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Trajectory Optimization of an Air Defense Missile with Model Predictive Control

Emre Saylam Engineer, Roketsan Missile Industries Inc., Air Defense Systems, 06780, Ankara, Türkiye. emre.saylam@roketsan.com.tr

Nebi Bulut Engineer, Roketsan Missile Industries Inc., Air Defense Systems, 06780, Ankara, Türkiye. nebi.bulut@roketsan.com.tr

Rüştü Berk Gezer Engineer, Roketsan Missile Industries Inc., Air Defense Systems, 06780, Ankara, Türkiye. bgezer@roketsan.com.tr

Fatih Turgel Engineer, Roketsan Missile Industries Inc., Air Defense Systems, 06780, Ankara, Türkiye. fatih.turgel@roketsan.com.tr

ABSTRACT

For an air defense missile, guidance method used in the different phases of the flight greatly shapes the performance of the system. In this paper, an optimal guidance method using model predictive control to increase effective range of an air defense missile is presented. The model used for the model predictive control includes a generic drag force modeling, which is variable with the missile altitude and velocity, missile thrust profile, which is not assumed constant, constrained acceleration commands, and realistic autopilot dynamics. In the proposed optimal guidance method, missile acceleration limits, desired interception point and required acceleration are used in the cost function by considering three-dimensional engagement dynamics. Effectiveness of this guidance method is compared with other summarized guidance laws through simulations.

Keywords: Model Predictive Control; Trajectory Optimization; Guidance;

Nomenclature

α, β	=	Flight path angles
$\hat{C}^{(A,B)}$	=	Transformation matrix from reference frame B to A
$\vec{P}_M$	=	Missile position vector
$\vec{V}_M$	=	Missile velocity vector
$\vec{a}$	=	Acceleration vector
$\vec{\omega}$	=	Angular velocity vector
ρ_{atm}	=	Atmosphere density
T	=	Thrust
g	=	Gravitational acceleration
m	=	Missile mass
C_x	=	Aerodynamic drag coefficient

1 Introduction

For a lock after launch air defense missile, until missile's seeker locks onto the target, missile is guided using information supplied by the ground radar. This phase is called "midcourse guidance" phase. For practical applications, there are several widely used midcourse guidance methods. These methods differ into classical guidance laws such as proportional navigation (PN) based guidance and modern guidance laws based such as optimal control guidance. According to Ref. [1], PN based guidance methods generally make use of target's position and velocity, but they do not take into account possible target maneuvers in guidance law design. Modern guidance laws do not directly depend on missile-target geometrical rule; instead, they are derived from mathematical representations of the missile system dynamics.

In the literature, for surface to air missiles there are many midcourse guidance methods. In Ref. [2], a time dependent bias, which is calculated using missile – target range, PN is used to create acceleration commands. In Ref. [3], a two stage PN-based guidance law for impact angle constrained is explained. During flight, proportional navigation gain is updated according to desired impact angle. On the other hand, the study in Ref. [4] uses optimal control to minimize energy based cost function while estimating time to go. In Ref. [5], three-dimensional trajectory shaping guidance law satisfying a terminal impact angle is proposed named as generalized explicit guidance (GENEX). Furthermore, in Ref. [6], a model predictive static programming method is applied to nonlinear systems by using a suboptimal control technique. While forming an energy efficient trajectory, it satisfies certain alignment constraints in azimuth and elevation channels.

In this paper, firstly, realistic system dynamics model is obtained for a generic air defense missile. This model is a real time representation of the missile's aerodynamics and propulsion characteristics under all flight conditions such as different altitudes. In order to make this model more realistic, autopilot dynamics are also taken into account. Next, nonlinear system dynamics model is represented in state space form. Then, all equality and inequality constraints are defined. In reality, every missile has a certain acceleration capability based on its flight dynamics and mechanical design. Therefore, control command calculated from guidance algorithm must be lower than this limit for throughout flight. This limit is an inequality constraint. In addition, in order to satisfy successful engagement, interception position is set as an equality constraint. Finally, time of flight is chosen as a hard constraint, to be able to solve nonlinear system of equations by using Model Predictive Static Programming approach. After expressing all constraints, Hamiltonian function is derived. By using Hilderth's quadratic programming Ref. [7] procedure on Model Predictive Static Programming (MPSP) and PN generated flight trajectory as an initial states to MPSP, optimization problem is solved and ideal trajectory is obtained.

This paper is organized as follows; in section 2, the problem is mathematically modelled and optimal missile guidance algorithm is obtained by using MPSP. In section 3, simulation results are presented. Finally, Section 4 concludes the paper.

2 Modelling

2.1 Mathematical Modeling of the Missile System Dynamics

In order to obtain missile dynamic model, it is required to define reference frames. For this study, two major reference frames are used. These are the inertial fixed reference frame (F_I) represented as $[\vec{u}_1{}^{(I)} \quad \vec{u}_2{}^{(I)} \quad \vec{u}_3{}^{(I)}]$ and the missile wind frame (F_w) represented as $[\vec{u}_1{}^{(W)} \quad \vec{u}_2{}^{(W)} \quad \vec{u}_3{}^{(W)}]$.

The $\vec{u}_1{}^{(W)}$ axis of the missile wind frame points in the direction of the velocity vector as given in Fig. 1. Here, α and β stands for flight path angles with respect to inertial fixed reference frame. These angles are going to be used to define flight trajectory.

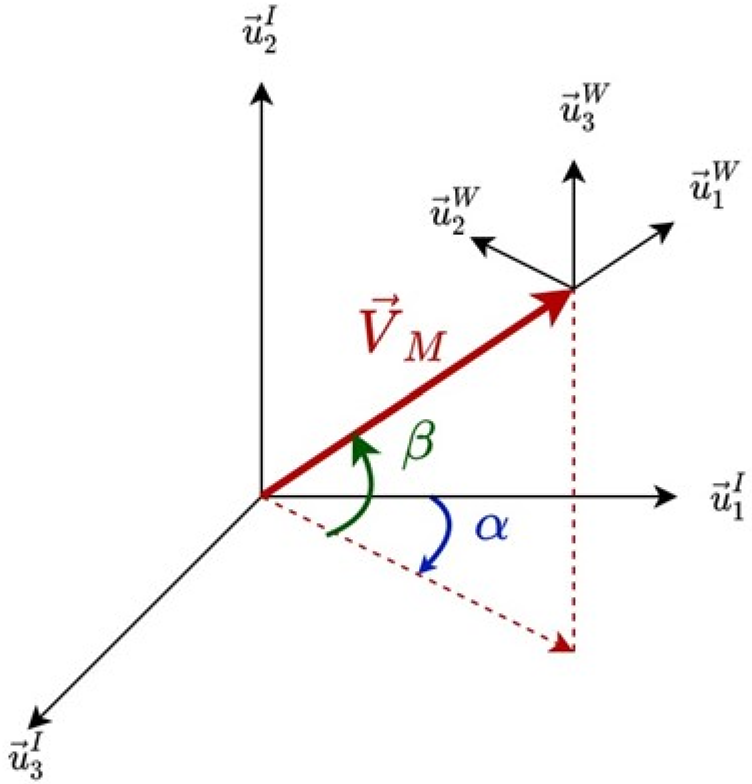

Fig. 1 Reference frames

By using these reference frames, transformation matrix from the wind frame to the inertial frame can be written as follows,

$$\hat{C}^{(I,W)} = \begin{bmatrix} \cos(\alpha)\cos(\beta) & -\sin(\alpha) & \cos(\alpha)\sin(\beta) \\ \sin(\alpha)\cos(\beta) & \cos(\alpha) & \sin(\alpha)\sin(\beta) \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \tag{1}$$

Acceleration of the missile is decomposed in the wind frame as follows by using the transport theorem;

$$\{\vec{a}_M\}^{(W)} = \{D_I(\vec{V}_M)\}^{(W)} = \{D_W(\vec{V}_M) + \vec{\omega}_{V/I} \times \vec{V}_M\}^{(W)} \tag{2}$$

Angular velocity of the wind frame with respect to the inertial frame $\vec{\omega}_{V/I}$ can be written in terms of rate of change of the flight path angles as;

$$\{\vec{\omega}_{V/I}\}^{(W)} = \begin{bmatrix} -\dot{\alpha}\sin\beta \\ \dot{\beta} \\ \dot{\alpha}\cos\beta \end{bmatrix} \tag{3}$$

Missile velocity and its derivative in the wind frame can be shown as;

$$\vec{V}_M^W = \begin{bmatrix} V_M \\ 0 \\ 0 \end{bmatrix}$$
$$\{D_W(\vec{V}_M)\}^{(W)} = \begin{bmatrix} \dot{V}_M \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

After combining equations (2), (3) and (4), acceleration relation of the missile is obtained.

$$\{\vec{a}_M\}^{(W)} = \begin{bmatrix} a^W_{M_x} \\ a^W_{M_y} \\ a^W_{M_z} \end{bmatrix} = \begin{bmatrix} \dot{V}_M \\ V_M\dot{\alpha}\cos\beta \\ -V_M\dot{\beta} \end{bmatrix} \tag{5}$$

By using the above relation, the rate of change of the flight path angles and the missile velocity are derived.

$$\dot{\alpha} = \frac{a^W_{M_y}}{V_M\cos\beta}, \qquad \dot{\beta} = -\frac{a^W_{M_z}}{V_M}$$
$$\dot{V}_m = \frac{T - 0.5\rho_{atm}V_M^2 S_{ref} C_x\left(a^W_{M_y}, a^W_{M_z}, V_M\right)}{m} - g\sin\beta \tag{6}$$

In addition, the rate of change of missile position is computed as follows:

$$\left\{D_I(\vec{V}_M)\right\}^{(I)} = \begin{bmatrix} \dot{P}_{M_x} \\ \dot{P}_{M_y} \\ \dot{P}_{M_z} \end{bmatrix} = \begin{bmatrix} V_M\cos\beta\cos\alpha \\ V_M\cos\beta\sin\alpha \\ -V_M\sin\beta \end{bmatrix} \tag{7}$$

To model the autopilot dynamics, the relation between acceleration command and realized accelerations are assumed as first order system model with time constant τ. In Laplace domain, assumed relation can be shown as:

$$\frac{a^W_{M_y}(s)}{a^W_{M_{y,c}}(s)} = \frac{a^W_{M_z}(s)}{a^W_{M_{z,c}}(s)} = \frac{1}{\tau s + 1} \tag{8}$$

In time domain this relation can be shown as:

$$\begin{aligned} \tau\dot{a}^W_{M_y}(t) + a^W_{M_y}(t) &= a^W_{M_{y,c}}(t) \\ \tau\dot{a}^W_{M_z}(t) + a^W_{M_z}(t) &= a^W_{M_{z,c}}(t) \end{aligned} \tag{9}$$

Assuming acceleration command is constant between $t_{(k)}$ and $t_{(k+1)}$, relation can be solved as:

$$
\begin{aligned}
a^W_{M_y}(k+1) &= a^W_{M_y}(k)e^{-\frac{\Delta t}{\tau}} + a^W_{M_y,c}(k)\left(1 - e^{-\frac{\Delta t}{\tau}}\right) \\
a^W_{M_z}(k+1) &= a^W_{M_z}(k)e^{-\frac{\Delta t}{\tau}} + a^W_{M_z,c}(k)\left(1 - e^{-\frac{\Delta t}{\tau}}\right)
\end{aligned}
\tag{10}
$$

According to Ref. [8] system dynamics model of the missile can be represented in discrete state form as:

$$
\bar{x}(k+1) = \bar{F}_k\left(\bar{x}(k), \bar{u}(k)\right) = \begin{bmatrix} P_{M_x}(k+1) \\ P_{M_y}(k+1) \\ P_{M_z}(k+1) \\ V_M(k+1) \\ \alpha(k+1) \\ \beta(k+1) \\ a^W_{M_y}(k+1) \\ a^W_{M_z}(k+1) \end{bmatrix}
$$

$$
\bar{x}(k+1) = \begin{bmatrix} P_{M_x}(k) + V_M(k)\cos\beta(k)\cos\alpha(k)\Delta t \\ P_{M_y}(k) + V_M(k)\cos\beta(k)\sin\alpha(k)\Delta t \\ P_{M_z}(k) - V_M(k)\sin\beta(k)\Delta t \\ V_M(k) + \frac{T - 0.5\rho_{atm}V(k)^2_M S_{ref} C_x\left(a^W_{M_y}, a^W_{M_z}, V_M\right)}{m}\Delta t - g\sin\beta(k)\Delta t \\ \alpha(k) + \frac{a^W_{M_y}(k)}{V_M(k)\cos\beta(k)}\Delta t \\ \beta(k) - \frac{a^W_{M_z}(k)}{V_M(k)}\Delta t \\ a^W_{M_y}(k)e^{-\frac{\Delta t}{\tau}} + a^W_{M_y,c}(k)\left(1 - e^{-\frac{\Delta t}{\tau}}\right) \\ a^W_{M_z}(k)e^{-\frac{\Delta t}{\tau}} + a^W_{M_z,c}(k)\left(1 - e^{-\frac{\Delta t}{\tau}}\right) \end{bmatrix}
\tag{11}
$$

In the above system dynamic equations, $C_X\left(a^W_{M_y}, a^W_{M_z}, V_M\right)$ stands for drag coefficient of the missile and it is assumed that it depends on missile's accelerations and velocity magnitude. Also, for simplicity, autopilot dynamics is modeled as a first order transfer function.

The input vector is defined in terms of acceleration commands as shown:

$$
\bar{u}(k) = \begin{bmatrix} a^W_{M_y,c}(k) \\ a^W_{M_z,c}(k) \end{bmatrix}
\tag{12}
$$

The output relationship of the state space equation is selected as follows, where, the missile's positions are considered as the outputs.

$$\bar{y}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{M_x}(k) \\ P_{M_y}(k) \\ P_{M_z}(k) \\ V_M(k) \\ \alpha(k) \\ \beta(k) \\ a_{M_y}^W(k) \\ a_{M_z}^W(k) \end{bmatrix} \tag{13}$$

2.2 Optimal Missile Guidance with Model Predictive Control under Acceleration Limit Constraints

The states and outputs of the system dynamics equation in nonlinear discrete form is given below. In addition, the step count term “k” in parentheses will be denoted as subscripts henceforth.

$$\begin{aligned}\bar{x}_{k+1} &= \bar{F}_k(\bar{x}_k, \bar{u}_k)\\ \bar{y}_k &= h(\bar{x}_k)\end{aligned} \tag{14}$$

where, variables are defined in the real space as $\bar{x}_k \in \mathbb{R}^{8\times1}$, $\bar{u}_k \in \mathbb{R}^{2\times1}$, $\bar{y}_k \in \mathbb{R}^{4\times1}$ and $k = 1, 2, \dots, N$ are the time steps. The primary objective is to obtain an optimum control input $\bar{u}_k$, $k = 1, 2, \dots, N-1$ such that the final output, $\bar{y}_N$ generated by the algorithm satisfies predefined output, $\bar{y}_N^*$ subject to certain constraints such as time of flight. Moreover, it is required to achieve this task with minimum control effort.

If outputs $\bar{y}_N$ are expanded about desired final position $\bar{y}_N^*$ by using Taylor series,

$$\bar{y}_N = \bar{y}_N^* + \left[\frac{\partial \bar{y}_N}{\partial \bar{x}_N}\right]\left(\bar{x}_N - \bar{x}_N^*\right) + H.O.T. \tag{15}$$

The nonlinear system dynamic equations are linearized with respect to a defined control input vector from $k = 1, 2, .., N$, as shown in Ref. [7]:

$$\bar{x}_{k+1} = \bar{F}_k(\bar{x}_k^*, \bar{u}_k^*) + \left[\frac{\partial \bar{F}_k}{\partial \bar{x}_k}\right]\Bigg|_{\substack{\bar{x}_k=\bar{x}_k^*\\ \bar{u}_k=\bar{u}_k^*}} (\bar{x}_k - \bar{x}_k^*) + \left[\frac{\partial \bar{F}_k}{\partial \bar{u}_k}\right]\Bigg|_{\substack{\bar{x}_k=\bar{x}_k^*\\ \bar{u}_k=\bar{u}_k^*}} (\bar{u}_k - \bar{u}_k^*) \tag{16}$$

$$\begin{aligned}d\bar{y}_N &\cong \bar{y}_N^* - \bar{y}_N\\ d\bar{x}_k &\cong \bar{x}_k^* - \bar{x}_k\\ d\bar{u}_k &\cong \bar{u}_k^* - \bar{u}_k\end{aligned} \tag{17}$$

The error above in the output can be expressed as:

$$d\bar{y}_N = Ad\bar{x}_1 + B_1 d\bar{u}_1 + B_2 d\bar{u}_2 + \ .. \ + B_{N-1} d\bar{u}_{N-1} \tag{18}$$

Where

$$\begin{aligned}A &= \left[\frac{\partial \bar{y}_N}{\partial \bar{x}_N}\right]\left[\frac{\partial \bar{F}_{N-1}}{\partial \bar{x}_{N-1}}\right]\left[\frac{\partial \bar{F}_{N-2}}{\partial \bar{x}_{N-2}}\right] \dots \left[\frac{\partial \bar{F}_1}{\partial \bar{x}_1}\right]\\ B_k &= \left[\frac{\partial \bar{y}_N}{\delta \bar{x}_N}\right]\left[\frac{\partial \bar{F}_{N-1}}{\partial \bar{x}_{N-1}}\right]\left[\frac{\partial \bar{F}_{N-2}}{\partial \bar{x}_{N-2}}\right] \dots \left[\frac{\partial \bar{F}_{k+1}}{\partial \bar{x}_{k+1}}\right]\left[\frac{\partial \bar{F}_k}{\partial \bar{u}_k}\right]\end{aligned} \tag{19}$$

In addition $d\bar{x}_1 = 0$ because initial states are given, there will be no error, which means:

$$d\bar{y}_N = \sum_{k+1}^{N-1} B_k d\bar{u}_k \tag{20}$$

The cost function can be expressed as:

$$J\left(\bar{u}_1, \bar{u}_2, \ .. \ \bar{u}_{N-1}\right) = \frac{1}{2}\sum_{k=1}^{N-1} \bar{u}_k^T R_k \bar{u}_k \tag{21}$$

In addition, this cost function can be written in the form of $d\bar{u}_k$,

$$J = \frac{1}{2} \sum_{k=1}^{N-1} \left(\bar{u}_k^* - d\bar{u}_k\right)^T R_k \left(\bar{u}_k^* - d\bar{u}_k\right) \tag{22}$$

After expressing cost functions above, control inputs must satisfy following inequality constraints:

$$G_k \bar{u}_k - \bar{W}_k \leq 0,\ k = 1, 2,\ \ldots, (N-1) \tag{23}$$

Where

$$G_k = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}, \bar{W}_k = \begin{bmatrix} a_k \\ a_k \\ b_k \\ b_k \end{bmatrix} \tag{24}$$

In this expression, a_k and b_k are representing acceleration capabilities of the missile. Moreover, there is an equality constraint to be fulfilled:

$$\bar{y}_k = \bar{y}_k^* \tag{25}$$

After considering equality and inequality constraints, the new cost function can be expressed as Hamiltonian function by using Lagrange multipliers:

$$\begin{aligned} J^* & \left(d\bar{u}_k, \bar{\lambda}, \bar{\rho}_1, \bar{\rho}_2,\ \ldots, \bar{\rho}_{N-1}\right) \\ &= \frac{1}{2} \sum_{k=1}^{N-1} \left(\bar{u}_k^* - d\bar{u}_k\right)^T R_k \left(\bar{u}_k^* - d\bar{u}_k\right) + \bar{\lambda}^T \left(d\bar{y}_N - \sum_{k=1}^{N-1} B_k d\bar{u}_k\right) \\ &+ \sum_{k=1}^{N-1} \bar{\rho}_k \left[G_k \left(\bar{u}_k^* - d\bar{u}_k\right) - \bar{W}_k\right] \end{aligned} \tag{26}$$

Where $\bar{\lambda}$ and $\bar{\rho}_k$ are Lagrange multipliers, k representing the time step. After using optimality conditions on Lagrange multipliers, according to Ref. [8] control inputs (guidance law) can be written as:

$$\bar{u}_k = R_k^{-1} B_k^T A_\lambda \bar{c}_\rho - R_k^{-1} G_k^T \bar{\rho}_k \tag{27}$$

Where

$$A_\lambda = -\left(\sum_{k=1}^{N-1} B_k R_k^{-1} B_k^T\right)^{-1},\ \bar{b}_\lambda = \sum_{k=1}^{N-1} B_k \bar{u}_k^*,\ \bar{c}_\rho = \sum_{k=1}^{N-1} B_k R_k^{-1} G_k^T \bar{\rho}_k \tag{28}$$

3 Simulation Results

In this section, the trajectory optimization of an generic dual-pulse air defense missile is presented. Optimized trajectory that was obtained with model predictive control method is applied to midcourse guidance.

Two different scenarios are chosen for comparison, and in both scenarios the algorithm is demonstrated with the convergence of the optimization problem under both equality and inequality constraints. Horizon length for the optimization problem is set as estimated time to go. For the results, final velocity is compared. The simulation terminates when range goes below 10 meters.

3.1 Scenario I

In the first scenario, missile's initial α and β values are 70° and 0° respectively. Target is moving with constant velocity and altitude towards missile's initial position. Iterations and final results is compared with PNG.

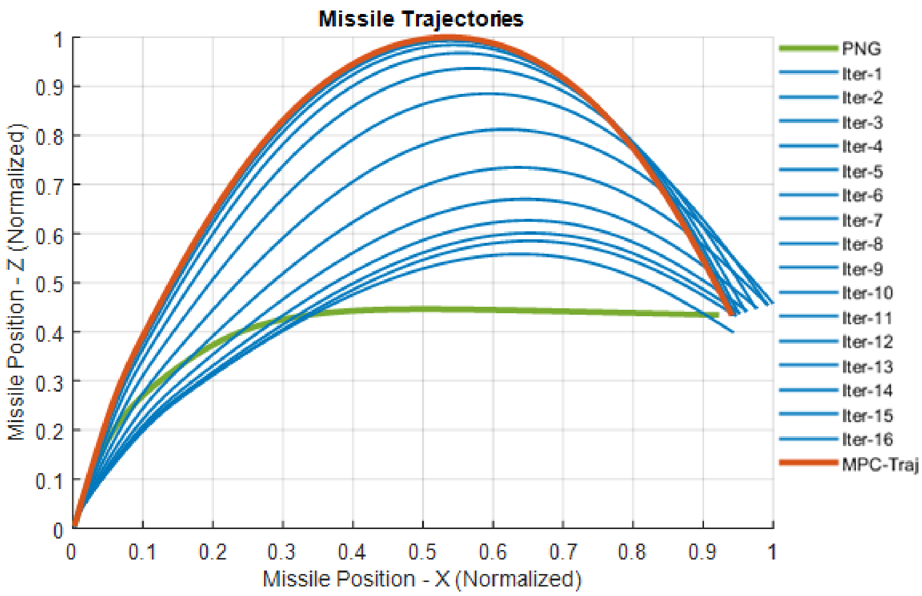

Fig. 2 Missile trajectories for each iteration in first scenario

In Fig. 2, the missile trajectories for each iteration are presented as normalized values. The green line shows the missile trajectory generated by the PN guidance. For the iterations, this PN generated trajectory is used as the initial trajectory that missile should track. The blue lines shows the MPC generated trajectories. After 16 iterations, the optimal trajectory (red line), that satisfy equality and inequality constraints, is obtained.

Iterations	Speed (Normalized)	Iterations	Speed (Normalized)
PNG	0.2500	Iteration-9	0.5400
Iteration-1	0.3200	Iteration-10	0.5585
Iteration-2	0.3300	Iteration-11	0.5662
Iteration-3	0.3400	Iteration-12	0.5699
Iteration-4	0.3500	Iteration-13	0.5717
Iteration-5	0.3800	Iteration-14	0.5726
Iteration-6	0.4200	Iteration-15	0.5729
Iteration-7	0.4800	Iteration-16	0.5732
Iteration-8	0.5200	MPC	0.5731

As can be seen from the table above, the normalized speed of the missile increases from the PN generated trajectory to the MPC generated. Due to MPC guidance law, the missile final speed is more than two times higher than PNG generated final speed.

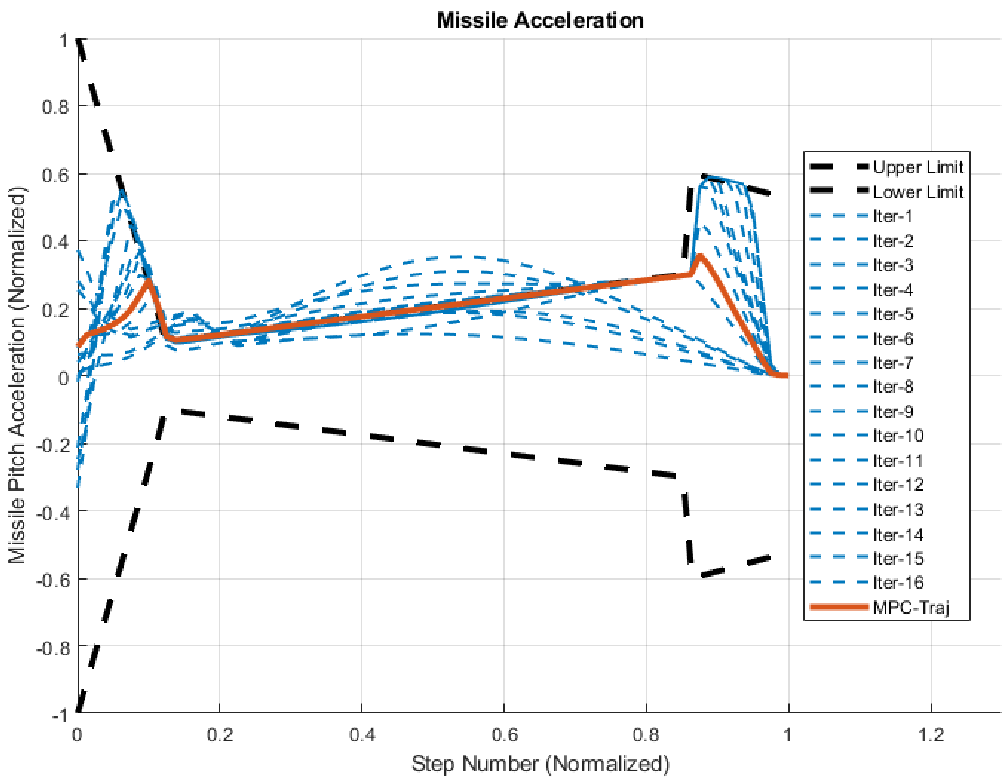

Fig. 3 Missile accelerations for each iteration

In Fig. 3 the missile pitch accelerations for each iteration can be seen for entire flight. Black dot lines represent inequality constraint on upper and lower acceleration limits. It should be noted that these bounds are not realistic and are only used as sample representations. The optimal missile trajectory generated by the MPC guidance must not exceed these bounds. The final iteration (MPC-Traj) satisfies this condition.

3.2 Scenario II

In the second scenario, missile's initial α and β values are 70° and 0° respectively. Target is moving with constant velocity and altitude in cross range as shown in Fig. 4. Results of this scenario is compared with both PNG and GENEX.

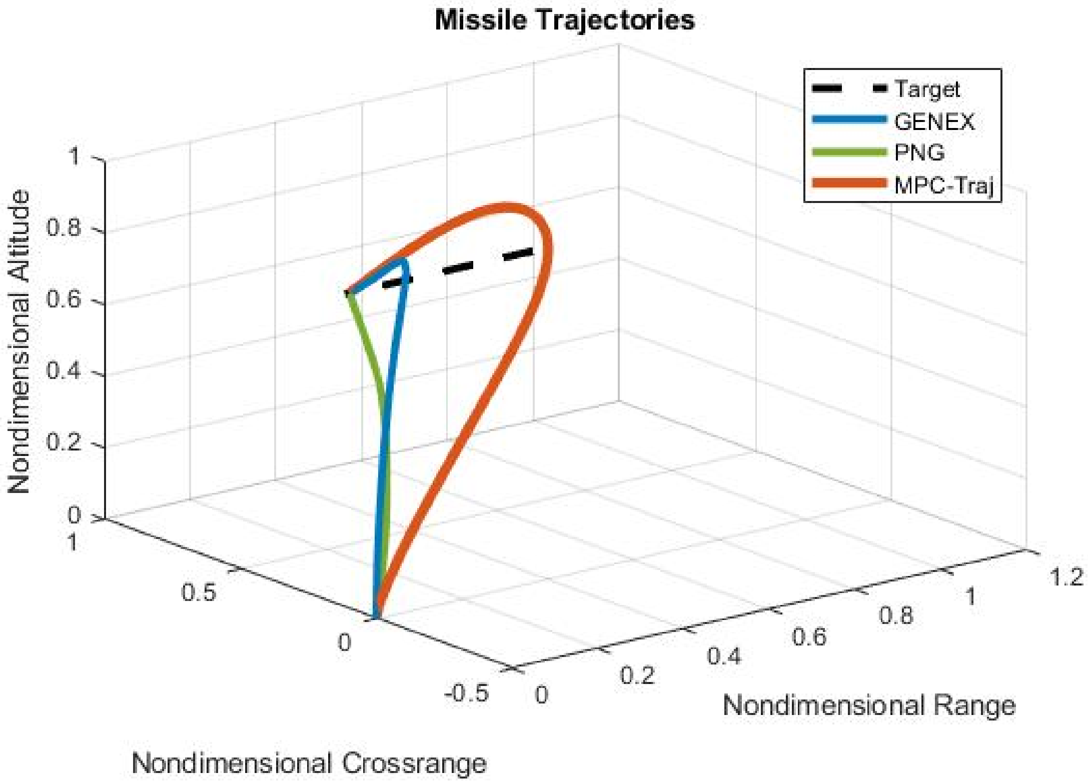

Fig. 4 3-D engagement for PNG, GENEX and MPC

In Fig. 4 the missile and target trajectories are presented as normalized values. The black dotted line shows the target trajectory, the blue line shows the missile trajectory for the GENEX, the green line shows the missile trajectory for PNG and the red line shows the missile trajectory for MPC.

Method	**Final speed (normalized)**
PNG	0.484
GENEX	0.634
MPC	1

As can be seen from the table above, MPC guided algorithm has the highest final speed. If the parameters of the GENEX algorithm changed, one may find better results than GENEX results above, but these parameters are highly scenario dependent. However MPC guided algorithm optimizes the best trajectory for each scenario under both equality and inequality constraints.

4 Conclusion

In the literature, there exist several approaches for applying optimization in the guidance algorithm. Most of these approaches make certain assumptions and use simplified kinematics models. In general, the simplifications or assumptions made can be summarized as follows;

- Velocity of the missile and its rate of change are constant
- The missile has unlimited acceleration capability under all flight conditions
- The flight mechanics of the missile are completely neglected.
- Atmospheric conditions are not modeled.

However, in this study, the system dynamics properties of the missile are considered, such as thrust profile, aerodynamic properties, and mass changes. In addition, acceleration limits and realistic autopilot dynamics are taken into account. By using this optimal guidance method, the missile follows an energy efficient trajectory. Then, the missile achieves a higher final velocity, which increases the missile's probability of hitting the target.

References

[1] Paul Zarchan. *Tactical and Strategic Missile Guidance*. AIAA, 1997. ISBN: 978-1563478741.

[2] Byung Soo Kim, Jang Gyu Lee, and Hyung Seok Han. Biased png law for impact with angular constraint. *IEEE Transactions on Aerospace and Electronic Systems*, 34(1):277–288, 1998. DOI: 10.1109/7.640285.

[3] Ashwini Ratnoo and Debasish Ghose. Impact angle constrained guidance against nonstationary nonmaneuvering targets. *Journal of Guidance, Control, and Dynamics*, 33(1):269–275, 2010. DOI: 10.2514/1.45026.

[4] Chang-Kyung Ryoo, Hangju Cho, and Min-Jea Tahk. Optimal guidance laws with terminal impact angle constraint. *Journal of Guidance, Control, and Dynamics*, 28(4):724–732, 2005. DOI: 10.2514/1.8392.

[5] Ernest J. Ohlmeyer and Craig A. Phillips. Generalized vector explicit guidance. *Journal of Guidance, Control, and Dynamics*, 29(2):261–268, 2006. DOI: 10.2514/1.14956.

[6] Prasiddha Nath Dwivedi, Abhijit Bhattacharya, and Radhakant Padhi. Suboptimal midcourse guidance of interceptors for high-speed targets with alignment angle constraint. *Journal of Guidance, Control, and Dynamics*, 34(3):860–877, 2011. DOI: 10.2514/1.50821.

[7] Clifford Hildreth. A quadratic programming procedure. *Naval Research Logistics Quarterly*, 4(1):79–85, 1957. DOI: 10.1002/nav.3800040113.

[8] Tuğba Bayoğlu and Gökcan Akalin. Reachability analysis of an aerodynamic interceptor with input constraint. In *AIAA SCITECH 2022 Forum*, Jan. 2022. DOI: 10.2514/6.2022-2039.

[9] Liuping Wang. *Model Predictive Control System Design and Implementation Using MATLAB*. Springer, 1st edition, 2009. ISBN: 978-1848823310.