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Field-of-View Limited Guidance Law for Maneuvering Targets Using Relative Virtual Frame Formulation

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ABSTRACT

In this paper, an analytical guidance law is developed for homing missiles with field-of-view (FOV) constraints to intercept a maneuvering target from the desired approach direction. Firstly, using a relative virtual reference frame, the original problem of a constant-speed missile intercepting a maneuvering target is transformed into an equivalent problem of a speed-varying missile pursuing a stationary target. Then, a new sliding surface is designed under the relative virtual reference frame. The desired approach direction and FOV constraint can be achieved by making the missile system states reach the sliding surface and then stay on it. Finally, the finite time control technique is employed to derive a feedback guidance law, and numerical simulations are conducted under various scenarios to validate the effectiveness of the proposed methods.

Keywords: Homing guidance; Terminal angle constraint; FOV constraint; Maneuvering target

Nomenclature

a_m, a_t	=	Missile acceleration, target acceleration
a_{rn}	=	Relative acceleration normal to the relative velocity vector
$\alpha, c_1, c_2, k_1, k_2$	=	Guidance parameters
e_q	=	Approach angle error
$\gamma_m, \gamma_t, \gamma_r$	=	Missile flight-path angle, target flight-path angle, relative flight-path angle
q, q_d	=	Line-of-sight angle, desired line-of-sight angle
r	=	Relative range
ρ, ρ_r	=	Speed ratio, relative speed ratio
s	=	Sliding surface
$\sigma_m, \sigma_t, \sigma_r$	=	Missile leading angle, target leading angle, relative leading angle
$\sigma_m^{max}, \sigma_r^{max}$	=	Maximum permissible values of leading angle and relative leading angle
T_s, T_f	=	Convergence time, impact time
v_m, v_t, v_r	=	Missile speed, target speed, relative speed
$\boldsymbol{V_m}, \boldsymbol{V_t}, \boldsymbol{V_r}$	=	Missile velocity vector, target velocity vector, relative velocity vector
W_1, W_2	=	Lyapunov functions

1 Introduction

Over the past decade, there has been significant interest in the development of advanced guidance laws to achieve terminal angle constraints on the target. The terminal angle constraint is required for homing missiles in order to maximize the warhead lethality and escape the defense zone of the target. In the meanwhile, the FOV constraint is also critical for practical applications because curved missile trajectories may let the onboard seeker lose the target, especially when the target is maneuvering. Given this context, the objective of this paper is to design an effective homing guidance law for intercepting maneuvering targets while satisfying the terminal angle and FOV constraints simultaneously.

Numerous research endeavors have been dedicated to the design of terminal angle-constrained guidance laws. In one of the earliest studies [1], an optimal guidance law with impact angle constraint was presented by solving a time-to-go weighted optimization problem. A nonlinear impact angle guidance law was developed in [2] based on the concept of two-phase proportional navigation (PN) guidance, where the look angle and acceleration constraints were satisfied via a switched guidance gain. In [3], the physical meaning of optimal impact angle guidance laws was investigated by introducing a new alternative form of the guidance command. In [4], an augmented plane pursuit guidance law was proposed for achieving a desired impact vector in three-dimensional (3D) space. The impact angle and time constraints were simultaneously satisfied in recent works by utilizing the look-angle tracking approach [5] and the augmentation of PN guidance [6]. Note that the aforementioned studies primarily focused on deriving guidance laws for stationary targets. The impact angle guidance problem against maneuvering targets is said to be more challenging. In [7], a new methodology for impact angle control was proposed to against various target motions by involving an estimated terminal flight-path angle. Recently, the relative reference frame was employed as an effective way to solve the impact angle control problem for maneuvering targets [8–12]. Under the relative reference frame, two nonlinear optimal impact angle guidance laws were derived in [8, 9] and the closed-form solutions of system states were also obtained without any linearizing approximations. In [10], the conventional PN guidance was applied to derive a polynomial-based impact angle guidance law, where the terminal virtual look angle was constrained to zero. In [11], the desired terminal angle was fulfilled by defining a virtual acceleration command that is related to the distance between the missile and target. In [12], a cooperative guidance law for attacking a maneuvering target with desired terminal angles was developed using the integral sliding manifold and fixed-time control approach.

The FOV constraint is important for practical implementation, as the target maneuver may cause the missile to lose the target. Consequently, it is worthwhile to develop advanced FOV-limited guidance laws to maintain the target lock-on. For the two-dimensional (2D) FOV-limited guidance problem, a look-angle profile that ensures the desired impact angle and time was introduced in [13]. For the 3D FOV-limited guidance problem, an impact angle guidance law was proposed in [14], where the length of flight trajectories was independent of missile speed variations. A cooperative circular guidance law was proposed in [15] with a biased command for controlling the time-to-go values and satisfying the FOV constraint. A unified FOV-limited guidance framework was introduced in [16], which can be used to limit the leading angle of most existing guidance laws. In [17], a novel backstepping guidance strategy and a nonlinear mapping approach were combined together to develop an impact angle guidance law under the seeker's FOV constraint. In [18], an analytical FOV-limited impact angle guidance law was introduced by decoupling the model into two components, effectively addressing the separate leading angles constraint problem under the 3D guidance model. In [19], an integral barrier Lyapunov function was utilized to derive a 2D FOV-limited impact angle guidance law for attacking maneuvering targets. In [20], a sliding surface that satisfies the impact angle and FOV constraints was introduced using the magnitude-limited sigmoid function. Furthermore, in [21], a new nonlinear error dynamics was derived to guide the missile to achieve the impact time, angle, and FOV constraints in 3D space.

As noted in the aforementioned studies, there is still a lack of advanced guidance laws capable of intercepting maneuvering targets while also meeting the terminal approach angle and FOV constraints. Therefore, this paper is in attempt to address such a challenging issue. Specifically, by introducing a relative virtual reference frame, the original problem of a constant-speed missile intercepting a maneuvering target is transformed into an equivalent scenario, where a speed-varying missile is engaging a stationary target. Then, a sufficient condition for satisfying the FOV constraint is derived and a new sliding surface is introduced under the relative virtual reference frame. Finally, a feedback guidance law is developed based on the finite-time control technique, which can achieve our guidance objectives by making the missile system states reach the sliding surface.

The rest of this paper is organized as follows. In Sec. 2, the engagement geometry and nonlinear kinematics are presented. In Sec. 3, a sufficient condition for meeting the FOV constraint is proposed and then a FOV-limited approach angle guidance law is developed. Simulation results validating the proposed guidance law are given in Sec. 4.

2 Problem Statement

The planar homing engagement geometry for a missile M and a maneuvering target T is depicted in Fig. 1, where (X_I, Y_I) denotes the inertial reference frame. In this paper, the missile and target are assumed to travel with constant speeds v_m and v_t, respectively, i.e., the missile and target control accelerations a_m and a_t are always perpendicular to their velocity vectors. Furthermore, the relative range and line-of-sight (LOS) angle between the missile and target are expressed as r and q, respectively. The flight-path angle and velocity leading angle of the missile and target are denoted by γ_m, γ_t, σ_m, and σ_t, respectively.

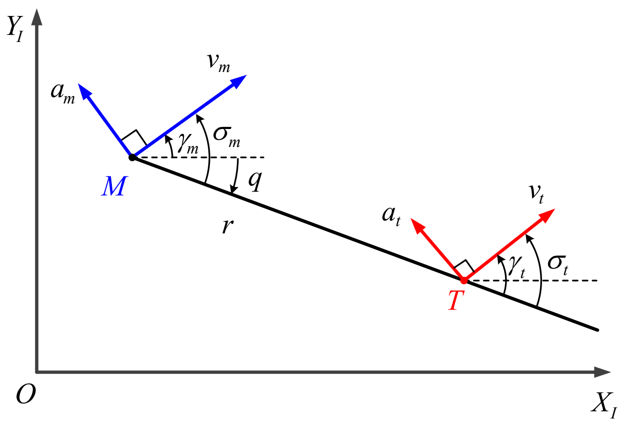


Fig. 1 Engagement geometry.

The nonlinear engagement kinematics can be given by

$$\dot{r} = v_t \cos\sigma_t - v_m \cos\sigma_m \tag{1a}$$

$$r\dot{q} = v_t \sin\sigma_t - v_m \sin\sigma_m \tag{1b}$$

$$\dot{\gamma}_t = a_t/v_t, \quad \dot{\gamma}_m = a_m/v_m \tag{1c}$$

$$\sigma_m = \gamma_m - q, \quad \sigma_t = \gamma_t - q \tag{1d}$$

The relative velocity vector $\boldsymbol{V_r}$ is defined as $\boldsymbol{V_r} = \boldsymbol{V_m} - \boldsymbol{V_t}$. The variables v_r and γ_r stand for the relative speed and relative flight-path angle, respectively. Then, the engagement kinematics in the reference virtual frame can be described as [8, 10, 11]

$$\dot{r} = -v_r \cos\sigma_r \tag{2a}$$
$$\dot{q} = -v_r \sin\sigma_r / r \tag{2b}$$
$$\dot{\gamma}_r = a_{rn}/v_r \tag{2c}$$
$$a_{rn} = a_m \cos(\gamma_r - \gamma_m) - a_t \cos(\gamma_r - \gamma_t) \tag{2d}$$

where $\sigma_r = \gamma_r - q$ is the relative velocity leading angle, a_{rn} represents the component of relative acceleration normal to the relative velocity vector. Accordingly, the original guidance problem of a constant-speed missile against a maneuvering target is transformed into the one against a stationary target. The main advantage of this transformation is that the relative leading angle can be utilized to facilitate the guidance law design for the maneuvering target.

The objective of this paper is to design a normal acceleration command a_m such that the missile can intercept the maneuvering target with a desired approach angle q_d, while satisfying the FOV constraint over the whole flight. Under the assumption of small attack angle and sideslip angle, the FOV constraint is equivalent to a constraint on the leading angle as $|\sigma_m| \leq \sigma_m^{max} < \pi/2$, where σ_m^{max} is the maximum permissible value of leading angle.

3 Guidance Law Design

In this section, we first introduce a sufficient condition for satisfying the FOV constraint under the relative virtual reference frame. Subsequently, a sliding surface based on this sufficient condition is proposed. If the sliding surface variable remains bounded and converges to zero before the final impact, both the desired approach angle and FOV constraint can be achieved. After that, a feedback guidance law that enforces the missile system states to reach the sliding surface is developed by utilizing the finite-time control technique.

3.1 FOV-Limited Sliding Surface

In this paper, we assume that the initial values of both the missile leading angle and relative leading angle are within the permissible set. In addition, it is supposed that the missile has a speed advantage over the target, i.e. $V_m > V_t$. This is a commonly employed assumption, as the typical speed of a rapid sailing warship is significantly lower than that of missiles. Before designing the sliding surface, we present the following lemma as a sufficient condition for the leading angle constraint.

Lemma 1 Considering the kinematics model (2), if the relative leading angle always satisfies $|\sigma_r| \leq \sigma_r^{max}$ where $\sigma_r^{max} = \sigma_m^{max} - \arccos\sqrt{1-\rho^2}$ and $\rho = v_t/v_m < 1$ is the speed ratio, then the missile can meet the FOV constraint.

Proof: According to the geometric relationship shown in Fig. 1, the relative speed and the relationship between γ_m and γ_r can be calculated as

$$v_r = \sqrt{v_m^2 + v_t^2 - 2v_m v_t \cos(\gamma_m - \gamma_t)} \tag{3a}$$
$$\gamma_m = \gamma_r + \arccos\delta \tag{3b}$$

where $\delta = (1 - \rho^2 + \rho_r^2)/(2\rho_r)$, $\rho_r = v_r/v_m$ is the relative speed ratio. According to Eq. (3a), the mathematical relationship among ρ and ρ_r can be expressed as

$$\rho_r = \sqrt{1 + \rho^2 - 2\rho\cos(\gamma_m - \gamma_t)} \tag{4}$$

From Eq. (4) and $\cos(\gamma_m - \gamma_t) \in [-1, 1]$, we can conclude that $\rho_r \in [1-\rho, 1+\rho]$. The derivative of δ with respect to ρ_r can be expressed as

$$\frac{\partial \delta}{\partial \rho_r} = -\frac{1-\rho^2}{2}\frac{1}{\rho_r^2} + \frac{1}{2} \tag{5}$$

For $\rho_r \in [1-\rho, 1+\rho]$, the behavior of δ can be investigated using Eq. (5) as follows

$$\partial\delta/\partial\rho_r < 0, \quad \rho_r \in (1-\rho, \sqrt{1-\rho^2}) \tag{6a}$$

$$\partial\delta/\partial\rho_r = 0, \quad \rho_r = \sqrt{1-\rho^2} \tag{6b}$$

$$\partial\delta/\partial\rho_r > 0, \quad \rho_r \in (\sqrt{1-\rho^2}, 1+\rho) \tag{6c}$$

According to the above three cases, it can be deduced that δ reaches its minimum value $\delta_{min} = \sqrt{1-\rho^2}$ when $\rho_r = \sqrt{1-\rho^2}$; and δ reaches its maximum value $\delta_{max} = 1$ when $\rho_r = 1 \pm \rho$. Recalling the geometric relation defined in relative virtual frame, we have

$$\sigma_m = \gamma_m - q = \gamma_r + \arccos\delta - q = \sigma_r + \arccos\delta \tag{7}$$

Thus the FOV constraint $|\sigma_m| \le \sigma_m^{max}$ is equivalent to

$$|\sigma_r + \arccos\delta| \le \sigma_m^{max} \tag{8}$$

By utilizing the inequality $|\sigma_r + \arccos\delta| \le |\sigma_r| + |\arccos\delta|$, we establish a sufficient condition for Eq. (8)

$$|\sigma_r| + |\arccos\delta| \le \sigma_m^{max} \tag{9}$$

It is obvious that as long as Eq. (9) remains valid throughout the engagement process, the FOV constraint can be satisfied. Substituting δ_{min} into Eq. (9), we obtain a sufficient condition as employed in this paper for the FOV constraint, which is expressed as $|\sigma_r| \le \sigma_m^{max} - \arccos\delta_{min}$. That completes the proof of Lemma 1.

Remark 1 As a sufficient condition for meeting the FOV constraint, Lemma 1 converts the constraint on the missile leading angle into a constraint on the relative leading angle, which facilitates the design of FOV-limited guidance laws for maneuvering targets. Furthermore, the condition of $\rho_r \in [1-\rho, 1+\rho]$ is equivalent to $v_r \in [v_m - v_t, v_m + v_t]$. Due to $\rho << 1$, the variation range of the relative speed v_r is sufficiently small. Thus, it is reasonable to replace the relative speed v_r with $v_m - v_t$ or $v_m + v_t$. The subsequent derivations are conducted under the assumption that the relative speed remains constant.

To proceed with our analysis, a variable represents the relative speed perpendicular to the LOS is formulated as

$$V_q = r\dot{q} = -v_r \sin\sigma_r \tag{10}$$

The constraint on the relative leading angle can be transformed into a constraint on the variable V_q

$$\left|V_q\right| \le v_r \sin\sigma_r^{max} = \kappa \tag{11}$$

where σ_r^{max} has been defined in Lemma 1. Taking the time derivative of V_q, we have

$$\dot{V}_q = -\dot{r}\dot{q} - a_{rn}\cos\sigma_r \tag{12}$$

For the FOV-limited approach angle guidance problem, a sliding surface is defined as

$$s = c_1 \frac{V_q}{\kappa^2 - V_q^2} + c_2 e_q \tag{13}$$

where $e_q = q - q_d$ is the approach angle error, $c_1 > 0, c_2 > 0$ are guidance parameters. The sliding surface as given by Eq. (13) is referred to as the FOV-limited sliding surface in this paper.

To the best of our knowledge, the proposed FOV-limited sliding surface (13) may has the following advantages:

1) If the FOV constraint is always satisfied throughout the engagement, the sliding surface s remains bounded; otherwise, s will become singular. Hence, the FOV constraint condition $|V_q| \le \kappa$ is equivalently expressed as the boundedness of the sliding surface s.
2) If the missile system states reach the sliding surface and stay on it before the interception, the approach angle error dynamics can be found as

$$\dot{e}_q = -c_2 e_q (\kappa^2 - V_q^2)/(c_1 r) \tag{14}$$

It is obvious that on the sliding surface, the approach angle error e_q will converge to zero automatically. Therefore, the approach angle guidance problem is address by enforcing the missile states reach the designed sliding surface.

Therefore, the guidance problem with approach angle and FOV constraints has been transformed into a stabilization control problem in the context of the sliding surface (13). In the next section, we will introduce a feedback guidance law to ensure that the missile system states can reach the sliding surface before interception.

3.2 FOV-Limited Approach Angle guidance law

In this subsection, we utilize the proposed FOV-limited sliding surface (13) to formulate a feedback guidance law. Taking the time derivative of s leads to

$$\dot{s} = c_1 \mu \dot{V}_q + c_2 \dot{e}_q \tag{15}$$

where $\mu = (\kappa^2 + V_q^2)/(\kappa^2 - V_q^2)^2$. Using the concept of finite-time control [22], we propose a FOV-limited approach angle guidance law

$$a_{rn} = -\frac{\dot{r}\dot{q}}{\cos\sigma_r} + \frac{c_2 \dot{q} + k_1 s + k_2 \operatorname{sgn}^{\alpha}(s)}{c_1 \mu \cos\sigma_r} \tag{16}$$

where $k_1 > 0, k_2 > 0, 0 < \alpha < 1$ are parameters to adjust the sliding surface convergence rate. Substituting the designed guidance law (16) into Eq. (12) results in

$$\dot{V}_q = -\frac{c_2 \dot{q} + k_1 s + k_2 \operatorname{sgn}^{\alpha}(s)}{c_1 \mu} \tag{17}$$

Then the time derivative of sliding surface (13) can be derived as

$$\dot{s} = -k_1 s - k_2 \operatorname{sgn}^{\alpha}(s) \tag{18}$$

Consider a Lyapunov function $W_1 = 0.5s^2$. Taking the time derivative of W_1 and substituting Eq. (16) into it, we find

$$\dot{W}_1 = s\dot{s} = -k_1 s^2 - k_2 |s|^{\alpha+1} = -2k_1 W_1 - \sqrt{2}^{\alpha+1} k_2 W_1^{\frac{\alpha+1}{2}} \tag{19}$$

According to the results in [22], the above equation implies that the missile system states can converge to the sliding surface at T_s and stay on it until the end of the engagement, and the convergence time T_s is obtained as

$$T_s \leq \frac{1}{k_1(1-\alpha)} \ln(1 + \frac{k_1}{k_2}(2W_1(0))^{\frac{\alpha-1}{2}}) \tag{20}$$

where $W_1(0)$ is the initial value of the Lyapunov function.

Note that the actual guidance command for the missile can be reformulated from Eq. (2d) as

$$a_m = \frac{a_{rn} + a_t \cos(\gamma_r - \gamma_t)}{\cos(\gamma_r - \gamma_m)} \tag{21}$$

in which the target acceleration a_t is assumed to be measurable to the missile seekers or radar systems. Owing to the utilization of the relative leading angle, the FOV constraint is achieved without the need for complex switching logic or multi-stage guidance designs. This approach ensures smooth guidance commands and facilitates practical implementation.

Remark 2 The convergence time T_s is determined by both the guidance parameters and initial launch conditions. As a result, the mission designer needs to select the guidance parameters c_1, c_2, k_1, and k_2 carefully to ensure $T_s < T_f$ in practical applications. Recalling the results in Remark 1, the minimum impact time can be calculated as $T_{f,min} = r_0/(v_m + v_t)$, where r_0 is the initial relative range. Thereby, $T_{f,min}$ can be used as the settling time of the sliding surface, i.e., $T_s = T_{f,min}$, which provides a guideline for parameter selection.

Remark 3 Most existing guidance laws with FOV constraints only aim to maintain the leading angle within its permissible interval $[-\sigma_m^{\max}, \sigma_m^{\max}]$. Nonetheless, the leading angle could still approach the upper or lower bounds of $[-\sigma_m^{\max}, \sigma_m^{\max}]$ during the guidance procedure. This will increase the risk of violating the FOV constraint, especially when the target is maneuvering or when there are external disturbances. As indicated in Lemma 1, if $\sigma_m > 0$, the actual maximum value of the leading angle is always smaller than the maximum permissible value $\sigma_m^{\max}$, and vice versa. In other words, there will be safety margins between the peak values of leading angle and the bounds of $[-\sigma_m^{\max}, \sigma_m^{\max}]$. This property can be considered as an advantage of the proposed guidance law.

3.3 Analysis of Proposed Guidance Law

First, we analyze the dynamics of the leading angle on the sliding surface. Differentiating σ_r and substituting $s = \dot{s} = 0$ into it, we have

$$\dot{\sigma}_r = \dot{\gamma}_r - \dot{q} = -\frac{c_2}{c_1 r \mu} \tan \sigma_r \tag{22}$$

Consider a Lyapunov function $W_2 = 0.5\sigma_r^2$. Taking the time derivative of W_2 and substituting Eq. (22) into it, we find

$$\dot{W}_2 = \sigma_r \dot{\sigma}_r = -\frac{c_2}{c_1 r \mu} \sigma_r \tan \sigma_r \leq 0 \tag{23}$$

Therefore, after reaching the sliding mode surface, the relative leading angle will automatically converge to zero. From Eq. (16) and Eq. (21), it can be observed that there are no singularity issues with the proposed guidance law. After that, one can further deduce from Sec.3.1 and Sec. 3.2 that $s, \dot{s}, e_q, \dot{q}$ will all converge to zero when $r \to 0$. Substituting the above conclusions into the proposed guidance law yields

$$\lim_{r\to 0} a_{rn} = 0, \quad \lim_{r\to 0} a_m = a_t \frac{\cos(\gamma_r - \gamma_t)}{\cos(\gamma_r - \gamma_m)} \tag{24}$$

The preceding discussion addresses the singularity of our guidance law and illustrates that the normal acceleration of the relative speed acceleration a_{rn} can converge to zero at the moment of impact. Additionally, as shown in Eq. (24), the missile acceleration at the final moment is only dedicated to compensating for the target maneuvers.

4 Numerical Simulations

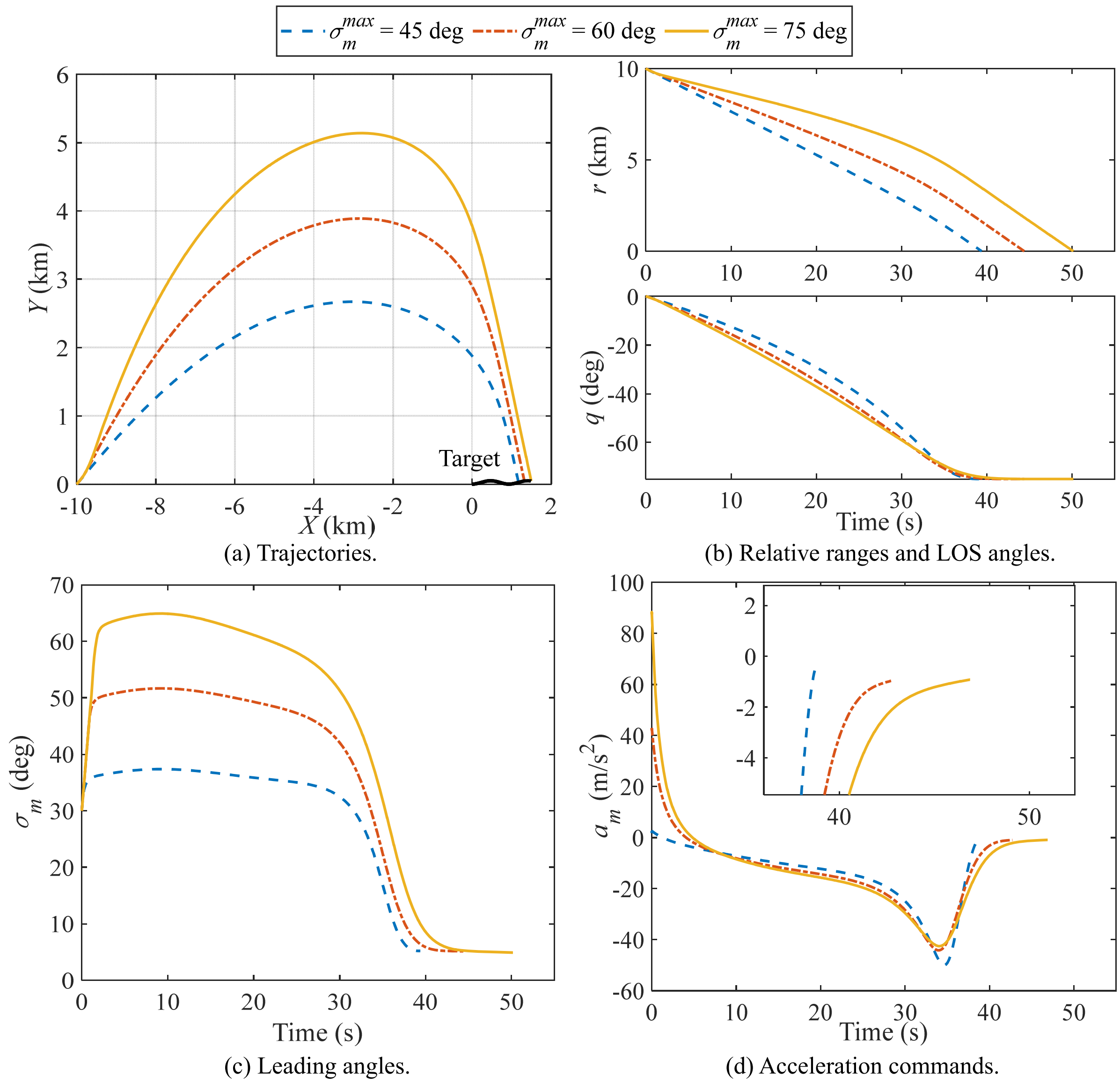


Fig. 2 Simulation results for the proposed guidance law under various FOV constraints.

In this section, we demonstrate the effectiveness of the proposed FOV-limited guidance law through a series of numerical simulations. For all simulation cases, the initial relative range between the missile and target is selected as 10 km, and the initial positions of the missile and target are selected as $(-10, 0)$ and $(0, 0)$ km. The constant speed of the missile and the target are 330 and 30 m/s, respectively. The same parameter values, $c_1 = 30$, $c_2 = 1$, $k_1 = 1$, and $k_2 = 1$ are employed throughout this section. The missile maximum acceleration is selected as 100 m/s^2, and the simulation stops when the relative range is smaller than 1.0 m.

In the first group of simulations, the initial flight-path angle of the missile and target are 30° and 0°, respectively, and the target performs a cosine maneuver $a_t = \cos(t/5)$ m/s^2. The desired approach angle is fixed to -75°, whereas the FOV constraint varies from 45° to 75°. Under these conditions, we present the simulation results for the FOV-limited approach angle guidance law in Fig. 2. As can be observed from Fig. 2a and 2b, all three missiles successfully intercept the maneuvering target while satisfying FOV and approach angle constraints. In Fig. 2c, the designed sliding surface effectively prevents the leading angle from exceeding its maximum permissible values. Furthermore, as shown in Fig. 2d, the missile accelerations remain continuous throughout the whole engagement. When a large magnitude of FOV constraint is selected, the trajectory turns away as soon as the missile is launched, resulting in a larger magnitude of initial acceleration commands and a longer missile trajectory. As the relative leading

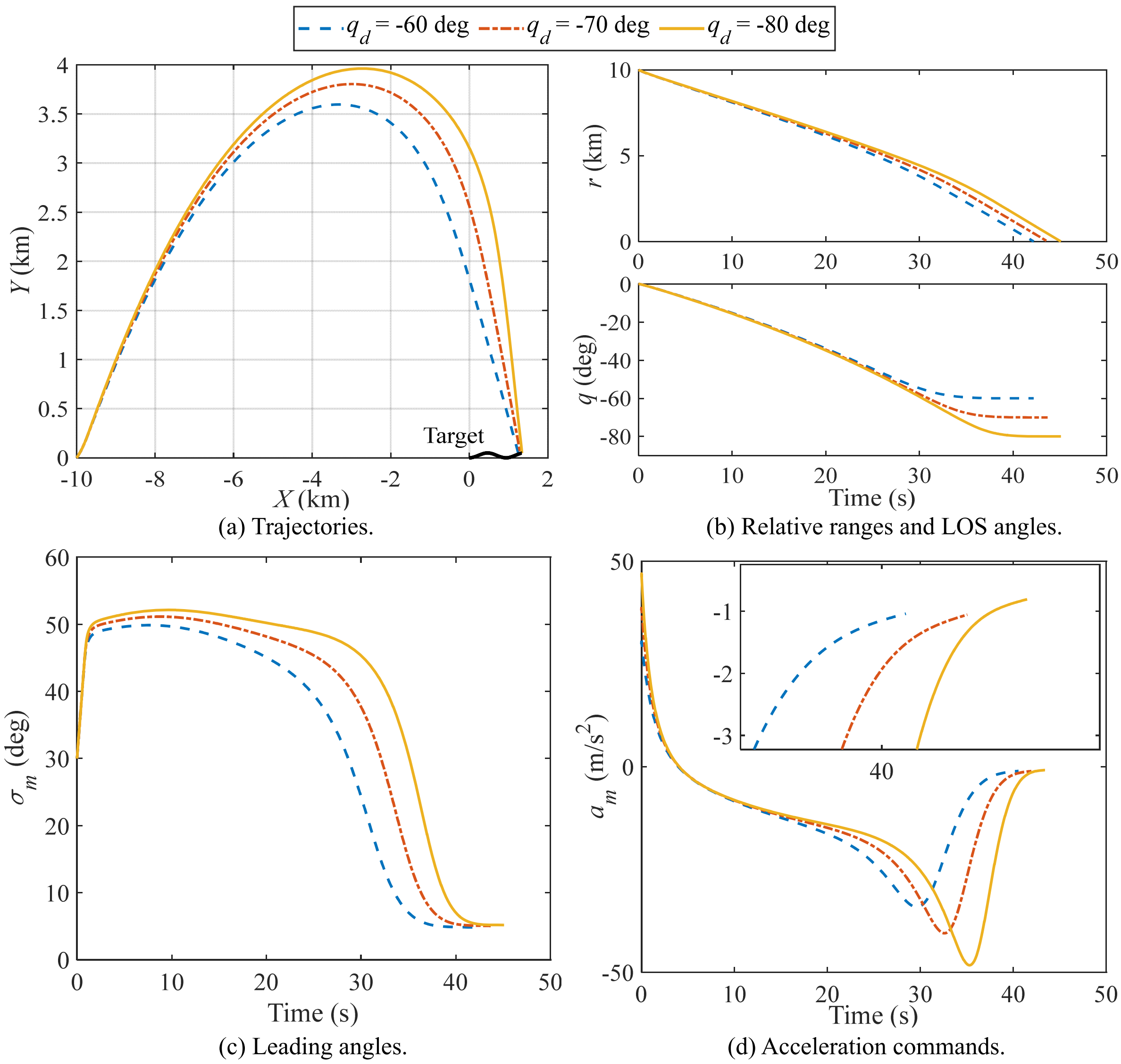


Fig. 3 Simulation results for the proposed guidance law under various approach angles.

angles converge to zero, acceleration commands leave only the component necessary for compensating the target maneuver.

In the second group of simulations, the FOV-limited approach angle guidance law is utilized to achieve various desired approach angles under a selected FOV constraint of 60°. All other simulation conditions are the same as those used in the first set of simulations. As depicted in Fig. 3, in each scenario, the missile performs essential maneuvers and successfully intercepts the target from desired directions. Even though the target is maneuvering, the FOV-limited approach angle guidance law can prevent the leading angle from exceeding its maximum permissible values. Furthermore, in Fig. 3d, it is illustrated that larger peak acceleration commands are generated as the desired approach angles increases. These simulation results demonstrate the effectiveness of the proposed FOV-limited guidance law in confining the missile leading angle and generating smooth guidance commands.

To further validate the capture capability of the proposed FOV-limited approach angle guidance law, different target maneuvers are introduced in this group of simulations. The target maneuver patterns are provided in Table 1. The FOV constraint is selected as 60° and the desired approach angle is selected as -60°. Considering the simulation conditions in Table 1, the results of the proposed FOV-limited approach angle guidance law for different target maneuvers are shown in Fig. 4. The simulation results in Fig. 4

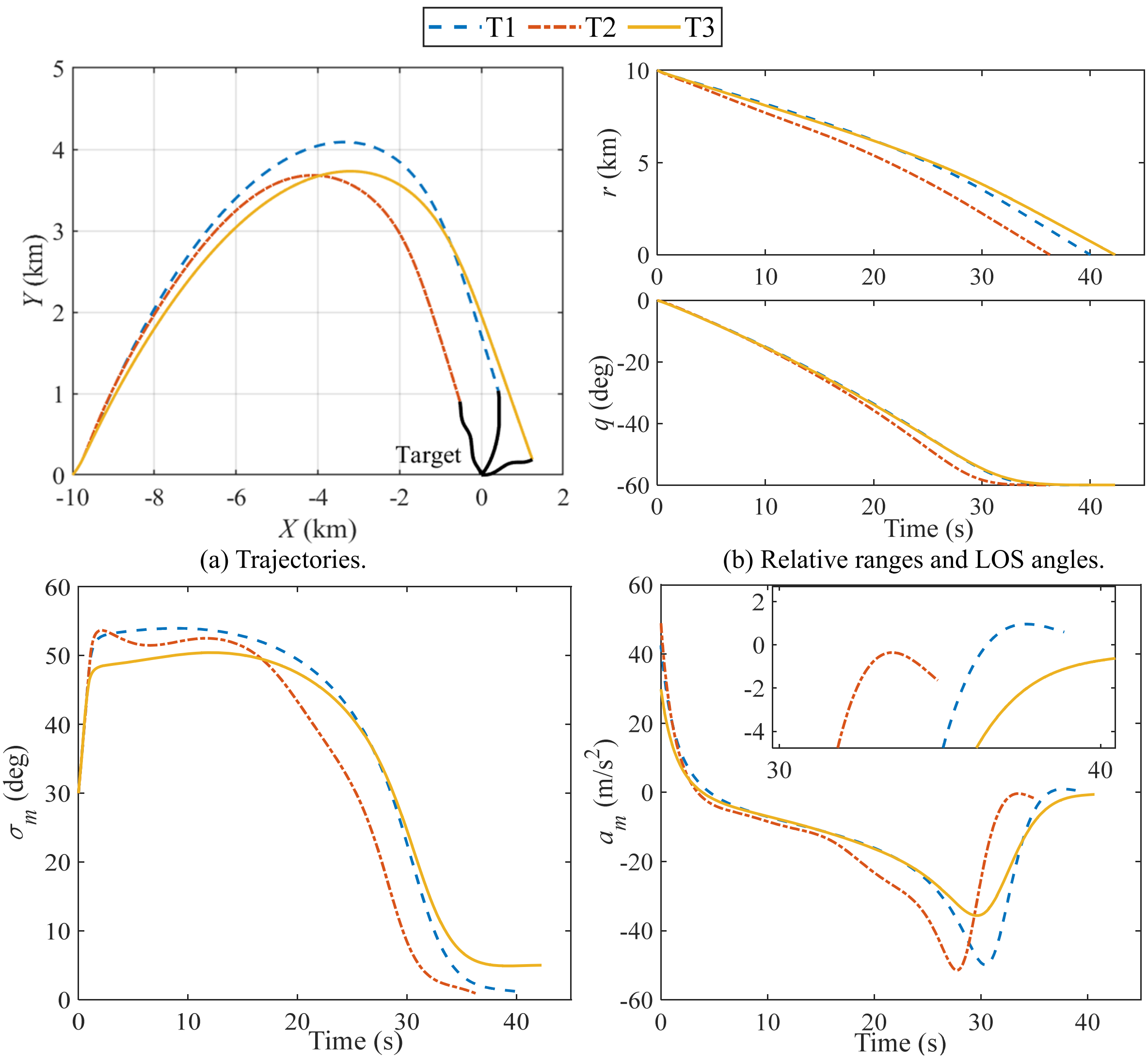
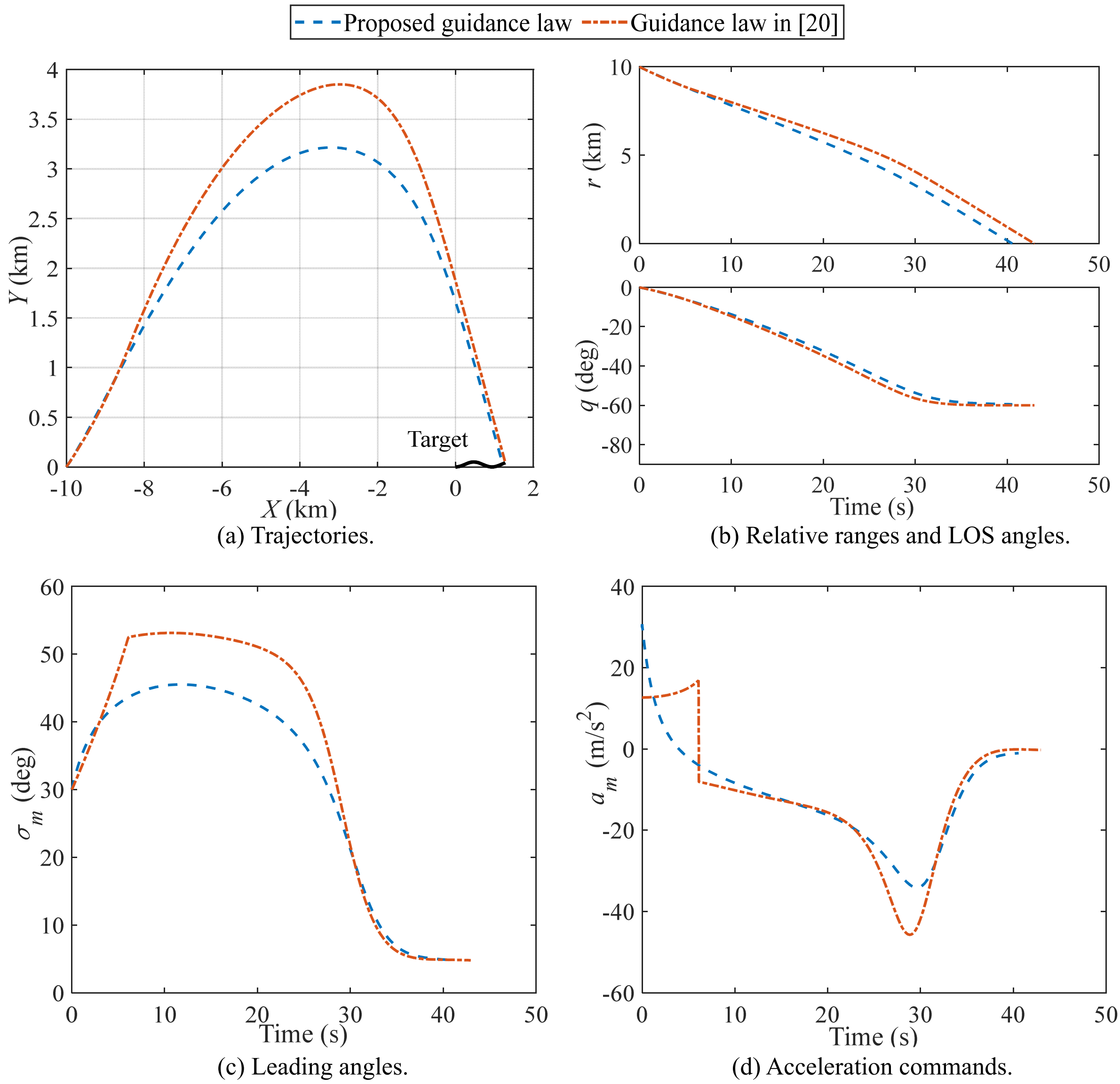


Fig. 4 Simulation results for the proposed guidance law under various target maneuvers.

Table 1 Simulation conditions

Target	Initial flight-path angle γ_t (deg)	Target acceleration a_t (m/s^2)
T1	30	1
T2	120	4*cos($t/3$)
T3	0	1*sin($t/3$)

demonstrate that the finite-time guidance law can enforce the missile to satisfy the desired approach angle and FOV constraint for various target maneuvers.



(a) Trajectories. (b) Relative ranges and LOS angles. (c) Leading angles. (d) Acceleration commands.

Fig. 5 Simulation results of comparison study.

A FOV-constrained guidance law was developed in [20] to achieve desired approach angles on maneuvering targets. To demonstrate the superiority the proposed guidance law, its performance is compared with that of the advanced guidance law of [20]. The simulation results of our comparison study are shown in Fig. 5. It can be observed that both guidance laws satisfy the desired approach angle and confine the leading angle within 60°. However, as seen in Fig. 5(d), the guidance command of the guidance law in [20] is discontinuous, which may degrade the accuracy of the inner-loop attitude controller. In contrast, our guidance law generates smooth guidance commands and does not encounter any singularities.

5 Conclusion

This paper proposed an analytical guidance law to achieve the desired approach direction and FOV constraint for the problem of a homing missile against a maneuvering target. A relative virtual reference frame was used to transform the original FOV-limited guidance problem for intercepting a maneuvering target into an equivalent problem against a stationary target. Subsequently, within the relative virtual reference framework, a novel sliding surface was designed based on the relative leading angle. The desired approach direction and FOV constraint were realized by guiding the missile system states to reach and stay on the sliding surface. Finally, a feedback guidance law was derived utilizing the finite-time sliding control approach. Numerical simulations demonstrated that the proposed method has a superior ability to capture maneuvering targets from desired directions and confine the missile leading angle within its maximum permissible range. However, the impact time control against maneuvering targets is required for further study to form a salvo attack with multiple missiles.

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