

# **Enhanced Inertial Navigation System for Missile Applications Using Seeker, RF Data Link and Radar Altimeter Measurements**



#### *ABSTRACT*

**In the domain of missile applications, the integration of the Global Positioning System (GPS) technology has proven pivotal in improving the accuracy and reliability of Inertial Navigation Systems (INS). Nevertheless, the susceptibility of GPS signals to blockage, particularly within adversarial operational theaters, presents a noteworthy challenge. This paper presents a novel approach to enhance the precision of Inertial Navigation System (INS) solutions when GPS signals are unavailable. The proposed method integrates a seeker for Line of Sight (LOS) angle, an RF data link for range information, and a radar altimeter for measuring the distance between the missile and the ground. Combining these technologies makes the INS solution accurate even during GPS signal unavailability. The proposed approach capitalizes on the utilization of three supplementary data sources: the seeker, responsible for providing the line-of-sight (LOS) angle, the RF data link, providing the range information between the missile and the platform, and the radar altimeter, providing the distance between the missile and the ground. The proposed methodology involves the integration of these three measurements with the INS framework with the help of the Extended Kalman Filter (EKF). One of the key advantages of this method is its ability to function independently of explicit landmark positioning. The effectiveness of the proposed approach is demonstrated through comprehensive simulation studies, revealing substantial improvements in the missile navigation accuracy and the targeting precision. This research underscores the potential significance of incorporating seeker, RF data link and radar altimeter measurements within the INS solution, offering enhanced guidance capabilities in scenarios where GPS signals may be malfunction, thereby contributing to the advancement of missile guidance technology in challenging operational environments.**

**Keywords:** Inertial Navigation Systems Improvement, RF Data Link, Radar Altimeter, Seeker, Extended Kalman Filter.

## **Nomenclature**





## **1 Introduction**

The advancements in the technology have led to the increased sophistication and intelligence of guided missiles. However, the realm of navigation applications has remained relatively unchanged in practice, with Inertial Navigation Systems (INS) relying on Global Positioning System (GPS) signals whenever they are available. Unfortunately, various factors such as signal jamming, receiver issues, or satellite malfunctions can disrupt GPS signals, causing INS solutions to diverge over time [9]. This paper's primary objective is to explore the potential of utilizing a seeker as an external measurement source to enhance INS solutions. The seeker can be directed toward a landmark along the missile's trajectory, using Line-Of-Sight (LOS) angle and/or LOS rate measurements to improve the accuracy of the INS solution.

It is worth noting that the concept of using a seeker for INS assistance is not new, as previous researchers have explored and proposed various solutions. Some studies, like Ref. [1] and Ref. [2], have integrated the seeker during the midcourse phase to enhance the INS accuracy, employing the Extended Kalman Filtering (EKF) to fuse the LOS angles and the LOS rates with the INS. Notably, Ref. [2] does not assume the knowledge of the landmark's position, unlike Ref. [1], but it does not correct the missile's position either. Another approach, as seen in Ref. [3], utilizes the LOS angles and the indirect Kalman Filtering to enhance the INS solution by forming indirect measurements through the comparison of raw and estimated measurements, with the GPS incorporated into the Kalman Filtering structure. In Ref. [4], the LOS angles and the radar altimeter data are employed to estimate INS errors in a cruise missile using EKF, assuming the knowledge of the landmark's position. Additionally, Ref. [5] establishes a relationship between the LOS angles and the INS error model, improving the INS solution with a square root Kalman filter that incorporates both measured and calculated LOS angles. Veth, Ref. [6], used binocular stereopsis which provide the range from the vehicle to the landmark to integrate the image and the inertial measurements to improve navigation solutions. In Ref. [14], INS solution is improved by integrating seeker and RF datalink measurements with inertial measurements.

Radar altimeters have been a subject of study in INS for several decades and are commonly used for the terrain-aided navigation, including terrain contour matching (TERCOM) systems Ref. [9]. Notably, Ref. [7] proposes three nonlinear Kalman filtering techniques for the terrain-aided navigation, while Ref.



[8] introduces hierarchical elevation map clustering to enhance the INS solution. Additionally, Ref. [10] employs terrain-aided navigation for unpowered tactical missiles, integrating radar altimeter data and terrain slope information within a Kalman Filter to estimate navigation errors. Ref. [11] addresses the implementation challenges associated with radar altimeters in terrain-aided navigation. In Ref. [13], the radar altimeter is merged with the seeker and the inertial measurements to improve navigation solution. However, proposed method of Ref. [13] is not robust against the slope of the surface.

To address a limitation found in Ref. [1,3-6], where the landmark's position must be preloaded into the missile's database before flight. This study utilizes RF data link and radar altimeter to overcome beforementioned limitation. By merging seeker, RF datalink, and radar altimeter measurements, INS solutions can be improved with robust initialization. It is important to clarify that, for the sake of simplicity in system analysis, it is assumed that the missile's attitude is perfectly known, and the INS derives the missile's position and velocity solely from accelerometer measurements.

## **2 INS Aiding Formulation**

### **2.1 System Model**

The states of the system are given as follows

$$
\bar{X} = \begin{bmatrix}\n\delta x_m \\
\delta y_m \\
\delta z_m \\
\delta v_x \\
\delta v_y \\
\delta v_y \\
b_x \\
b_y \\
b_z \\
\delta x_L \\
\delta y_L \\
\delta z_L\n\end{bmatrix}
$$
\n(1)

The states are three missile position errors, three missile velocity errors, three accelerometer biases, and three landmark position errors.

When the landmark is assumed to be stationary, continuous system equations can be written as

$$
\begin{aligned}\n\delta \dot{x}_m \\
\delta \dot{y}_m \\
\delta \dot{v}_x \\
\delta \dot{v}_z \\
\delta \dot{v}_z \\
\delta \dot{v}_z \\
\delta \dot{y}_x \\
\delta \dot{v}_y \\
\delta \dot{v}_z \\
\delta \dot{y}_x \\
\delta \dot{v}_y \\
\delta \dot{v}_z \\
\delta \dot{z}_z\n\end{aligned}
$$
\n(2)



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$$
\dot{\bar{X}} = \hat{F}\bar{X} \tag{3}
$$

To make the problem easier,  $\hat{C}^{(i,b)}$  (Direction Cosine Matrix (DCM) from the body frame to the inertial frame) is assumed to be known. The discrete form of the system is as follows

$$
\bar{X}_{k+1} = \hat{\phi}_k X_k \tag{4}
$$

Where the system transition matrix,  $\hat{\phi}_k$ , is obtained as follows

$$
\hat{\phi}_k = \hat{I}_{12x12} + \hat{F}d_t \tag{5}
$$

Where  $d_t$  is the time interval.

#### **2.2 Measurement Model**

#### *2.2.1 Seeker Measurement Model*

In this study, the seeker is assumed to provide the Line-of-Sight (LOS) angle directly. In real applications, it is known that the seeker provides angular position of the target with respect to the body. However, since  $\hat{C}^{(i,b)}$  is known, it is a valid assumption. The seeker measurements are as follows

$$
\lambda_{az} = \operatorname{atan} \left( \frac{y_L - y_m}{x_L - x_m} \right) + \eta_{\lambda} \tag{6}
$$

$$
\lambda_{el} = \text{atan}\left(\frac{z_L - z_m}{\sqrt{(x_L - x_m)^2 + (y_L - y_m)^2}}\right) + \eta_{\lambda} \tag{7}
$$

Where  $\lambda_{az}$ ,  $\lambda_{el}$  are azimuth and elevation LOS angles, respectively.  $\eta_{\lambda}$  is as uncorrelated, zero-mean additive Gaussian noise with  $\eta_{\lambda} \sim N(0, \sigma_{\lambda})$ .

Seeker measurement equations are valid for a yaw-pitch gimbal system. Measurement equations for the other type of gimbal systems (e.g. pitch-yaw, roll-yaw etc.) should be derived.

#### *2.2.2 Radar Altimeter Measurement Model*

For modelling the radar altimeter measurement model, it is assumed that the landmark is located on the ground. The radar altimeter measurement are as follows

$$
h = abs(zL - zm) + \etah
$$
\n(8)

where h is the altitude difference between the landmark and the missile, and  $\eta_h$  is as uncorrelated, zero-mean additive Gaussian noise with  $\eta_h \sim N(0, \sigma_h)$ .

#### *2.2.3 RF Data Link Measurement Model*

The data link provides the distance between the missile and the platform antenna. The position of the platform is assumed to be known, which is a valid assumption, since the data link can carry the related information to the missile. The RF data link measurement model is as follows

$$
r = \sqrt{(x_m - x_p)^2 + (y_m - y_p)^2 + (z_m - z_p)^2} + \eta_r
$$
\n(9)

Where  $x_p$ ,  $y_p$ ,  $z_p$  are the position of the platform, and  $\eta_r$  as uncorrelated, zero-mean additive Gaussian noise with  $\eta_r \sim N(0, \sigma_r)$ .



#### *2.2.4 Linearization of Measurement Models*

The Extended Kalman Filter (EKF) is used in this study as an estimator. The EKF requires linearized relationship between states and measurement equations. Measurement equations are linearized by taking derivatives of the measurement equations with respect to the states. Jacobian matrix of the measurement equations are as follows

$$
\hat{H} = \begin{bmatrix}\n\frac{\partial \lambda_{az}}{\partial x_m} & \frac{\partial \lambda_{az}}{\partial y_m} & \frac{\partial \lambda_{az}}{\partial y_x} & \frac{\partial \lambda_{az}}{\partial y_y} & \frac{\partial \lambda_{az}}{\partial y_z} & \frac{\partial \lambda_{az}}{\partial b_x} & \frac{\partial \lambda_{az}}{\partial b_y} & \frac{\partial \lambda_{az}}{\partial b_z} & \frac{\partial \lambda_{az}}{\partial y_L} & \frac{\partial \lambda_{az}}{\partial y_L} \\
\frac{\partial \lambda_{el}}{\partial x_m} & \frac{\partial \lambda_{el}}{\partial y_m} & \frac{\partial \lambda_{el}}{\partial z_m} & \frac{\partial \lambda_{el}}{\partial y_x} & \frac{\partial \lambda_{el}}{\partial y_y} & \frac{\partial \lambda_{el}}{\partial y_z} & \frac{\partial \lambda_{el}}{\partial b_x} & \frac{\partial \lambda_{el}}{\partial b_y} & \frac{\partial \lambda_{el}}{\partial b_z} & \frac{\partial \lambda_{el}}{\partial x_L} & \frac{\partial \lambda_{el}}{\partial y_L} \\
\frac{\partial \lambda_{em}}{\partial x_m} & \frac{\partial \lambda_{em}}{\partial y_m} & \frac{\partial \lambda_{em}}{\partial z_m} & \frac{\partial \lambda_{em}}{\partial y_x} & \frac{\partial \lambda_{em}}{\partial y_y} & \frac{\partial \lambda_{em}}{\partial y_z} & \frac{\partial \lambda_{em}}{\partial b_x} & \frac{\partial \lambda_{em}}{\partial b_y} & \frac{\partial \lambda_{em}}{\partial b_z} & \frac{\partial \lambda_{em}}{\partial y_L} & \frac{\partial \lambda_{el}}{\partial z_L} \\
\frac{\partial \tau}{\partial x_m} & \frac{\partial \tau}{\partial y_m} & \frac{\partial \tau}{\partial z_m} & \frac{\partial \tau}{\partial y_x} & \frac{\partial \tau}{\partial y_y} & \frac{\partial \tau}{\partial y_z} & \frac{\partial \tau}{\partial b_x} & \frac{\partial \tau}{\partial b_y} & \frac{\partial \tau}{\partial b_z} & \frac{\partial \tau}{\partial x_L} & \frac{\partial \tau}{\partial y_L} \\
\frac{\partial \lambda_{el}}{\partial x_m} & \frac{\partial \lambda_{el}}{\partial y_m} & \frac{\partial \tau}{\partial z_m} & \frac{\partial \tau}{\partial y_x} & \frac{\partial \tau}{\partial y_y} & \frac{\partial \tau}{\partial y_z} & \frac{\partial \lambda_{el}}{\partial b_x} & \frac{\partial \lambda_{el}}{\partial b_y} & \frac{\partial \lambda_{el}}{\partial b
$$

Where

$$
R_H = \sqrt{(x_L - x_m)^2 + (y_L - y_m)^2}
$$
 (12)

$$
R_T = \sqrt{(x_L - x_m)^2 + (y_L - y_m)^2 + (z_L - z_m)^2}
$$
\n(13)

By merging all measurement equations, the measurement vector is defined as

$$
\bar{z} = \begin{bmatrix} \lambda_{az} \\ \lambda_{el} \\ h \\ r \end{bmatrix}
$$
 (14)

The system and the measurement models are obtained. These models can be used in the Extended Kalman Filter to improve the INS solution. The initialization procedure for the proposed EKF is same with Ref. [13]. The difference between the missile and the landmark is assumed to be obtained by the radar altimeter. Although the surface is not flat, the landmark position initialization is expected to converge to the real value fast enough Ref. [14].

## **3 Simulation and Results**

The proposed methodology is tested by the air-to-ground scenario in the MATLAB-Simulink environment. The missile is released from the platform at 3000 m altitude with 300 m/s initial velocity. The seeker is locked on the landmark which is on the way to the target. Even though the seeker is locked on the target; the missile keeps going to the target. The LOS rates obtained from the seeker in the midphase is used to improve the INS solution, not to hit the target. The target is located at 20 km in downrange, and 5 km in crossrange. The landmark is located at 15 km in downrange, and 3 km in crossrange. For the seeker, the limited field of regard is considered. The error parameters used in the simulation are given in [Table 1.](#page-5-0)



#### <span id="page-5-0"></span>**Table 1 Error Parameters**



### **3.1 Single Run Results**

The error parameters given in the [Table 1](#page-5-0) are used for a scenario given before. The EKF results are shown in the Fig. 1. The EKF algorithm starts estimating error parameters at 5 s(3 s after store separation). The reason for the late initialization of the EKF is that at the beginning of the flight waiting the autopilot algorithms to stabilize the missile first is more crucial for the entire mission. At around 50 s, the measurement updates are stopped because of gimbal limits. It is assumed that the gimbal cannot exceed 25 degrees for both axis (pitch and yaw). When one of the gimbal angles reaches to limit, the lock cannot be maintained any more.

In the first figure of Fig. 1, It is seen that the position errors converge to and remain below 10 m in all axes. INS-Only errors are much higher that the proposed method. That is why the INS-Only results cannot be seen in the figures. However, INS-Only errors are shown separately in the following section. In the second figure of Fig. 2, It is seen that the velocity errors converge to and remain below 1 m/s in all axes. The main reason for the INS errors is bias, which causes the INS results to diverge in time. In the third figure of Fig. 1, it is seen that the bias errors are successfully reduced. The last figure of Fig. 1 belongs to landmark position errors. Landmark position is trivial for the missile guidance, since the main objective of the missile is to hit the target, not the landmark. However, if the proposed method is used at the terminal phase of the guidance, landmark becomes target and estimating target position can be crucial when the lock is lost at the terminal phase. When the terminal phase is lost, it is a general approach to guide the missile to the last known target position.

As it is seen from the Fig. 1, not all of the covariance bounds diminish. The velocity and the bias covariance bounds diminish because these states are perfectly observable. However, the covariance bounds of the position of the missile and the landmark do not narrow or slightly enlarge. The position of the landmark and the missile depend on each other. In equations (6-8), the relation between the position states and the measurements are seen clearly. The seeker measurement provides the landmark angular position with respect to missile position. That is why the EKF cannot know where to add the errors.





**Fig. 1 Single run estimated errors**

### **3.2 Monte Carlo Results**

The proposed method is analyzed by 1000 runs in a Monte Carlo. The error parameters given in the Table 1 are taken as  $1 \sigma$  in the Monte Carlo simulation. The following figures show the RMSE of the proposed method and the INS-Only results.

In the first figure of Fig. 2, It is seen that even though the INS-Only position errors exceed 500 m, the position errors of the proposed method converge to and remain below 10 m in all axes. The velocity error of the INS-Only results diverges as it is seen from Fig. 2. This result is expected because the accelerometer bias (about 0.3 m/s2, 30 milli-g) is also integrated to calculate the velocity from the acceleration measurement. The velocity errors of the proposed method converge to and remain below 1 m/s in all axes. Since the bias errors of the accelerometer are well estimated, the velocity errors and the position errors do not increase drastically after the measurement updates are halted.





**Fig. 2 RMSEs of EKF and INS-Only results**

As shown in both single run and Monte Carlo runs, the proposed method can improve navigation results and remove the bias and initial errors. The method can be used for more than one landmark on the course. This method enhances the missile's ability to hit targets, particularly during extended flight durations.

## **4 Conclusion**

In conclusion, this paper has presented a novel methodology for enhancing the accuracy and reliability of the INS in missile applications, particularly in environments where GPS signals are not available. The proposed approach leverages three supplementary data sources: the seeker, the RF data link, and the radar altimeter, integrating them into the INS framework using the EKF. One notable advantage of this method is its independence from the explicit landmark positioning. Through comprehensive simulation studies, the research has demonstrated the significant improvements in the missile navigation accuracy and the targeting precision achieved by incorporating seeker, RF data link, and radar altimeter measurements within the INS solution. This highlights the potential significance of these measurements in enhancing missile guidance capabilities in scenarios where GPS signals may malfunction or be unavailable. In challenging operational environments, the methodology introduced in this paper contributes to the advancement of missile guidance technology, ensuring more reliable and precise navigation for critical missions. In this study, the missile attitude is assumed to be known



perfectly. This assumption should be investigated for real applications in the future works. Also, the missile attitude will be added to the problem formulation. In this work, LOS angle is used as the seeker measurement. However, the seeker also provides LOS-rate. LOS-rate measurement may be added in the future works to increase reliability and robustness of the filter.

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