EuroGNC Madrid, Spain May 5th-7th 2026

Computational Impact-Angle Guidance with Biased Proportional Navigation

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ABSTRACT

A trajectory-shaping technique for controlling the impact angle is presented, where the time-to-go information is not required. The objective is achieved by augmenting the proportional-navigation guidance command with bias addition. In the most basic case, the bias is constant, which means the target will be captured with the bias term still acting. The analytical formulation shows that the bias is embedded within an integral equation, which is why a numerical solution is needed. The computational guidance law that is herein proposed is feasible for online implementation. The performance is demonstrated with computer simulations.

Keywords: Trajectory Shaping; Impact Angle; Proportional Navigation; Bias; Computation

Nomenclature

r = range
v = speed
λ = line-of-sight angle
γ = flight-path angle
ε = look angle
a = acceleration
N = navigation gain
b = bias
τ = nondimensional time
ρ = nondimensional range
c = integration constant

1 Introduction

The primary requirement of the terminal guidance process is that it must lead the pursuer to the target. In addition to this, the application of interest might necessitate other objectives. For example, the secondary objective could be approaching the target from a specific direction. Depending on the application, being able to control the shape of the trajectory and consequently the impact geometry might mean one or a combination of the following: exploiting the weak points of the target, increasing the warhead effectiveness, avoiding directional defense mechanisms, adjusting the time of arrival, or reducing the collateral damage, etc.

Under the action of proportional navigation (PN) guidance, the impact angle, i.e. the final value of the path angle corresponding to the velocity vector of the pursuer, is determined by the initial conditions. This implies that it is possible to obtain a desired impact angle by constructing the required initial geometry. However, being possible does not mean being feasible. In fact, there will be limited authority on initial conditions in a realistic situation, if any at all. A viable alternative to adjusting the initial conditions is what is called "trajectory shaping" [1]. This phrase refers to the control action performed by the pursuer in order to modify its otherwise direct course towards the target.

Some of the trajectory-shaping guidance laws might readily be identified as a variant of PN. The first attempt in this direction seems to be the scheme proposed in [2], which attempts to enhance the PN law with the addition of a bias term as a function of the range to go, which is a problematic variable. In another noteworthy contribution [3], the pursuer, whose aerodynamic behavior is taken into account, is guided toward the stationary target by means of PN applied in two orthogonal planes with adaptive usage of the navigation gains. In [4], the guidance law sought convergence to a circular path by utilizing the PN law with a specific value of the navigation gain. Two-phased guidance schemes are proposed in [5] and [6], where the objective of the first phase is to provide proper initial angular conditions for the second phase governed by PN. The difference between these two studies is the way the first phases were handled: The former employed PN whereas a polynomial trajectory was followed in the latter. Another two-phased structure is presented in [7], where the PN guidance law is enhanced with bias addition during the first phase to be able to shape the trajectory.

Like [7], the current study originates from [8], where an *almost* thorough treatment of impact-angle control with biased PN (BPN) can be found. This work completes the unfinished business in [8] by proposing a computational method to find the constant bias required to achieve a desired impact angle against a stationary target. It will be shown that the bias appears inside an integral equation that cannot be solved analytically, which is why a numerical root-finding routine is utilized. The proposed technique, which does not need the time to go, is straightforward and can thus be implemented fairly easily. Simulation results confirm its effectiveness and show that its performance mimics optimality for specific values of the navigation gain.

2 Preliminaries

In this section, firstly, the planar engagement kinematics will be described. Secondly, the mathematics of the guidance law to be used for impact-angle control will briefly be introduced.

2.1 Engagement Kinematics

The engagement geometry between a pursuer P and a T is shown in Fig. 1, where the range between the objects is denoted by r. The pursuer, which is guided by the lateral acceleration a, has a speed of v, which is for now constant. The line-of-sight angle is denoted by λ. The flight path angle of the pursuer is represented by γ. The guidance geometry is governed by the following equations of motion:

$$\dot{r} = -v\cos(\gamma - \lambda) \tag{1a}$$

$$r\dot{\lambda} = -v\sin(\gamma - \lambda) \tag{2b}$$

Defining the look angle as

$$\varepsilon = \gamma - \lambda \tag{2}$$

Eqs. (1) can be rewritten as

$$\dot{r} = -v\cos\varepsilon \tag{3a}$$

$$r\dot{\lambda} = -v\sin\varepsilon \tag{3b}$$

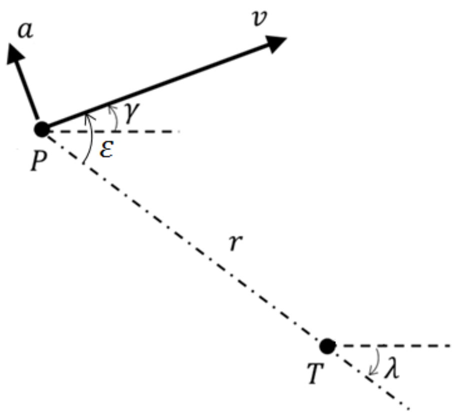


Fig. 1 Planar engagement geometry

2.2 Guidance Law

The guidance law to be utilized in this work is BPN, which is expressed as

$$a = \left(N\dot{\lambda} + b\right)v \tag{4}$$

where N is the navigation gain and b is the bias. Noting that a is responsible for rotating the velocity vector without changing its magnitude, the following equivalent form can be written:

$$\dot{\gamma} = N\dot{\lambda} + b \tag{5}$$

As done in [8] for a better understanding of the BPN kinematics, the non-dimensional time and range are now introduced as

$$\tau = \sigma b t \tag{6a}$$

$$\rho = \frac{\sigma b}{v} r \tag{6b}$$

where σ indicates the sign of b. By combining Eq. (5) and Eqs. (6), Eqs. (3) may be written as

$$\rho' = -\cos\varepsilon \tag{7a}$$

$$\varepsilon' = -(N-1)\frac{\sin\varepsilon}{\rho} + \sigma \tag{7b}$$

where $(\cdot)' = \frac{\mathrm{d}(\cdot)}{\mathrm{d}\tau}$. For investigating the analytical characteristic of the relative look angle, Eq. (7b) is divided by Eq. (7a) to obtain

$$\frac{\mathrm{d}\sin\varepsilon}{\mathrm{d}\rho} = (N-1)\frac{\sin\varepsilon}{\rho} - \sigma \tag{8}$$

The derivation of the solution to this differential equation is detailed in [9]. The solution for $N \neq 2$ happens to be as follows:

$$\sin\varepsilon = c\rho^{N-1} + \frac{\sigma}{N-2}\rho \tag{9}$$

As the initial range and look angle are known, for a given bias, the integration constant c can be obtained as:

$$c = \left(\sin\varepsilon_i - \frac{\sigma\rho_i}{N-2}\right)\rho_i^{1-N} \tag{10}$$

where the subscript i denotes the initial value. Moreover, the following can be shown to hold by combining Eqs. (2), (5), (6a), (7b), and (5):

$$\gamma' = -cN\rho^{N-2} - \frac{2\sigma}{N-2} \tag{11}$$

3 Impact-Angle Control

To relate the bias to the impact angle, Eq. (11) could be utilized. However, the derivative there is with respect to the nondimensional time, whose final value is not known. It should be with respect to the nondimensional range, whose final value must be zero. To achieve this transformation, Eq. (11) can be modified through Eq. (6a) and Eq. (9) to obtain [9]

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\rho} = \frac{\mathrm{d}\gamma}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}\rho} = \frac{N\sin\varepsilon/\rho - \sigma}{\sqrt{1-\sin^2\varepsilon}} = g(\rho) \tag{12}$$

where g is a function of ρ, that is $g(\rho)$, because $\varepsilon = \varepsilon(\rho)$ as indicated in Eq. (9). Assuming a successful engagement, the terminal path angle can be given via the definite integral of Eq. (12) as

$$\gamma_f = \gamma_i + \int_{\rho_i}^{0} g(\theta)\,d\theta \tag{13}$$

where θ is a dummy variable. Here, the following quantity might be introduced:

$$f = \gamma_i - \gamma_f + \int_{\rho_i}^{0} g(\theta)\,d\theta \tag{14}$$

The root of f gives a desired bias term b that will lead to the desired impact angle. Given a desired impact angle and initial conditions, a numerical method, e.g., Newton's method utilizing the gradient can be used to find the root of Eq. (14). To this end, its derivative with respect to b is computed as

$$\frac{\mathrm{d}f}{\mathrm{d}b} = \frac{\mathrm{d}}{\mathrm{d}b}\int_{\rho_i}^{0} g(\theta)\,\mathrm{d}\theta \tag{15}$$

Using the Leibniz integral rule, Eq. (15) is further deduced as

$$\frac{\mathrm{d}f}{\mathrm{d}b} = -g(\rho_i)\frac{\partial\rho_i}{\partial b} + \int_{\rho_i}^{0} \frac{\partial g(\theta)}{\partial b}\mathrm{d}\theta \tag{16}$$

where the partial derivative $\partial g(\theta)/\partial b$ can be expressed as

$$\frac{\partial g(\theta)}{\partial b} = \frac{\partial g(\theta)}{\partial(\sin\varepsilon)} \frac{\partial(\sin\varepsilon)}{\partial c} \frac{\partial c}{\partial \rho_i} \frac{\partial \rho_i}{\partial b} \tag{17}$$

Furthermore, the partial derivatives in Eq. (17) can be readily obtained via Eqs. (12), (9), (10), and (6b) respectively:

$$\frac{\partial g(\theta)}{\partial(\sin\varepsilon)} = \frac{N/\rho - \sigma\sin\varepsilon}{(1-\sin^2\varepsilon)^{3/2}} \tag{18a}$$

$$\frac{\partial \sin\varepsilon}{\partial c} = \rho^{N-1} \tag{18b}$$

$$\frac{\partial c}{\partial \rho_i} = \sigma\rho_i^{1-N} + (1-N)(\sin\varepsilon_i)\rho_i^{-N} \tag{18c}$$

$$\frac{\partial \rho_i}{\partial b} = \frac{\sigma}{v} r_i \tag{18d}$$

For solving b from Eq. (14), three parameters need to be known: $\varepsilon_i, r_i, \gamma_f$. Here, γ_f is the desired impact time, which is the user input to the algorithm. It is worth noting that this root-finding approach is suitable for online bias update as the previous value would enable a warm start for finding the root. Such an online-update approach could help overcome disturbances.

4 Simulations

To demonstrate the practical usefulness of the proposed constant-bias PN guidance law, a ground-to-ground scenario is considered. The stationary target takes its place 5 km away from the starting point of the pursuer, which has a speed of 250 m/s and an initial path angle of 15 °. The desired impact angle is -90 °. In each run below, the bias value is computed once at the initial time by finding the root of Eq. (14), where the numerical value of the integral in Eq. (16) is obtained through the MATLAB command *integral*, which uses global adaptive quadrature.

Before detailing the results of the engagement simulations, it might be instructive to see how the total control effort, which is defined as $0.5\int a\,\mathrm{d}t$, varies with the navigation gain. To this end, a batch run is conducted to obtain the curve in Fig. 2. Here, it is observed that the optimal value for this scenario happens to be $N \approx 3.8$. This value is not quite close to $N = 3$, which is the well-known optimal value for unbiased PN in a linear setting.

It should be noted that $N = 3.8$, where the corresponding bias that will result in vertical impact is $b = 9.64$ °/s, is the optimal gain for impact-angle control with BPN; it will not produce the optimal acceleration signal. To see how it compares to the optimal solution, Fig. 3 can be investigated, where the optimal curve is the product of FALCON.m [10], a freeware software program for numerical optimization. It is seen that the results are comparable, yet BPN issues slightly higher accelerations at the beginning and the end of the engagement. This is not surprising because BPN is known to be equivalent to optimal control when the kinematics is linear.

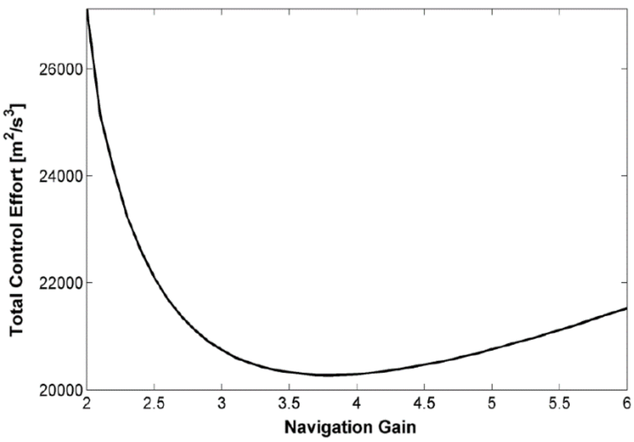


Fig. 2 Total effort as a function of navigation gain

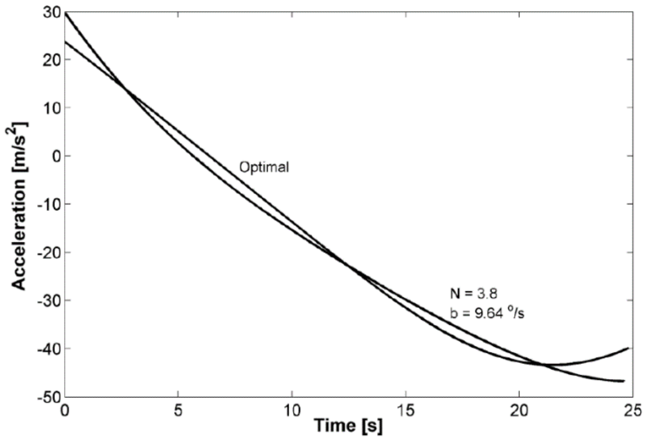
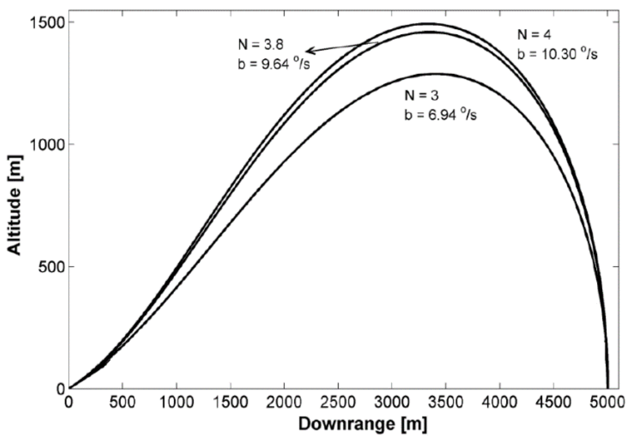
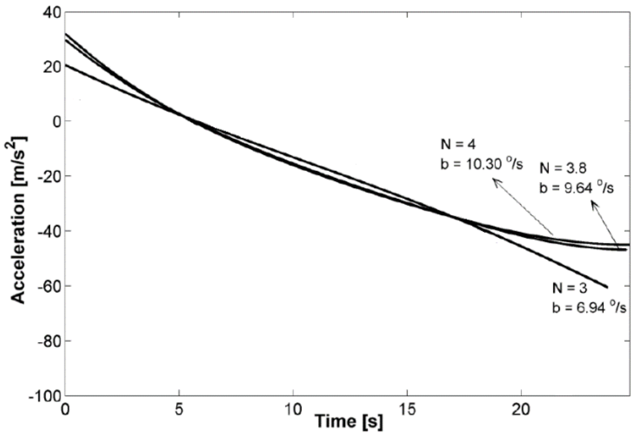


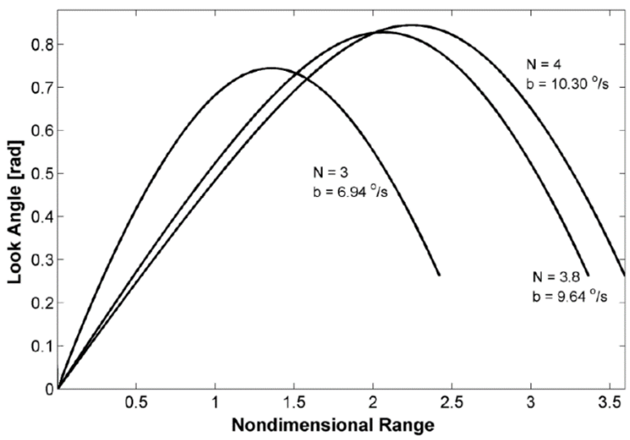
Fig. 3 Optimal control vs. BPN



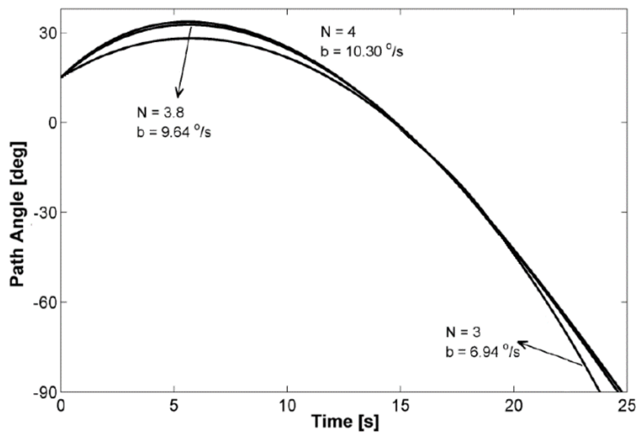
a) Altitude vs. downrange



b) Acceleration vs. time



c) Look angle vs. nondimensional range



d) Path angle vs. time

Fig. 2 Simulation results with different navigation gains

The detailed simulation results for $N = 3$ (with $b = 6.94$ °/s) and $N = 4$ (with $b = 10.30$ °/s), which are common guidance gains, are presented in Fig. 3. The results with the optimal value identified above as $N = 3.8$ are also included, and as would be expected, these are quite similar to what $N = 4$ leads to. Fig. 3a displays that a lower trajectory is produced by a lower navigation gain since it requires lower bias; the less the bias the more initial dominance PN will have. In accordance with this, the acceleration histories in Fig. 3b show that the effort demanded by $N = 3$ is initially less but the price paid for this is the increased terminal effort, which is a clear disadvantage. Fig. 3c presents the look angle variations with respect to the nondimensional range, where the temporal progression is from right to left. Because it has the least lofted trajectory, $N = 3$ requires the least amount of look-angle capacity. Lastly in Fig. 3d, it is confirmed that vertical impact is indeed achieved perfectly by all pursuers.

5 Conclusion

The impact-angle control method introduced in this work is formulated by appending a bias term to the proportional navigation guidance law without utilizing the time to go. The bias appears inside an integral equation that cannot be solved analytically, which is why a computational routine is utilized. This routine, which is merely a root-finding process, is enabled by the closed-form availability of partial derivatives associated with a function that relates the bias to the impact angle. It is shown that the performance of constant-bias PN is comparable to that of the optimal solution. The feasibility and effectiveness of the proposed technique is tested using simulations with a stationary target. The results confirm that it is a viable candidate to serve as a practical guidance law.

Acknowledgments

We thank Haichao Hong for his contribution in the mathematical formulation.

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