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Application of a Mixed H_2/H_∞ Synthesis Approach to the Control Problem of the LISA Mission

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ABSTRACT

The Laser Interferometer Space Antenna (LISA) mission from the European Space Agency (ESA) poses particularly interesting challenges in terms of Guidance, Navigation, and Control (GNC), given the accuracy requirements it entails, together with significant constraints in terms of sensing and actuation, and the associated non-trivial Dynamics, Kinematics, and Environment (DKE), the latter being addressed in a companion paper. On the one hand, since several of the GNC requirements are described in the frequency-domain, and some parameters of the spacecraft model are uncertain, robust H_∞ control is a natural candidate technique to address this type of problem. On the other hand, the H_2 norm optimization allows formulating the problem as that of minimizing the energy on specific channels, which is relevant for fuel consumption reduction and for improving the conditions for the scientific experiments execution. Hence, this paper describes a preliminary application of a mixed H_2/H_∞ control design solution to the LISA mission, discussing the advantages and limitations of this type of technique.

Keywords: dynamics, robust control, GNC, LISA

1 Introduction

The Laser Interferometer Space Antenna (LISA) mission, a large-class mission from the European Space Agency (ESA), aims to be the pioneer gravitational wave observatory in the realm of space exploration [3]. This ambitious objective hinges on the deployment of three satellites, positioned at a vast separation of 2.5 million kilometers from one another, arranged in a rotating triangular configuration. These satellites orbit 50 million kilometers away from Earth, following a Heliocentric Earth Trailing Orbit (HETO). Equipped with highly sensitive laser and interferometry-based sensors, each satellite detects the smallest perturbations within the triangular formation, which originate from the low-frequency gravitational waves. It is the substantial scale of this partially virtual spatial structure that enables the observation of gravitational waves, unveiling the distortions in the fabric of spacetime.

Being able to sense such small perturbations is translated into very stringent requirements for the Guidance, Navigation, and Control (GNC) system [1], [2]. For this reason, ESA issued an invitation to tender for the LISA GNC development [7], with the consortium led by SENER and participated by DEIMOS being one of the selected teams for this activity. Within this project, entitled DFACS-L3GWO, DEIMOS' responsibilities included the DKE modeling, in addition to the investigation of a complementary control design approach, which is described in the sequel.

Since several of the GNC requirements of LISA (cf. [6], [5]) are expressed in the frequency-domain, and considering the presence of uncertainties in some of the key spacecraft model parameters, but also taking into account that, at least for the science phase, the dynamics can be considered approximately Linear Time-Invariant (LTI), robust H_∞ control emerges as a fitting approach for tackling this precise challenge (c.f. [7], [9]). However, it is also in general desirable that the resulting control solution generates smooth commands, in particular to minimize fuel consumption or actuator wear. Hence, the well-known mixed H_2/H_∞ control approach was selected by DEIMOS to tackle this problem.

Since the selected approach is model-based, it requires sufficiently accurate models to proceed with the next steps of the design. Hence, the first challenge posed by LISA in terms of control design is the modeling of the DKE, which is particularly difficult given the several orders of magnitude between the positions of the spacecraft and the small displacements of the test masses inside each of them, which are used to sense the perturbations caused by the gravitational waves. This was addressed in detail within DFACS-L3GWO and the main results are provided in a companion paper [10].

With the DKE models defined, the control problem can be setup by selecting the appropriate control loops, followed by the adoption of the preferred controller synthesis routines. While toolboxes, such as `systune` from Mathworks®, already exist that allow for setting H_2 and H_∞ requirements and automatically optimize the controller gains, for the selected fixed controller structure, the approach adopted here resorts to the optimization of the controller without a particular structure. This is well-known to transform the optimization within the H_∞ problem convex, although such convexity is lost for the H_2/H_∞ case [8].

It is highlighted that, while the results presented here were obtained within the DFACS-L3GWO project and using the simulation facility developed for this study, the preliminary control design described next was not part of a review by ESA nor by SENER.

The remainder of this paper is organized as follows: Section 2 is devoted to the description of the mixed H_2/H_∞ control approach adopted; the Monte-Carlo results are provided in Section 3; finally, the conclusions are summarized in Section 4.

2 Mixed H_2/H_∞ Controller Synthesis

2.1 Introduction

In this section, a mixed H_2/H_∞ control design approach is applied to the problem of stabilization and reference tracking of DFACS-L3GWO. The process starts with the linearization of the dynamics, which are validated against the nonlinear models described in the companion paper [10]. These linearized dynamics are used, together with the so-called dynamic weights (design tuning nobs), to implement the design interconnection. An iterative design loop was implemented between the synthesis of the controller and the analysis – both in the frequency- and the time-domain – to tune the weights so that the requirements are attained. In this paper, the focus is given to describing the methodology and the associated advantages and limitations, rather than to the detailed controller tuning.

While H∞ constraints are suitable to ensure robust stability and performance for a family of plants (typically described in a Linear Fractional Transformation (LFT) fashion), H_2 constraints also entail remarkable properties that are particularly relevant for the drag-free control problem, since:

- the energy of the tracking error signal shall be sufficiently small for the scientific experiment to be feasible, which can be naturally posed as an optimization problem to minimize the H_2 norm of the operator from the disturbances to the tracking error.
- requirements on the sensitivity to measurement noise can be directly formulated by using the H_2 norm, as it provides the means to obtain the optimal solution, under the standard assumption of independent, zero-mean, white noise processes.

The overall controller synthesis methodology can be structured into the following three main steps:

- Modeling: The derivation of a reliable model is of paramount importance for control design. The model adopted within the work reported in this paper is described by using the LFT framework, which is particularly suitable for robust control design. This step also includes the validation of the models against the nonlinear dynamics.
- Controller design/synthesis: This step is where the controller is actually computed, by using, in this case, a mixed H_2/H_∞ control design approach. The proper definition of the so-called dynamic weights, which are used to shape the frequency response of the closed-loop system, is key.
- Analysis: The closed-loop system is finally evaluated with the controller obtained in the previous step. It is typical to iterate between these last two steps, either manually or by using global optimization approaches.

The fine tuning of the control design weights is an iterative process that can require a significant effort, especially when nonlinearities are present and poorly modeled, and/or when the characteristics of the disturbances are not known exactly. To address these problems, DEIMOS has developed its own control design tools, implementing the following, non-mutually-exclusive control design weights tuning approaches:

Figure 2-1: Main approaches for controller synthesis weights tuning

An example of the type of interconnection typically used to synthesize controllers based on the minimization of norms of transfer functions is depicted in Figure 2-2, where the most common shapes of the associated synthesis weights are qualitatively illustrated.

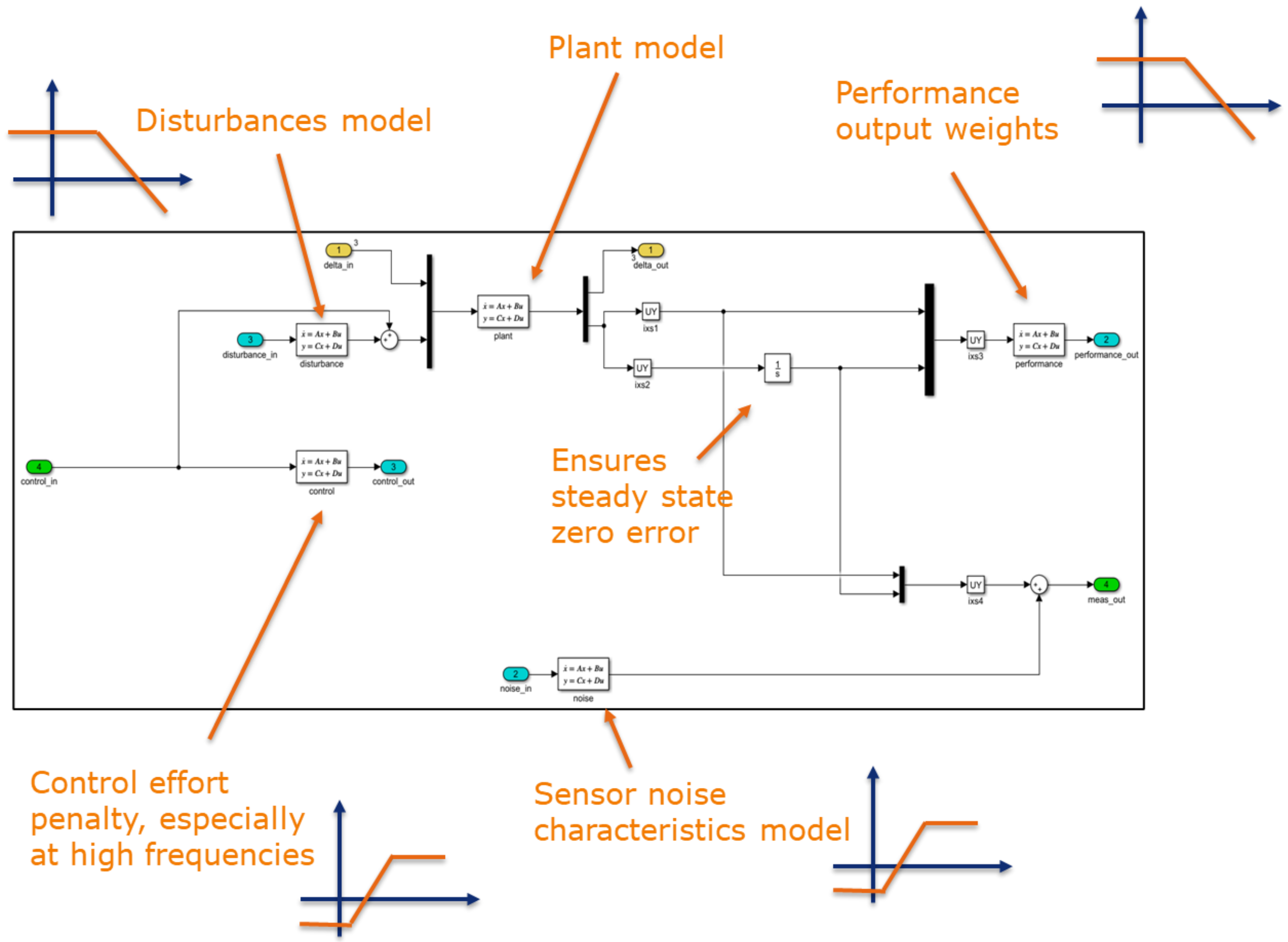

Figure 2-2: Controller synthesis interconnection example

Once the interconnection is formulated, an optimization problem is solved so as to minimize the *gain* from the selected inputs (disturbances and measurement noise, and possibly outputs of a delta-block representing model uncertainty) to the desired performance outputs. As an example, Figure 2-3 depicts the case in which a mixed H_2/H_∞ controller is to be obtained.

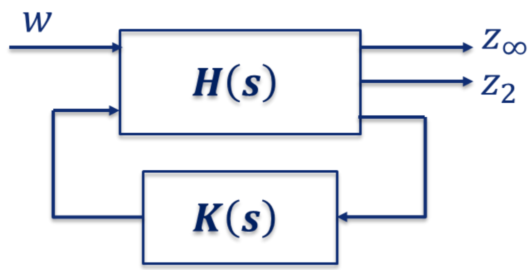

Figure 2-3: Mixed H_2/H_∞ optimization problem formulation

In this case, the controller synthesis can be formulated as

$$K = \underset{K}{\arg\min}\ \psi \left\| T_{z_\infty \leftarrow w} \right\|_\infty + \beta \left\| T_{z_2 \leftarrow w} \right\|_2$$

where $T_{y \leftarrow w}$ denotes an operator taking w to y, and ψ and β weigh the importance of each channel. While the solution to the determination of the controller K_∞ that minimizes $\left\| T_{z_\infty \leftarrow w} \right\|_\infty$, as well as the solution to the determination of the controller K_2 that minimizes $\|T_{z_2 \leftarrow w}\|_2$, are both convex optimization problems (with different well-known methods available to solve them), the mixed problem (cf., e.g. [8]) becomes non-convex and thus its solution is not so straightforward. The solution adopted in this paper resorts to solving a semi-definite program, subject to Linear Matrix Inequalities (LMIs), formulated as

$$\begin{bmatrix} A_c'X_c + X_cA_c & X_cB_{1c} & C_{1c}' \\ B_{1c}'X_c & -\gamma I & D_{1c}' \\ C_{1c} & D_{1c} & -\gamma I \end{bmatrix} < 0$$

$$\begin{bmatrix} A_cX_c + X_cA_c' & X_cB_{c2} \\ B_{c2}'X_c & -I \end{bmatrix} < 0, \begin{bmatrix} X_c & C_2' \\ C_2 & Z_{c2} \end{bmatrix} > 0, \ Tr(Z_{c2}) < \alpha^{1/2}, \ D_{c2} = 0$$

in which α is the H_2 norm and γ is the H_∞ norm of the obtained closed-loop system. It is highlighted that, for this specific formulation of the problem, an additional constraint is posed, so that the problem becomes convex. In particular, it is assumed that the Lyapunov matrix (i.e., the positive definite matrix of the quadratic Lyapunov function considered) is the same for the minimization of both norms. This makes the problem tractable, at the cost of increasing the conservatism (i.e., non-optimality) of the solution. For the sake of reproducibility of the results, the `h2hinfsyn` function of MATLAB® has been used. Alternative solutions include `systune`, which in addition allow to enforce a given structure to the controller. The preferred option here was to allow the controller to be unstructured, in order to enable a better tradeoff between the two types of criteria, without the influence of suboptimality that could stem from a fixed controller structure.

Knowledge on the characteristics of the disturbances is of utmost importance, as the controller will be guaranteed to satisfy the requirements only if the *actual* disturbances have the frequency-domain behavior considered during design. Being any physical system effectively impacted by nonlinear dynamics, equivalent linear models of the disturbances are non-trivial to be obtained, at least with a sufficient level of representativity. Hence, a follow-up is typically adopted that resorts to global optimization methods to improve the closed-loop properties.

As illustrated in Figure 2-1, two broad classes of weight tuning approaches can be adopted:

- **Weights optimization**: in this case, a global optimizer is used to determine the weights for the controller synthesis, such that a given criterion is minimized. This criterion can be defined by means of a cost function with different terms, including penalties on the violation of requirements, robustness properties provided by the controller, or even the outcomes of time-domain assessments. The fact that, once the optimizer selects the weights, a controller synthesis technique such as structured or unstructured H_∞ synthesis, μ-synthesis, H_2 or mixed H_2/H_∞, is used, allows a priori stability and/or performance guarantees to be obtained. A drawback of this method is the fact that, except for the structured H_∞ synthesis, high order controllers may be generated.
- **Controller parameters optimization**: an alternative approach is to optimize the parameters (gains) of the controller directly. The cost function (i.e., criterion) can be defined as in the previous case, but no guarantees are provided – not even in terms of closed-loop stability – a priori, i.e., before the current cost is computed. One of the main advantages of this technique, implemented, for instance, by `systune` in MATLAB, is the fact that the structure (and, therefore, the order) of the controller can be fixed.

L3GWO poses a challenging control problem that can be addressed by a divide-and-conquer approach: rather than designing a single controller for all the modes and all the control loops, the control design started with the definition of control modes, that can be seen as sub-modes of the GNC modes.

For each control loop, a mode is defined by a unique combination of sensors, actuators, and controller gains; if one of those changes, it is considered that the control mode (or sub-mode) is also changed.

2.2 Science Mode S/C Attitude Controller

In the sequel, focus is given to the so-called Science Mode of the S/C attitude control loop [4], although the process adopted was the same for the remaining control loops and sub-modes. While the objective is to design mixed H_2/H_∞ controllers for the aforementioned reasons, we first focus on the design of an H_∞ controller that is able to meet the closed-loop requirements. The starting point is, therefore, to list those requirements, described as frequency-domain constraints. For the present control loop and GNC mode, these constraints can be interpreted as follows: the Amplitude Spectral Density (ASD) of the output of the closed-loop system, driven by the exogenous disturbances and sensor noises, needs to be below the curve in Figure 2-4. Models of the sensors and actuators [5], and the navigation system, were derived, in order to obtain approximations of the associated ASD, which is key for the controller synthesis setup, as they were used to define the input weighting functions in Figure 2-5.

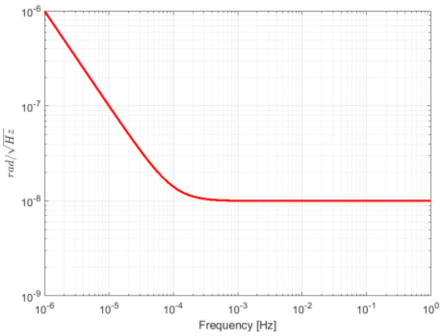

Figure 2-4: ASD constraints for the S/C attitude tracking error

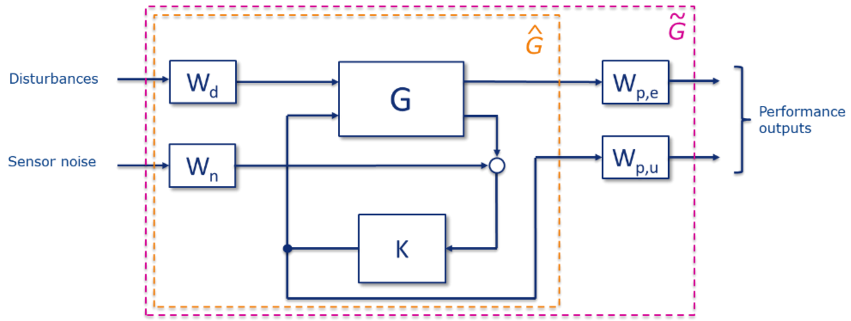

Figure 2-5: Simplified interconnection view for controller synthesis

The interconnection illustrated in Figure 2-5, in which $W_p = \text{blkdiag}(W_{p,e}, W_{p,u})$, G is the plant model, and K is the controller, it is straightforward to conclude that, if $\|\tilde{G}\|_\infty < 1$, then

$$\|\tilde{G}\|_\infty = \|W_p\hat{G}\|_\infty < 1 \Rightarrow \|\hat{G}\|_\infty < \frac{1}{\|W_p\|_\infty}$$

Hence, the closed-loop system is compliant with the requirements, embedded in W_p and in the augmented $\tilde{G}$ plant. For this specific case, $W_{p,e}$ is modeled as a transfer function whose shape is the inverse of the one represented in Figure 2-4. Moreover, $W_{p,u}$ is defined as a constant set to 0.2, so that a 1 nrad tracking error generates a torque command smaller than 5 nNm, thus keeping the actuators sufficiently far from saturation. Although not needed for this specific control loop, it is also common for $W_{p,u}$ to be frequency-dependent, especially so that commands with a high-frequency content are penalized.

Once a controller meeting the requirements is obtained, one can pursue further reducing the “energy” of selected error signals. This can be performed, as described above, by weighing also the H_2-norm of desired performance channels. In this case, we consider minimizing the H_2-norm from the disturbances to the $W_{p,u}$ outputs (i.e., the control effort, ultimately aiming to minimize fuel consumption). Varying the weight on the H_2-norm, when compared to the H_∞-norm, leads to the following results.

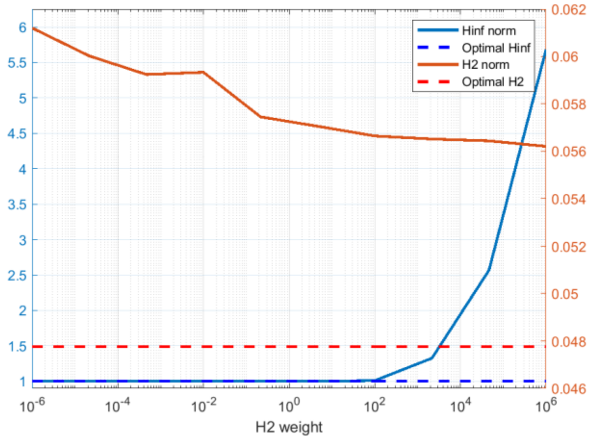

Figure 2-6: Tradeoff of H_2 vs. H_∞ norm

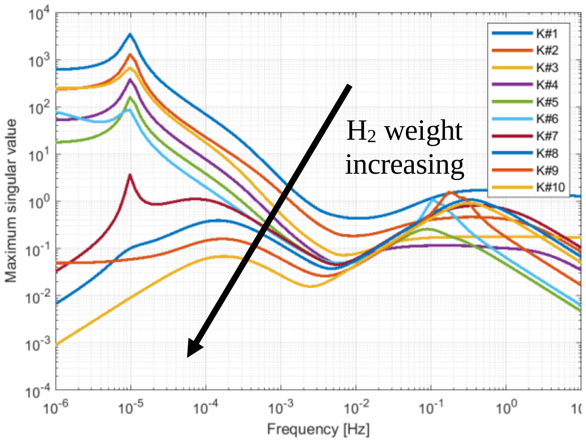

Figure 2-7: Controller frequency response evolution with increasing H_2 weight

As illustrated in Figure 2-6, increasing the H_2-norm weight ultimately impacts the H_∞-norm. As a consequence of that, also the controller bandwidth gets reduced, so as to minimize the control effort – cf. Figure 2-7. However, it can also be seen that for intermediate values of both weights a balanced design is obtained, in which $\gamma \approx 1$ and with an H_2-norm that is around 25% above the optimal value (compared to an H_2-norm of more than 50% above the optimum for the pure H_∞ design). These are promising results, as overall an H_2-norm reduction can be obtained, without jeopardizing the requirements satisfaction, at least in the linear realm. In practice, this can represent a better behavior in the frequency-domain, while possibly reducing power consumption.

To further assess this, a time-domain simulation was performed and the MPS torques, depicted in Figure 2-8 to Figure 2-10, analyzed. For this scenario, a mixed H_2/H_∞ controller with a small (10^{-6}) weight on the H_2 cost was selected, with this being sufficient for illustration purposes of the potential benefits stemming from a mixed synthesis approach.

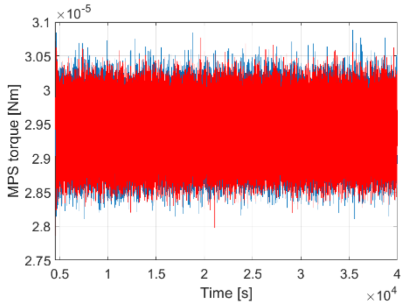

Figure 2-8: MPS torque around the x-axis (red: mixed H_2/H_∞, blue: H_∞)

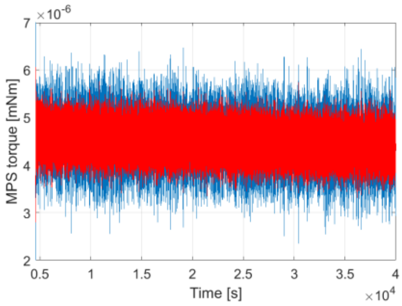

Figure 2-9: MPS torque around the y-axis (red: mixed H_2/H_∞, blue: H_∞)

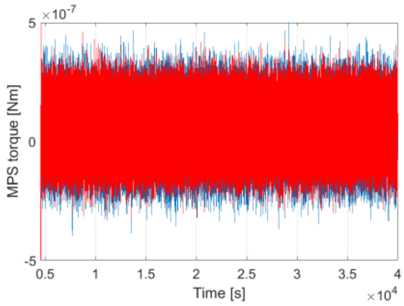

Figure 2-10: MPS torque around the z-axis (red: mixed H_2/H_∞, blue: H_∞)

For this specific case, the standard deviations of the MPS torques are as follows, having in mind that only a single (randomly generated) configuration of the plant was considered:

Table 2-1: Standard deviation of the MPS torques

	x-axis	y-axis	z-axis
Standard H_∞	0.30 µNm	0.44 µNm	0.09 µNm
Mixed H_2/H_∞	0.28 µNm (-7%)	0.27 µNm (-40%)	0.08 µNm (-10%)

Therefore, a reduction of the standard deviation of the MPS torque of up to 40% was observed, for this preliminary tuning, which can clearly be beneficial in terms of fuel consumption. However, these preliminary results, although being promising, would require a more detailed validation through, in particular, a Monte-Carlo testing campaign. Moreover, by being less aggressive in terms of attenuation of maximum amplifications of the closed-loop, the solution is naturally non-optimal in that sense, contrary to the standard H_∞ synthesis.

3 Simulation Results

A small, 10-shot, Monte-Carlo campaign, with 40,000 secs per shot, was performed, in order to have a preliminary insight on the robustness of the controllers designed for the Science Mode. The simulation results were obtained with the L3GWO simulation framework developed by SENER within the DFACS-L3GWO project, incorporating the DKE models developed by DEIMOS.

The ASD of the error variables of interest are illustrated in Figure 3-1 to Figure 3-3. It is stressed that the computation of the ASD at lower frequencies would require even longer simulation periods. In terms of S/C attitude control, it is clear from Figure 3-1 that the pointing error frequency-domain requirements are satisfied. While not shown here, the time-domain response is also well within the boundaries. Similarly, despite the observation of some resonance peaks, which are potentially dampened in a second iteration of the controller design, the requirements (depicted by the black lines) on the test masses maximum angular jitter and position error are clearly satisfied.

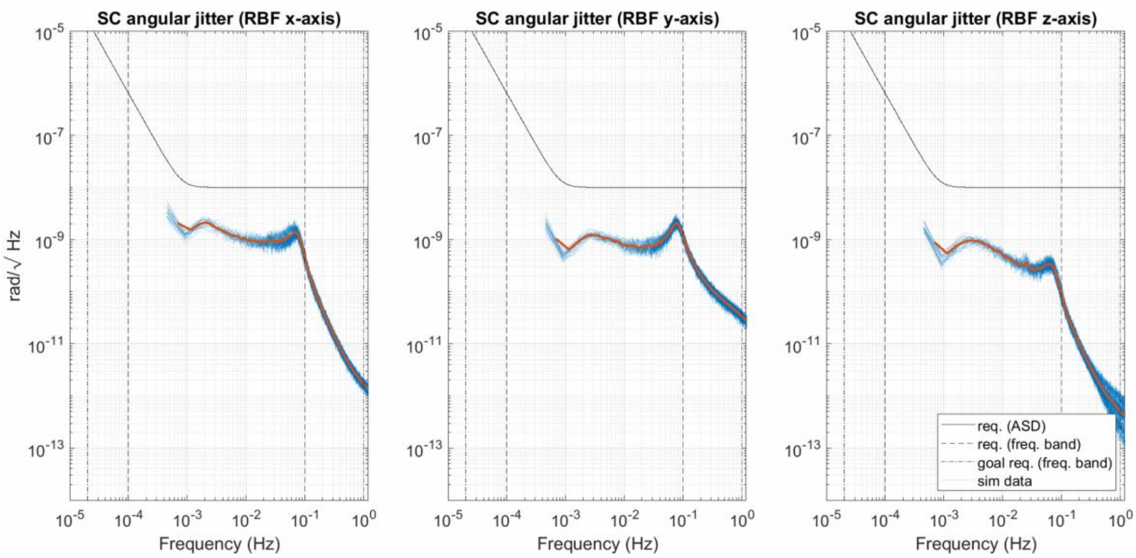

Figure 3-1: S/C angular jitter in the frequency–domain - SCI mode analysis: Monte-Carlo results

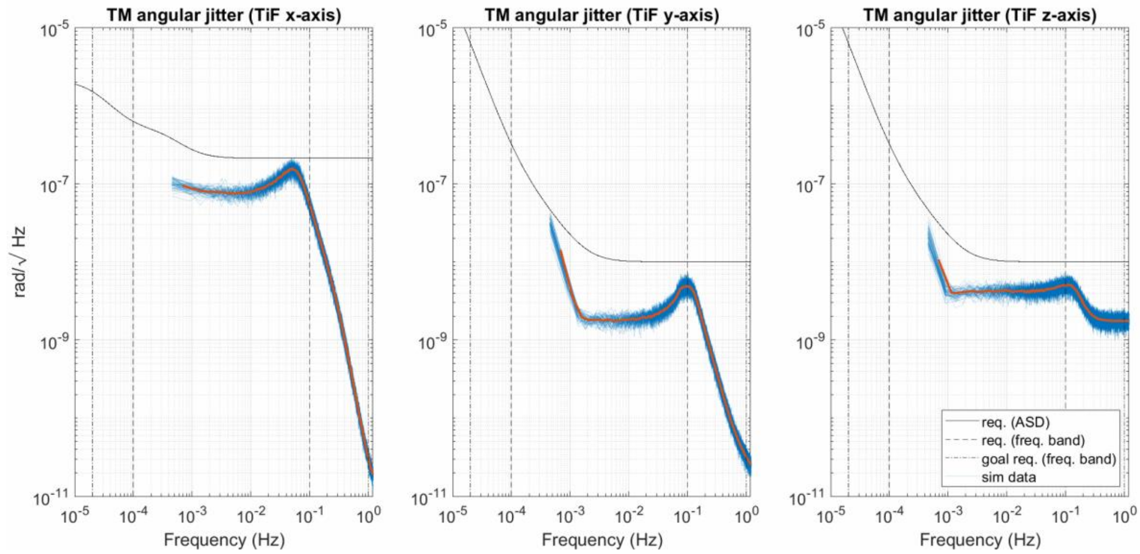

Figure 3-2: Test masses angular jitter in the frequency–domain - SCI mode analysis: Monte-Carlo results

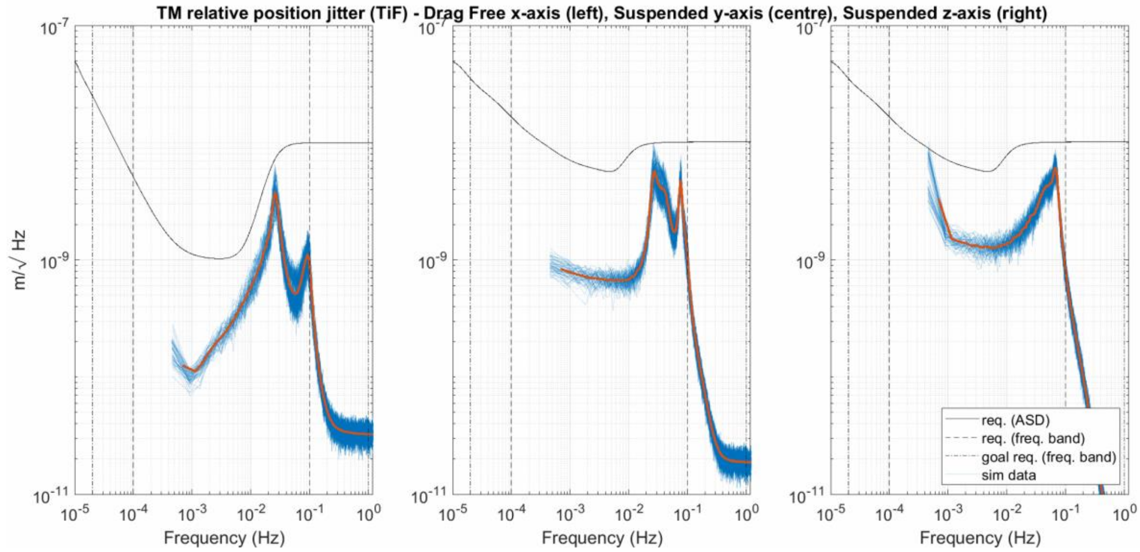

Figure 3-3: Test masses relative position errors in the frequency–domain - SCI mode analysis: Monte-Carlo results

4 Conclusions

This paper provided an overview of the fully functional control prototype and the associated preliminary analyses performed by DEIMOS for the LISA mission application scenario. For this preliminary prototype, all of the performance requirements in the Science Mode were satisfied, for all the frequency ranges, with no actuator saturation being observed for this GNC mode. Moreover, a preliminary assessment of the benefits of a mixed H_2/H_∞ design was provided, comparing an H_∞ controller with another obtained with a mixed H_2/H_∞ synthesis, for one of the control loops, in the science mode, and considering the same preliminary selection of tuning weights. A reduction of the standard deviation of the commanded signals of up to 40% was observed in a time-domain simulation, without any further iteration on the design weights.

In terms of future work, a detailed tuning of the controllers is recommended. While the results provided here are promising, the potential benefits of a mixed H_2/H_∞ design approach, in particular in terms of error energy minimization, can only be unleashed with another iteration on the tuning of the controllers. Moreover, the use of alternative tools such as `systune`, in which the H_2 objectives can be set as soft constraints, while allowing to enforce a particular structure of the controller, are also deemed as particularly relevant for this type of problem.

Acknowledgments

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References

[1] P. F. Gath et al., Drag free and attitude control system design for the LISA science mode, in Proceedings of the AIAA Guidance, Navigation and Control Conf. and Exhib., Hilton Head SC, 20-23 August 2007, AIAA 2007 No. 6731 (American Institute of Aeronautics and Astronautics, 2007).

[2] S.-F.Wu and D. Fertin, Spacecraft drag-free attitude control system design with quantitative feedback theory, Acta Astronautica 62, 668 (2008).

[3] K. Danzmann et al., LISA - Laser Interferometer Space Antenna. A proposal in response to the ESA call for L3 mission concepts, Mission Proposal (Max Planck Institute of Quantum Optics, Garching, DE, 2017).

[4] J.Wertz, Spacecraft Attitude Determination and Control, (2002).

[5] O. Jennrich, LISA technology and instrumentation, Class. QuantumGrav. 26, 153001 (2009).

[6] Schumaker B. Disturbance reduction requirements for LISA. (2003).

[7] R. Sánchez Maestro, J. Veenman, J. Cardín, J. Grzymisch, V. Preda, GNC Functional Architecture Design and Implementation of the LISA Drag Free Control System, ESA GNC-ICATT 2023.

[8] Scherer, C. W. (1996). Mixed H2/H∞ control for time-varying and linear parametrically-varying systems. International Journal of Robust and Nonlinear Control, 6(9-10), 929-952.

[9] Navarro-Tapia, D., & Marcos, A. (2023, August). Legacy-Based Design of the LISA Mission Science Control Mode. In 2023 IEEE Conference on Control Technology and Applications (CCTA) (pp. 412-417). IEEE.

[10] Cojocaru, A.F. & Lapusneanu, A. & Rosa, P. & Sánchez Maestro, R. & Cardin, J. & Veenman, J. & Preda, V. & Grzymisch, J. (2024). Development of the Dynamics, Kinematics, and Environment (DKE) Models for L3GWO. In CEAS EuroGNC Conference, Bristol, UK (Submitted).