight using different potatial functions at different intervals of flight. This feature might be desired in order to escape from air defence systems. Prediction of the swarm trajectory becomes more difficult when it is compared to a single potential function case.

Formation transformation scenario with 1 informed agent and 6 naïve agents is simulated. Target is located at 4000 meters away on the ground at the heading direction. Informed munition is released from 3000 meters altitude and uninformed agents are also released from the same altitude. The informed agent is located at the center of the horizontal axis and naïve agents are placed to left and right of the informed agent. The distance between agents is 10 meters for all. The informed agent is released 0.1 seconds before the naïve agents. Flight is divided into 3 different intervals. At each interval different potential function coefficients are used. Potential function coefficients used at different intervals are shown in the Table 2.

Table 2: Potential Function Coefficients for Different Intervals

a=0.25	a=0.025	a=0.025
b=50	b=15	b=50
c=40	c=5	c=5
$d_{eq} = 14.5$	$d_{eq} = 5.5$	$d_{eq} = 12.2$

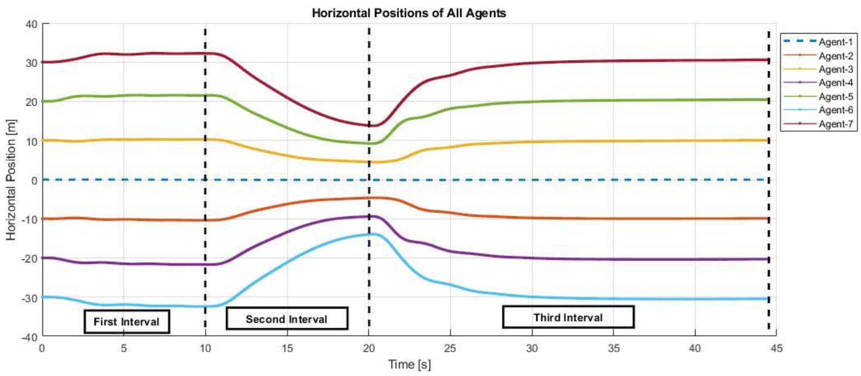

Fig. 19. Formation Transformation Scenario Horizontal Positions

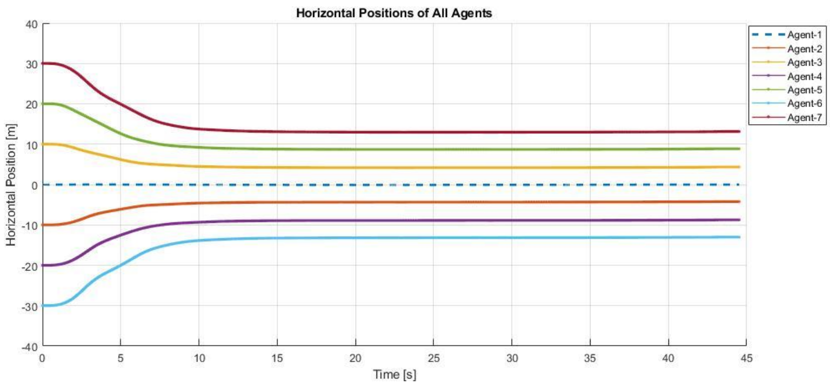

Fig. 20. Horizontal Positions with One Potential Function

An equilibrium point(d_{eq}) of a potential function is directly determined by coefficients a,b and c. The equilibrium point can be calculated for the case when there are only two agents but it is also effective when there are more than 2 agents. The wideness of the swarm is directly affected by the equilibrium point of the potential functions. At the first interval swarm converges to a larger distribution and then at the second interval agents come closer. At the last interval again, the agents spread away from each other. Fig. 20. shows the trajectory of all agents if the swarm use only the potential function at the second interval under the same initial conditions. Agents directly follow the related equilibrium positions in that case and the future positions of an agents can easily be predicted when it is compared to the formation transformation case.

3 Conclusion

Cooperative guidance methods are designed for different systems in time. Controlling multiple agents simultaneously is not also favorable but sometimes essential for UAV's, spacecrafts, unmanned ground vehicles, robots, quadrotors. Using multiple vehicles at the same time develops operational efficiency, decreases mission time, and enhances overall success of the mission.

The main objective of the study is to show that the communication between munitions can be used to shape the formation of these munitions during flight as it is used for quadrotors, UAVs, and ground vehicles. The communication is established using radio frequency datalink added to the system. Using the advantage of communication, some of the agents might be used as a cheaper munition with less equipment on them. Positions of neighbor agents are actively used to determine the position of all naïve agents. Potential function method is applied according to the attraction and repulsion forces on each naïve agent. The desired horizontal position is calculated and these positions are commanded to related autopilots.

Application of this method prevents agents to hit each other and also go away from each other. Therefore, it provides collision avoidance and uniform arm flight while going through the target. Acceleration commands of the formation control algorithm are less than $25\ m/s^2$ for informed agent and less than $40\ m/s^2$ for naïve agents at the pitch axis at the beginning and it decreases around gravity. On the other hand, at the yaw axis acceleration commands are around $0\ \ m/s^2$ with small arrangement commands. When the acceleration commands are considered it can be said that this method can be a useful solution to control the formation and guide a munition swarm.

Different simulation outputs are created for one informed and various number of naïve agents. At the vertical axis naïve agents follow the trajectory of the informed agent even when they are released from different altitudes. For all cases, swarm agents tend to fly in a regular combination and converge at some point of the flight. It is clearly seen that the munitions converge to a uniform formation later when the number of naïve agents increases. This is expected because there are more interactions to settle when there are more agents.

Formation control algorithms are one of the most important parts of cooperative guidance methods. The formation of the swarm may easily be changed with potential function constants. Moreover, the formation might be reshaped during the flight with the usage of different constants at different time intervals. In this study, formation changing with the potential function method is implemented to munition swarm for the first time in the literature. This property may increase the chance of reaching the target since it is harder to identify the motion of the members and it is harder to predict the future position of the munition.

Proposed formation control method is considered as a way of controlling no-thrust-air-to-ground munition swarm. This method may be supported with speed control methods using aerodynamic data and impact time control algorithms for the swarm munition would be investigated as a future work. Likewise, accelerometer, gyroscope, and data link equipment may be modeled with measurement errors and data transfer delay and effect of these may be studied in the future.

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