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Enhanced Impact Vector Guidance: Addition of Impact Time Using Bézier Curves and Virtual Targets

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ABSTRACT

Missiles with high speed and maneuverability utilize various guidance algorithms to reach their intended targets. While some algorithms are designed solely for target acquisition, these algorithms can be diversified based on desired impact angles or times. The Impact Vector Guidance (IVG) algorithm ensures a missile collides with a stationary or moving target at the desired angle. In this study, the objective is to preserve the impact angle performance of the existing IVG guidance law while adding impact time capabilities using Bézier curves and a virtual target approach. Bézier curves are mathematical curves generated using basis functions and control points to form parametric polynomials. Initially, virtual targets are created using Bézier curves, and the guidance law borrowed from the literature enables impact angle constraints while chasing the virtual target. Impact time selection is made possible through the use of virtual targets. Consequently, by integrating impact time constraints into IVG, successful strikes on stationary targets are achieved. The application of this newly obtained three-dimensional guidance law called Enhanced Impact Vector Guidance (EIVG) is presented for different scenarios.

Keywords: Bézier curves; impact vector guidance; impact time guidance, virtual target

Nomenclature

$C(u)$	=	Bézier Curve
$C^{'}(u)$	=	Derivative of Bézier Curves
u	=	Knot Value
$t, t_f, t_{des},$	=	Time, Final Time, Desired Final Time
$B_{i,n}$	=	Bernstein Polynomials
P_i, Q_i	=	Control Points
a_m, a_t	=	Normal Accelerations of Missile and Target
V_m, V_t	=	Velocities of Missile and Target

σ_m, σ_t	=	Look Angles of Missile and Target
λ	=	LOS angle
r	=	Range
ζ	=	Tuning Parameter
θ_m, ψ_m	=	Euler Angles for Missile
θ_t, ψ_t	=	Euler Angles for Target
θ_f, ψ_f	=	Final Euler Angles for Missile
ω_r	=	Angular rate of LOS vector
V_c	=	Closing Speed
N	=	Guidance Gain
u_f, u_r, u_v	=	Unit Vectors for Final speed, Range vector, and Current Speed
π, α	=	Constants for Sigmoid

1 Introduction

Designing algorithms for successfully directing and guiding a missile from one point to a target is a highly important aspect of operational performance. Guidance algorithm or guidance law is one of these methods that enable vehicles like missiles to reach their intended targets [1]. Guidance laws can simply aim to achieve reaching the target, or they can aim to reach the target at a desired angle, at a specified time, or both. Furthermore, these laws can be designed in two or three dimensions depending on the requirements. While designing 2D guidance laws is relatively easier, developing 3D guidance systems is more difficult due to the increasing complexity of the engagement dynamics.

One of the well-known guidance laws for both 2D and 3D engagement scenarios is Proportional Navigation (PN), which emerged in the mid-twentieth century [2]. The simple and elegant formulation of this law gives the ability to penetrate both stationary and moving targets successfully. It aims to guide a missile towards a target by adjusting its acceleration proportionally to the rate of change of the line of sight (LOS) vector. Depending on the performance criteria or operational demands, the successful penetration capabilities of the PN may not cover all the requirements. Some of these requirements may be the impact time and impact angle constraints for an engagement. The impact time problem for a missile guidance perspective is firstly mentioned in the work [3]. The work decomposes the acceleration into two parts: one for reducing the miss distance while the other part for obtaining the required trajectory length. By doing so, researchers propose a guidance law capable of making salvo attacks in 2D scenarios. Researchers are also focused on 3D impact time guidance law [4]. The work uses a composite guidance law and regulator term for impact time error. The polynomial shaping approaches are widely utilized in impact time guidance laws, and the work [5] and [6] shapes the look angle and range as polynomials, respectively. In addition to the desired impact time, final angle constraints are widely studied in the guidance community. The work [7] takes advantage of polynomial shaping methods in order to achieve an impact angle guidance law when there is a need to limit the look angle value. 3D impact angle guidance laws are also studied in [8] and [9]. Both methods are designed for 3D engagements and can handle target maneuvers. The performance of the [9] is superior to [8] by looking at the results proposed in the work. Dealing with the impact angle and impact time constraints combined also draws attention. The work [10] obtains a guidance law for moving targets by shaping the look angle. Feedback linearization and Bézier curves are used in [11] to achieve a guidance law that can meet the impact time and angle constraints. Additionally, nonlinear control methods like Sliding Mode Control are used in [12], [13], [14], and [15]. In the work [16], a virtual target approach is proposed. The guidance law has two stages; the first stage adjusts the impact angle in finite time, and the second one is the PN. Similarly, a two-stage guidance law with a virtual target approach is presented in [17]. The dual virtual target concept resulting in an impact time control guidance law is proposed in [18].

This study presents the development of a guidance law for 3D engagement problems, specifically targeting impact time and angle. In the existing literature, researchers have primarily focused on employing analytical methods, nonlinear control techniques, and optimal control techniques to address this problem. While these proposed methods are well-designed and effective, the associated design procedures tend to be complex. The main motivation behind this research is to establish a guidance law that achieves satisfactory performance without introducing unnecessary design complexities. To this end, the guidance law known as Impact Vector Guidance (IVG) in [9], developed for an impact angle problem, is borrowed and augmented with virtual target concepts to incorporate impact time control. In order to incorporate impact time control into this guidance law, a Virtual target with a predetermined position is utilized by employing Bézier curves. These curves are generated using control points and basis functions using the boundary conditions. This approach allows for the attainment of curves of desired sizes while preserving impact angle capabilities. By determining the virtual target positions in each time instance based on the desired time of the collision, a trajectory is established for the virtual target along the Bézier curves. Thus, the missile will chase the virtual target till the virtual and real targets coincide at the end of the engagement.

The rest of the paper is structured as follows. In Chapter 3 of the study, an introduction to Bézier curves is provided, followed by the derivation of the polynomials for the 4th-order Bézier curve employed in this work. Subsequently, in Chapter 4, a detailed explanation of the borrowed IVGr guidance law is presented, accompanied by illustrative figures. Furthermore, the computation of initial conditions for the Bézier curve and a detailed description of the virtual target and its trajectories are given. The proposed method is provided and discussed in this chapter before the simulations take place. In Chapter 5, simulations and results for two different scenarios are presented using the proposed guidance law, followed by their analysis and interpretation before the conclusion.

2 Bézier Curves

Bézier curves are parametric polynomials that enable describing a curve by linear combinations of its control points for a given degree. These curves consist of control points and their corresponding basis functions. The initial and final points of the curve are the first and last control points and define the beginning and end of the curve. Intermediate control points determine the overall geometry of the curve with the help of the basis functions, which are the functions responsible for interpolation between the control points. The formula for an n^{th} order Bézier curve is given in Eq. (1). More information can be found in [19].

$$C(u) = \sum_{i=0}^{n} B_{i,n}(u) P_i \quad 0 \le u \le 1 \tag{1}$$

In Eq. (1), n is the degree, P_i is the i^{th} control point of $n+1$ control point, $B_{i,n}$ is the basis function (Bernstein Polynomials) for i^{th} control point P_i, and u denotes the knot parameter. The basis function in Eq. (1) is calculated as in Eq. (2).

$$B_{i,n}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i} \tag{2}$$

The fundamental effect of basis functions is to define the contribution of each control point to the Bézier curves for degree n. Since basis functions are parametric functions of u for given n, the contribution distributions of different sets of control points are the same for specified degrees at any knot value. The variation of the Bézier curve for the same degree and different control points is illustrated in Fig. 1. From Fig. 1 and the equation of basis functions, it is obvious that the Bézier curves can not be modified locally since all of the basis functions are nonzero at all knot values.

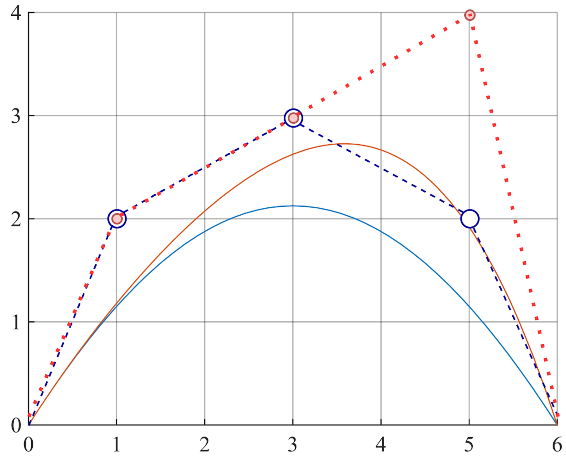


Fig. 1 4th Degree Bézier curve and the effect of control point location

In addition, it is also possible to express the derivatives of Bézier curves in a similar manner. For an nth degree Bézier curve, the first derivative of the curve is of degree $(n-1)$. Eq. (3) provides the formulas used to obtain the derivative of the Bézier curve. For higher order derivatives, the Eq. (3) can be repeated up to $(n-1)^{th}$ derivative by treating the derivative curve as a new original curve and so on.

$$\begin{aligned} Q_i &= n(P_{i+1} - P_i) \\ C'(u) &= \sum_{i=0}^{n-1} B_{i,n-1}(u) Q_i \end{aligned} \tag{3}$$

where $C'(u)$ is the derivative of the original curve $C(u)$ with respect to u and Q_i is the control points of the $C'(u)$.

In this study, it was decided that the 4^{th} order Bézier curve is sufficient for the application. In this case, the Bézier curve and its derivative with respect to u were obtained as in Eqs. (4) and (5), respectively.

$$C(u) = (1-u)^4 P_0 + 4(1-u)^3 u P_1 + 6(1-u)^2 u^2 P_2 + 4(1-u) u^3 P_3 + u^4 P_4 \tag{4}$$

$$\frac{dC}{du} = C'(u) = 4(1-u)^3 (P_1 - P_0) + 12(1-u)^2 u (P_2 - P_1) + 12(1-u) u^2 (P_3 - P_2) + 4u^3 (P_4 - P_3) \tag{5}$$

3 The Enhanced Impact Vector Guidance

In this section, the proposed virtual target approach will be presented after the necessary preliminaries about the 3D engagement geometry and the terminal requirements, as well as an impact angle guidance law from the literature to be enhanced. The 3D engagement geometry used in the design of the 3D guidance law is presented in Fig. 2. In this configuration, there are two components of missile acceleration. Both of these accelerations are perpendicular to the velocity vector of the missile. As a result, the magnitude of the missile's velocity remains constant throughout the flight, meaning that these two accelerations only cause changes in the direction of the missile. The addition of target movement, as in Fig. 2, increases the complexity of the engagement. Therefore, in this study the target movements are neglected for the sake of simplicity.

Upon examining Fig. 2, it is evident that the system is highly complex, and it can be inferred that the kinematic equations of the system would also be quite intricate [20]. Well-known PNG deals with these

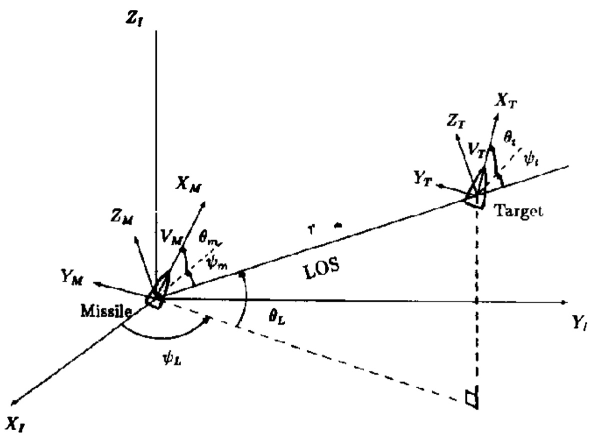


Fig. 2 3D engagement geometry [20]

complex dynamics by a simple Eq. 6.

$$\boldsymbol{a}_{\text{PNG}} = N\omega_r \times \boldsymbol{v} \tag{6}$$

where ω_r is the LOS rate vector, N is the navigation gain, and $\boldsymbol{v}$ is the velocity vector of the missile. The LOS rate vector can be formulated as Eq. 7.

$$\omega_r = \frac{\boldsymbol{r} \times \dot{\boldsymbol{r}}}{r^2} \tag{7}$$

Although the 3D engagement can be done even for moving targets, the terminal impact time and angle constraints can not be met using PNG. This terminal constraints are visualized in Fig. 3. In this figure, the orientation of the missile's speed v can be expressed by means of two components, ψ and θ.

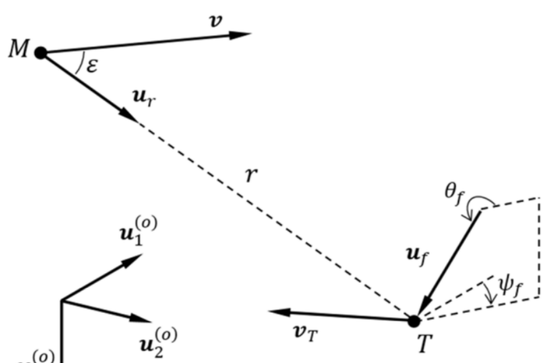


Fig. 3 Terminal constraints for 3D engagements [9]

The angle between LOS vector r and $\boldsymbol{v}$ is called look angle ξ, unit vectors u_f and u_r are the final velocity direction and LOS vector directions respectively [9]. Therefore, the ψ_f and θ_f are the terminal constraints in addition to the terminal time constraint t_{des}. Thus, the orientation of the missile velocity component can be written as in Eq. 8 using terminal angle constraints.

$$u_f = \begin{bmatrix} \cos\psi_f \cos\theta_f \\ \cos\theta_f \sin\psi_f \\ -\sin\theta_f \end{bmatrix} \tag{8}$$

The work carried out by Erer [9] proposed two novel guidance laws that can deal with such angle constraints. These guidance laws are designed as range and velocity-based and are named IVGr and IVGv, respectively. IVGr has been used in this study, and a related guidance command is given in Eq. 9.

$$a_{\mathrm{IVG}-r} = v_c \left\{ (N+1)\omega_r + (N-1)\frac{v_c}{r}\cos^{-1}\left(u_f \cdot u_r\right) \frac{u_f \times u_r}{\left|u_f \times u_r\right|} \right\} \times u_v \tag{9}$$

where ω_r is los rate vector, u_f is unit vector of final velocity, u_v is unit vector of current velocity, u_r is unit vector for LOS vector, V_c is closing speed, and N is guidance gain.

In this guidance law, the LOS vector is brought to the desired impact angle at the impact time. In this case, the angles of the final velocity vector denoted as ψ_f and θ_f, are equalized to the final LOS vector angles. The performance of the IVGr for impact angle constraints is satisfactory. In this study, the objective is to further incorporate the determination of the impact time into this method. To accomplish this, a Bézier curve is initially designed to be used with the virtual target concept. Subsequently, a virtual target is created along this Bézier curve. Virtual targets continue to be generated until reaching the actual target. In the final step, the virtual target coincides with the actual target. The positions of virtual targets along the curve are determined based on the desired impact time constraints. A more comprehensible explanation of this method is provided in Fig. 4.

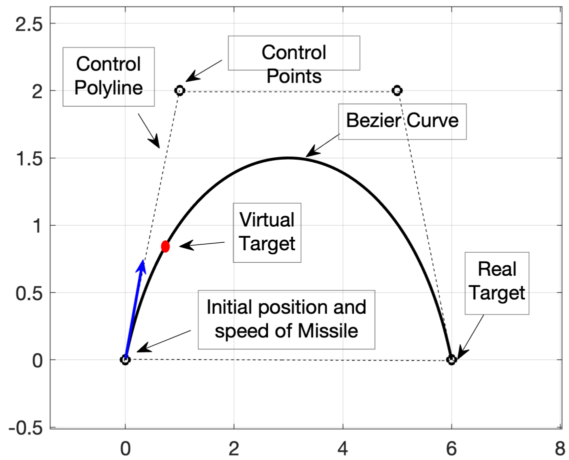


Fig. 4 Main idea behind EIVG methodology

The main motivation of the proposed virtual target approach is to achieve the desired impact time by increasing the total path length of the trajectory in a systematic way. In each time step, instead of the actual target T, a virtual target VT is created along the Bézier curve, and the missile chases the VT till it coincides with T at the end of the engagement. In Fig. 4, a simple snapshot of such Virtual target and Bézier curve is given for an instant.

For the proposed method, a fourth-degree Bézier curve, denoted as Eq. (4), is employed to create a virtual target's possible locations.

$$C(u) = (1-u)^4 P_0 + 4(1-u)^3 u P_1 + 6(1-u)^2 u^2 P_2 + 4(1-u) u^3 P_3 + u^4 P_4$$

The calculation of the control points in this equation is performed using the boundary conditions that define the engagement. These conditions are provided in Eqs. (10) and (11) for the 3D case and as given in Fig. 3.

$$V_{init} = \boldsymbol{v} \tag{10}$$

$$\mathbf{V}_{final} = |\boldsymbol{v}|u_f \tag{11}$$

Eq. (10) provides the initial velocity of the missile, while Eq. (11) represents the final velocity vector for an impact angle demand. In Eq. (11), the magnitude of the missile's velocity remains constant, and only its direction changes. The control point values are calculated using Eq. (12) with Eqs. (10) and (11).

$$\begin{aligned} P_0 &= P_m \\ P_4 &= P_f \\ P_1 &= P_0 + (t_{des}/n)V_{init} \\ P_3 &= P_4 - (t_{des}/n)V_{final} \\ P_2 &= 0.5(P_1 + P_3) \end{aligned} \tag{12}$$

where P_i, $0 \leq i \leq n$ is the control points, P_m is the position of the missile, and P_f is the target. Note that, if the subscript of P is a number, it is called a control point. The degree and desired impact times are n and t_{des}, respectively. In each time step, t_{des} is equal to the remaining time to go. P_1 and P_3 is calculated using Eq. 3, and the t_{des}/n arises from the time parameterization.

In addition, determination of the virtual target's position on the designed Bézier curve is an important challenge. Since a Bézier curve $C(u)$ is defined for $0 \leq u \leq 1$, the position of the virtual target $P_{vt} = C(u_v)$ should also be defined with a value within the range of $0 \leq u_v \leq 1$. The only known boundary condition is that, in the final step, the position of the virtual target should coincide with the position of the real target. Additionally, the position of the virtual target should be shaped in such a way that the virtual target motion does not cause sudden changes in the acceleration commands. For this reason, the transition between the virtual target positions in each step should be designed with a smooth function. After considering these requirements, it was concluded that the sigmoid function would meet the necessary requirements, and it is given in Eq. (13).

$$f(x) = \frac{1}{1 + e^{-\pi(x-\alpha)}} \tag{13}$$

Some modifications are made to the sigmoid function in order to prevent early engagement with the virtual target. As a result, the calculation of the u_v value that determines the position of the virtual target is obtained as shown in Eq. (14).

$$u_v(t) = 0.01 + \frac{0.99}{(1 + e^{-\pi(t-\alpha)})} \tag{14}$$

where α is the position of turn and π is the slope of the sigmoid function and t is time.Through Eq. (14), both desired requirements of the design are fulfilled. In other words, a smooth transition is achieved for each step, ensuring a continuous and gradual change, and in the final step, the virtual target coincides with the actual target.

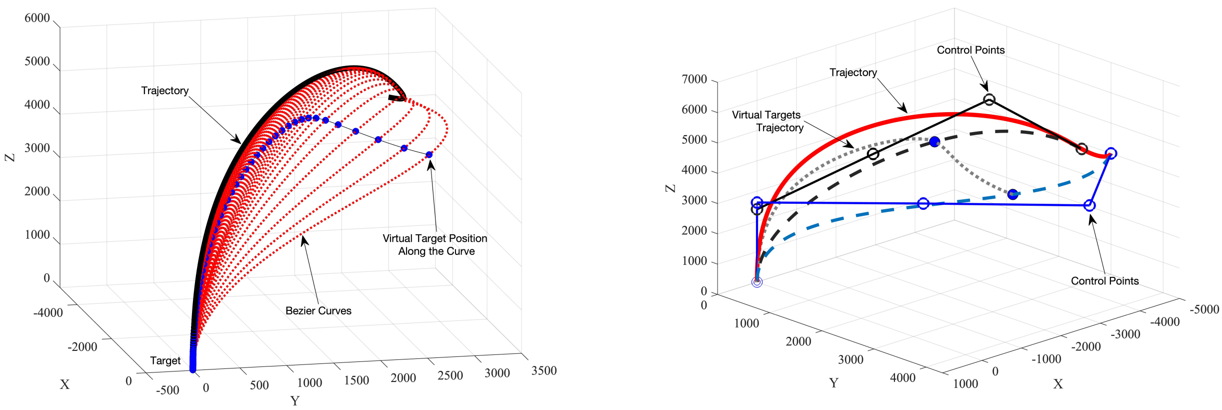
The proposed methodology operates under the assumption of a constant slope and fine-tunes the position of the turn for each individual engagement. However, due to the high complexity of the proposed method and its relatively early stage of development, an alternative offline algorithm is suggested, resembling the previously mentioned guidance laws. In the given Algorithm 1, offline simulations are made till the time error is within the given tolerance ϵ. The proposed algorithm brings some drawbacks mainly because of its offline nature and model dependent tuning parameters. The offline algorithms are not robust and the disturbance or target movement can decrease the performance. Using the proposed algorithm combined with IVGr is called Enhanced Impact Vector Guidance (EIVG) since the impact time capabilities are added via the Virtual Target approach. The offline algorithm limits the usage to stationary targets, while the original IVGr can engage with moving targets. Thus, the trade-off between impact time control and engaging with a moving target is made in this study.

Algorithm 1 Determination of α

input: $\alpha_l, \zeta, t_{des}, \epsilon$
output: α
for $i < n$ **do**
  $e_0 \leftarrow e(\alpha_l)$
  if $|e_0| < \epsilon$ **then** $\alpha \leftarrow \alpha_l$
    return α
  else
    $\alpha_l \leftarrow \alpha_l + \zeta \frac{e_0}{t_{des}}$
  end if
end for

Where n is maximum number of iteration, $\zeta = 0.3$ and $\alpha_l < 1$

The principle of the developed guidance law is clearly illustrated in Fig. 5. The Bézier Curves generated in each step during a flight are shown in Fig. 5a. The virtual targets are defined on these curves and are shown as blue circles. The graph shows that the virtual target position coincides with the actual target at the end of the simulation with the help of the sigmoid function. The basic idea of the developed guidance law can be summarized as chasing the virtual target defined on the Bézier curves according to the desired impact time at each step using the IVGr guidance law. Thus, impact time control is added to the IVGr guidance law, which already has the capability of impact angle control. Fig. 5b shows the Bézier curves and virtual target positions for two instants. The blue hollow circles are the control points for the Bézier Curve, which is shown as a blue dashed line. The virtual target at that instant is the blue mark on the Bézier curve. As the missile traverses its trajectory, shown as red, another Bézier curve is created using new control points shown as black hollow circles. The virtual target is found on the newly created black dashed line using the sigmoid function. The complete virtual target position, shown as the grey dotted line, is obtained by creating a Bézier curve at each instant and finding the virtual target position on it using the sigmoid function.



(a) Bézier curves and resulting virtual target trajectory. (b) Bézier curves and corresponding control points.

Fig. 5 Virtual target approach details a) Bézier curves and corresponding virtual target trajectory b) Construction of Bézier curves using control points

4 Simulations and Results

In this section, the application of the developed guidance algorithm for two different cases has been presented and compared with the IVGr guidance algorithm. The initial conditions for these cases are provided in Tab. 1.

Table 1 Initial Conditions of Cases

Parameters	Case 1	Case 2
Target Initial Position (m), P_T	$(0, 0, 0)$	$(0, 0, 0)$
Missile Initial Position (m), P_M	$(-5000, 3000, 4000)$	$(-5000, 3000, 4000)$
Missile Velocity (m/s), V_M	300	300
Missile Launch Angle (rad), (θ_i, ψ_i)	$(0, -\pi/2)$	$(0, \pi/6)$
Desired Impact Angle (rad), (θ_f, ψ_f)	$(0, \pi/6)$	$(\pi/2, \pi/6)$
Desired Impact Time (s), t_f	$(30s, 35s, 40s)$	$(30s, 35s, 40s)$

Using these initial values, the IVGr guidance law was first applied for Case 1, followed by the implementation of the new guidance law utilizing a virtual target. In Fig. 6, the trajectories generated by both the IVGr guidance law and the new guidance law for three different t_f values are depicted. Upon examining the figure, it can be observed that the new guidance law achieves both the desired angle and the desired time of arrival at the target.

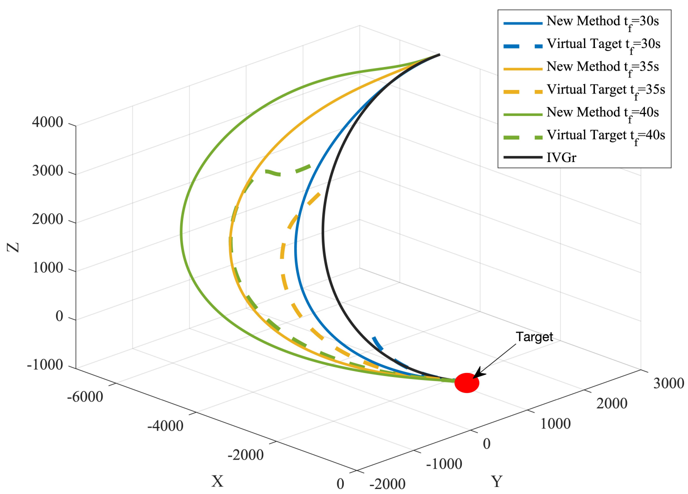


Fig. 6 3D trajectories of Case-1 by using IVGr and a new method for different desired impact time

Fig. 7 illustrates the variations of the missile's velocity, acceleration, look angle, and Euler angles during the flight duration. Upon examining Fig. 7, it can be observed that the missile's velocity remains constant throughout the flight duration. When analyzing the look angle graphs, it is evident that the look angle reaches 0 at the desired time. The Euler angle values, on the other hand, approximately follow similar curves. Terminal angle constraints are also met. Upon observing the magnitude of the ϕ angle, it can be seen that as the desired impact time increases, there is a slight deviation from 0; however, this deviation is on the order of 10^{-9}. Lastly, when examining the desired acceleration graph, it is apparent that IVGr requires the least acceleration, but as the desired impact time increases, the expected acceleration of the missile also increases.

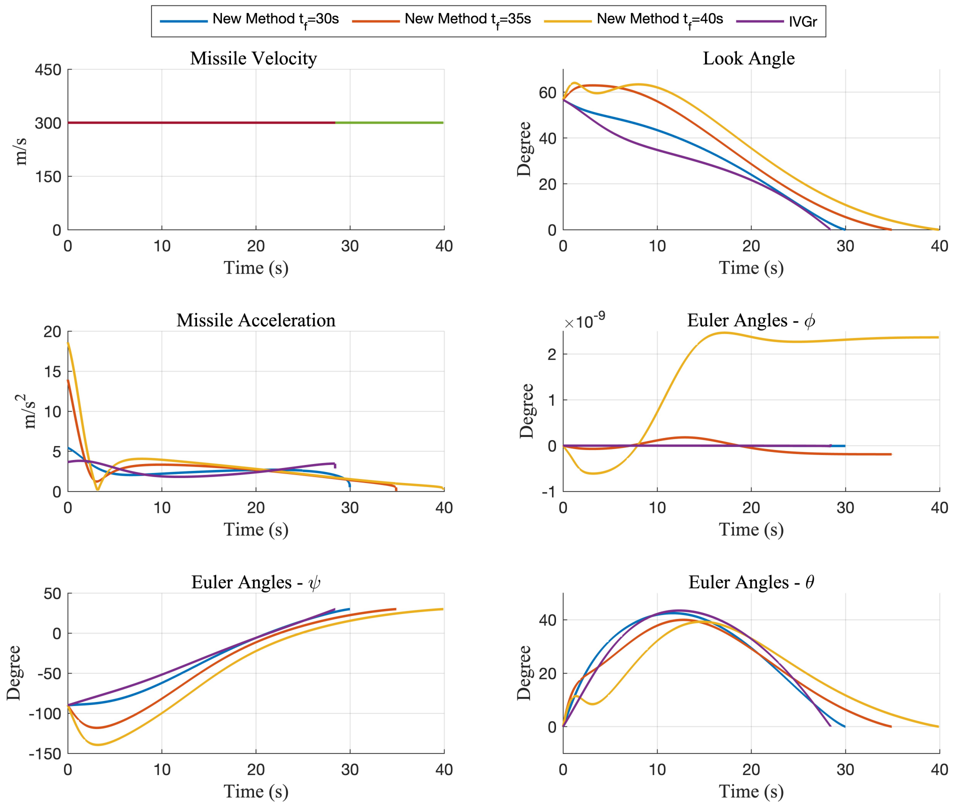


Fig. 7 Graphs of missile velocity, acceleration, look angle, and Euler angles for Case-1

The graphs obtained by applying the guidance laws under the conditions of Case 2 are provided in Figs. 8 and 9. As shown in Fig. 8, for all conditions, the missile reaches the target. The virtual targets, which enable the missiles to follow distinct trajectories, are also depicted in Fig. 8. Examining the figure, it can be observed that the missiles are launched with the same initial launch angle and then chase their corresponding virtual targets.

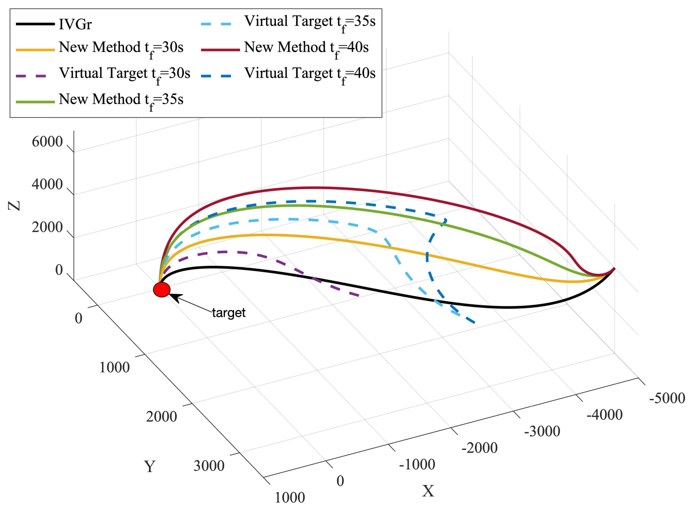


Fig. 8 3D trajectories of Case-2 by using IVGr and a new method for different desired impact time

Upon examining Fig. 9, similar observations can be made as in the Case-1. The missile's velocity remains constant, and the look angle becomes zero at the desired impact time. As the desired impact time

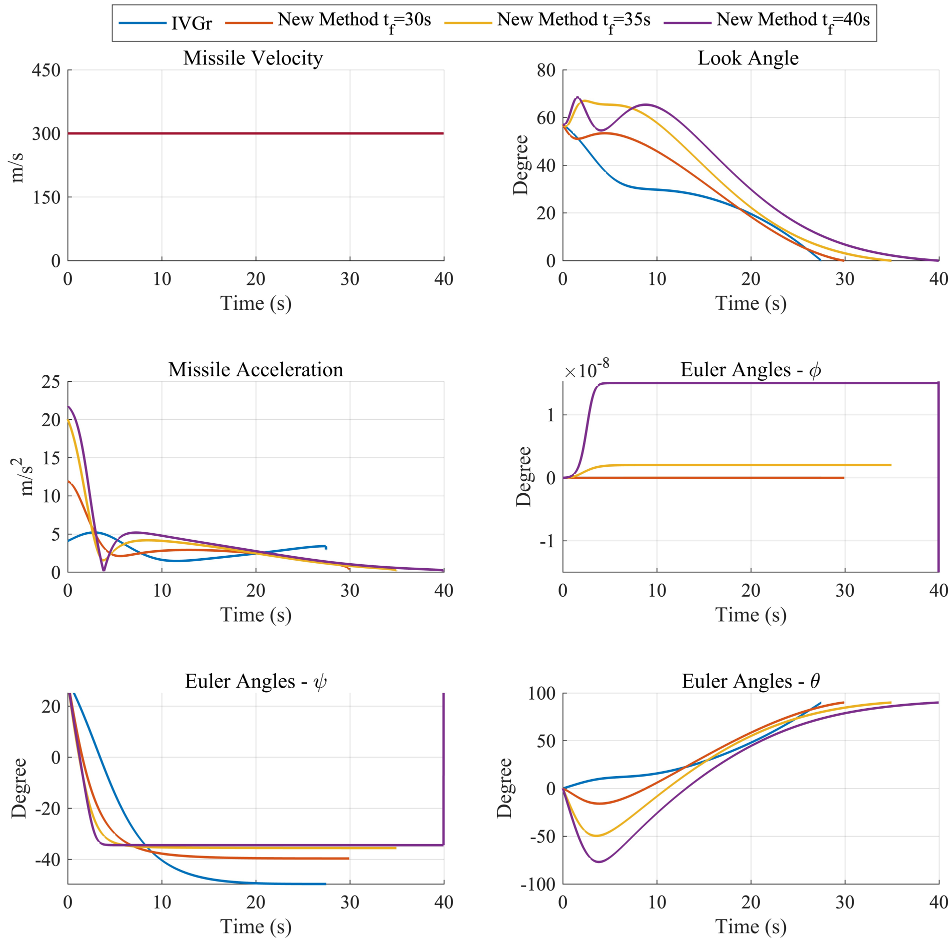


Fig. 9 Graphs of missile velocity, acceleration, look angle, and Euler angles for Case-2

increases, the initial acceleration applied to the missile also increases. The impact angle requirement of $\theta_f = -90deg$ is met. The ψ_f demand was not satisfied since it does not change the u_f, thus the error while $\theta_f = -90deg$.

5 Conclusion

In this study, the integration of impact time to the IVGr guidance law, which is a guidance law that provides control over the impact angle in three-dimensional engagements, has been conducted using Bézier curves and virtual targets. In the literature, complex methods such as analytical techniques and optimal control approaches are commonly employed to achieve control over impact time and angle. Here, the aim is to present a non-complex method utilizing Bézier curves and virtual targets for three-dimensional engagements. Upon examining the results, it can be observed that the desired impact times and angles are achieved. Although the performance of the EIVG is satisfactory, it is evident that as the desired impact time increases, the expected acceleration of the missile also increases. This result may be undesired for real-world applicability. The other drawback of the proposed method is the offline procedure. Therefore, the method is sensitive to disturbance and target maneuvers. Possible future works include eliminating the offline algorithm in order to get a more robust method and eliminating the higher acceleration demands at the beginning of the engagement. Another possible future work can be use this offline algorithm as an initial guess of a nonlinear receding horizon method and looking the engagement dynamics from a control problem perspective.

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