Impact Vector Guidance

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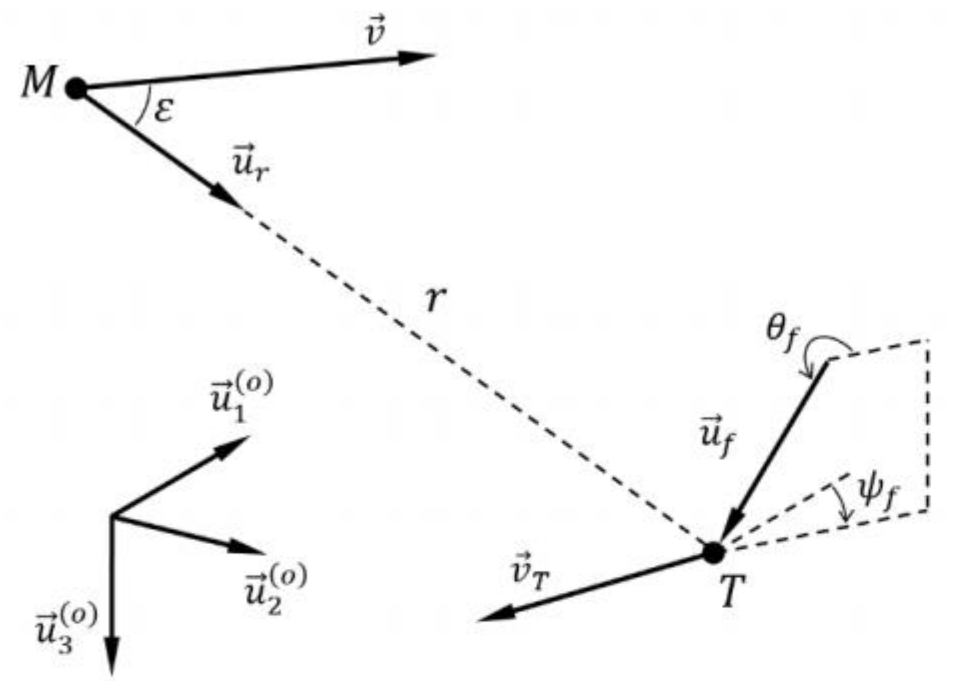
Impact Angle & Impact Vector

As their names imply, the objective of 2D methods is to achieve a specified impact angle whereas 3D methods aim to obtain a specified impact vector in space.



Target Sets

- Stationary
- Moving
- Maneuvering

State of the Art

Impact Angle

- Biased PN
- Lyapunov
- Sliding Mode
- SDRE
- NDI

Impact Angles in 3D

- GENEX
- Control Theory Based Methods

Structure

- Effective Pure PN (EPPN)
- Closed Form Solution of EPPN
- Comparison of EPPN & PPN
- Guidance Design
- Trajectory Shaping in 2D
- Trajectory Shaping in 3D
- Simulations

- Effective Pure PN
- Bias Vector

Effective Pure PN

navigation ratio

missile speed

$$a_{\mathrm{PPN}} = Nv\dot{\lambda}$$

$$a_{\mathrm{TPN}} = N'v_c\dot{\lambda}$$

effective navigation ratio

closing speed

$$a = v\dot{\gamma}$$

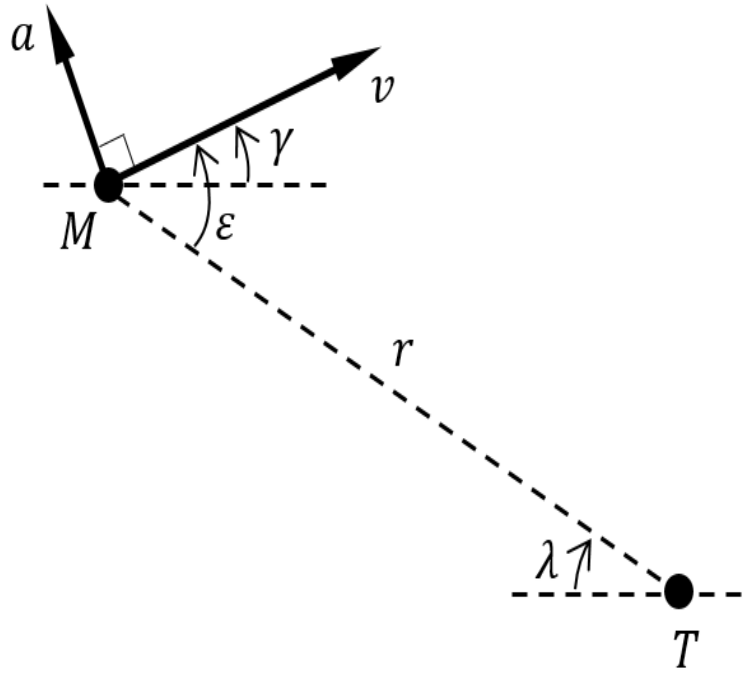
$$\dot{\gamma}_{PPN} = N\dot{\lambda}$$

$$\dot{\gamma}_{EPPN} = N\frac{v_c}{v}\dot{\lambda}$$

$$a_{\mathrm{EPPN}} = Nv_c\dot{\lambda}$$

Closed Form Solution of Effective Pure PN

As in the case of PPN, whose solution against a moving target is in the form of infinite series, but it is straightforward against a stationary target.



$$\dot{r} = -v\cos\varepsilon$$

$$r\dot{\lambda} = -v\sin\varepsilon$$

$$\varepsilon = \gamma - \lambda$$

$$v_c = -\dot{r} = v\cos\varepsilon$$

$$\dot{\gamma} = N\cos\varepsilon\,\dot{\lambda}$$

$$r\dot{\varepsilon} = v\sin\varepsilon\,(1 - N\cos\varepsilon)$$

$$\frac{d\varepsilon}{dr} = \frac{N\sin\varepsilon - \tan\varepsilon}{r}$$

$$\int_{r_i}^{r}\frac{dr}{r} = \int_{\varepsilon_i}^{\varepsilon}\frac{d\varepsilon}{N\sin\varepsilon - \tan\varepsilon}$$

$$\rho = r/r_i$$

$$c\rho = \frac{(\tan(\varepsilon/2))^{1/(N-1)}}{\{(N+1)(\tan(\varepsilon/2))^2 - (N-1)\}^{1/(N^2-1)}}$$

c: integration constant

Comparison of EPPN & PPN

The acceleration is normalized as

EPPN

$$\alpha = \frac{N}{\sin \varepsilon_i} \frac{\sin \varepsilon \cos \varepsilon}{\rho}$$

PPN

$$\alpha = \frac{N}{\sin \varepsilon_i} \frac{\sin \varepsilon}{\rho}$$

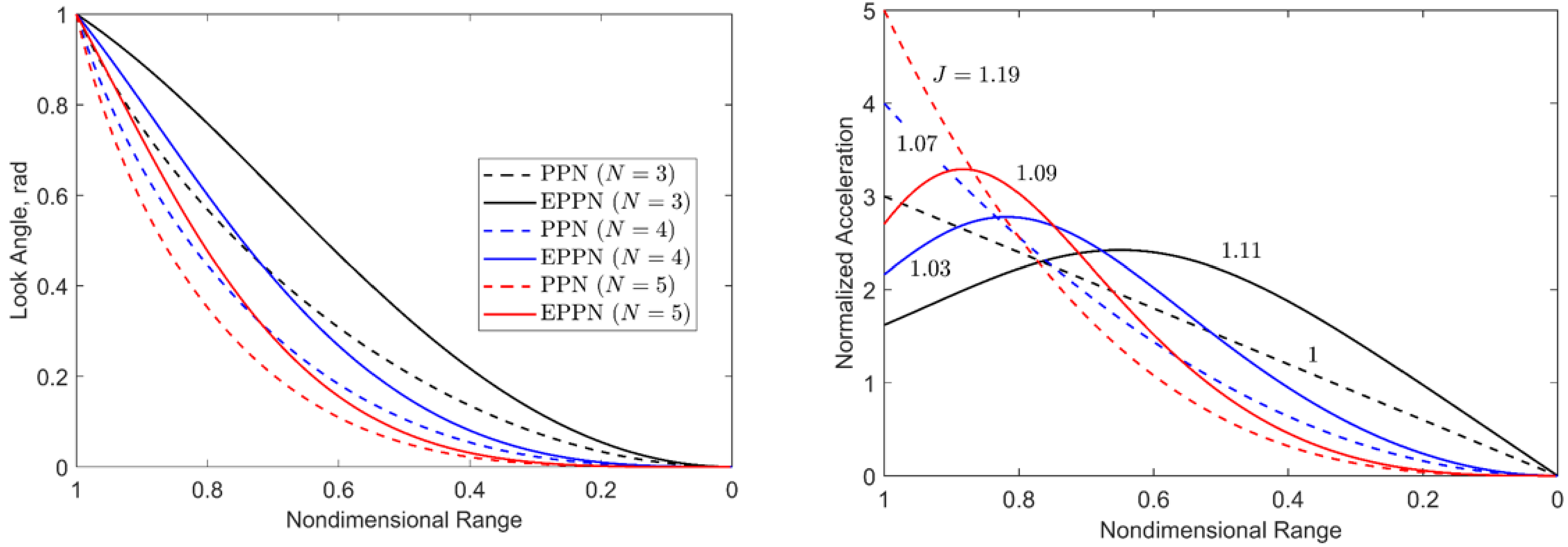
The values of the total control efforts normalized with respect to of PPN with N = 3:

$$J = \left(\int \alpha^2 \mathrm{d}\rho\right) / \left(\int \alpha^2 \mathrm{d}\rho |_{N=3}^{\mathrm{PPN}}\right)$$

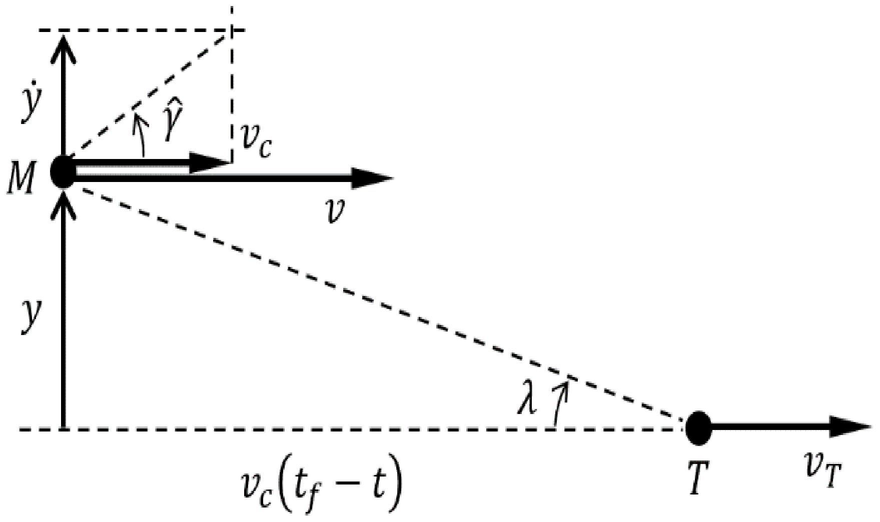
- Under certain circumstances, EPPN can be associated with lower maximum acceleration requirement and less total control effort.
- These circumstances are increased values of both the look angle and the navigation ratio, are also known to characterize trajectory-shaping guidance.

EPPN manifests itself as a powerful tool for impact angle control.

Effective Pure PN



Guidance Design



$$\lambda = -\frac{y}{v_c(t_f - t)}$$

y → vertical separation between missile and target

$v_c(t_f - t)$ → horizantal separation between missile and target

$$v_c = v - v_T$$

$$\dot{\lambda} = -\frac{\dot{y}}{v_c(t_f - t)} - \frac{y}{v_c(t_f - t)^2}$$

$$\hat{\gamma} = \frac{\dot{y}}{v_c}$$

relative flight path angle

$$\dot{\hat{\gamma}} = N\dot{\lambda} + b \longrightarrow \ddot{y} = a = v_c(N\dot{\lambda} + b)$$

Guidance Design

$$\dot{\hat{\gamma}} = N\dot{\lambda} + \boxed{b}$$ constant bias term

$$b = \frac{\hat{\gamma}_f - \hat{\gamma} - N(\lambda_f - \lambda)}{t_f - t}$$

$$\lambda_f = \hat{\gamma}_f$$

$$b_r = \dot{\lambda} + (N-1)\frac{\lambda - \lambda_f}{t_f - t}$$

r indicates that the angle error is formulated with respect to the LOS.

$$b_v = N\dot{\lambda} + (N-1)\frac{\hat{\gamma} - \hat{\gamma}_f}{t_f - t}$$

v indicates that it is formulated with respect to the velocity.

This fact might render this equation useless when the flight-path angle, rather than the relative one, is to have a desired final value against a moving target.

$$b_v = N\dot{\lambda} + (N-1)\frac{\gamma - \gamma_f}{t_f - t}$$

Trajectory Shaping in Two Dimensions

EPPN-based IA guidance laws

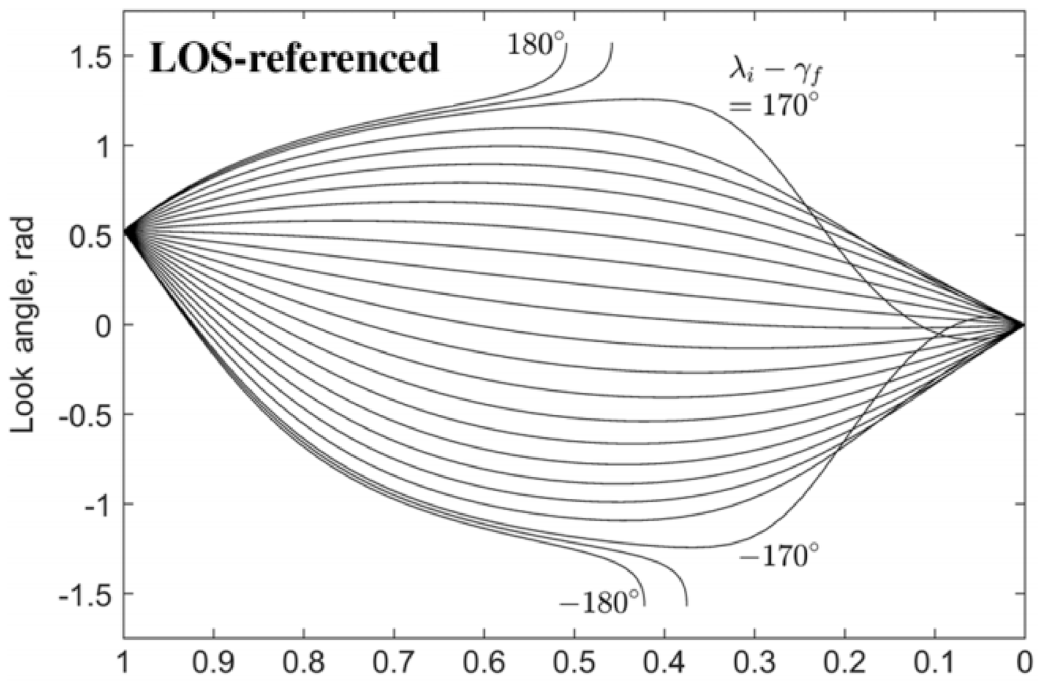
$$t_f - t = t_{go} \approx -\frac{r}{\dot{r}} = \frac{r}{v_c}$$

$$\dot{\gamma}_r = \frac{v_c}{v}\left\{\boxed{(N+1)}\dot{\lambda} + (N-1)\frac{v_c}{r}(\lambda - \lambda_f)\right\}$$

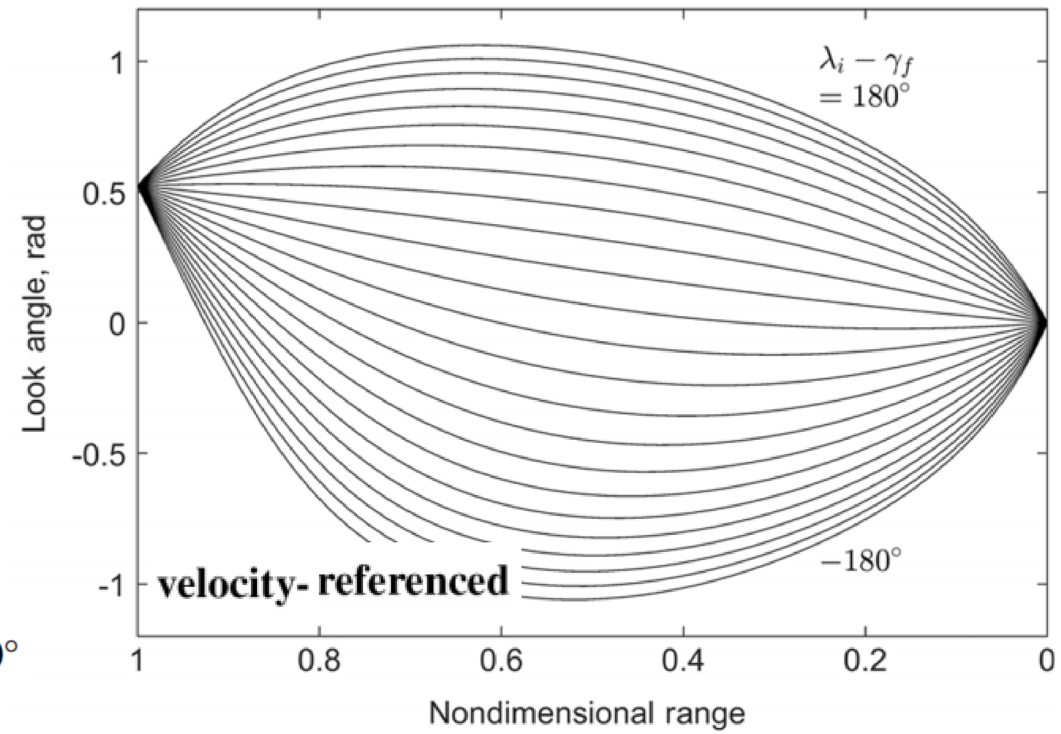
$$\dot{\gamma}_v = \frac{v_c}{v}\left\{\boxed{2N}\dot{\lambda} + (N-1)\frac{v_c}{r}(\gamma - \gamma_f)\right\}$$

It is also possible to have their PPN-based counterparts. However, these are left out because their trajectory-shaping performances happen to be rather poor in comparison with these guidance formulations.

Trajectory Shaping in Two Dimensions



$N = 3$
$\varepsilon_i = 30°$



$$\rho \frac{d\varepsilon}{d\rho}\Big|_{\dot{\gamma}_r} = (N+1)\sin\varepsilon - \tan\varepsilon - (N-1)\cos\varepsilon\left(\int_1^{\rho} \frac{\tan\varepsilon}{\rho} d\rho + \lambda_i - \gamma_f\right)$$

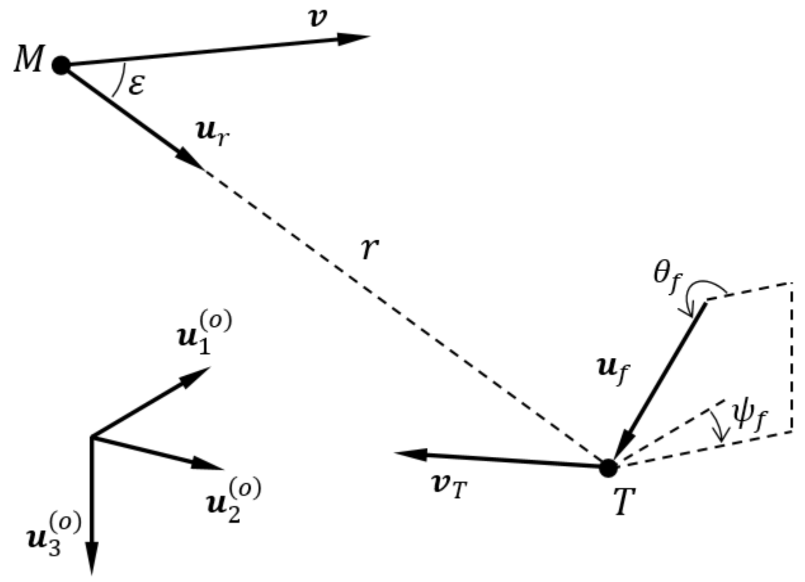
$$\rho \frac{d\varepsilon}{d\rho}\Big|_{\dot{\gamma}_v} = 2N\sin\varepsilon - \tan\varepsilon - (N-1)\cos\varepsilon\left(\varepsilon + \int_1^{\rho} \frac{\tan\varepsilon}{\rho} d\rho + \lambda_i - \gamma_f\right)$$

Trajectory Shaping in Three Dimensions

- The objective is to guide the missile in such a way that either the LOS vector $\boldsymbol{r}$ (with its unit vector $\boldsymbol{u}_r$) or the velocity vector $\boldsymbol{v}$ (with its unit vector $\boldsymbol{u}_v$) points in the same direction as $\boldsymbol{u}_f$ at the time of impact.

- The desired impact vector $\boldsymbol{u}_f$ may be defined in an observation frame $\mathcal{F}_o$ with axes $\boldsymbol{u}_{1,2,3}^{(o)}$ in terms of the yaw angle ψ_f and the pitch angle θ_f

$$\bar{u}_f^{(o)} = \begin{bmatrix} cos\,\psi_f\,cos\,\theta_f \\ cos\,\theta_f\,sin\,\psi_f \\ -sin\,\theta_f \end{bmatrix}$$



Trajectory Shaping in Three Dimensions

Ultimate objective is to rotate $\boldsymbol{u}$ onto $\boldsymbol{u}_f$

the direction of the bias term $\boldsymbol{u}_b = \dfrac{\boldsymbol{u}_f \times \boldsymbol{u}}{|\boldsymbol{u}_f \times \boldsymbol{u}|}$

$$\boldsymbol{a} = v_c \left\{ n\boldsymbol{\omega}_r + (N-1)\frac{v_c}{r}\delta\boldsymbol{u}_b \right\} \times \boldsymbol{u}_v$$

$\boldsymbol{u} = \boldsymbol{u}_r$ or $\boldsymbol{u} = \boldsymbol{u}_v$

$N + 1$ or $2N$

impact-angle error

$$\delta = cos^{-1}(\boldsymbol{u}_f \cdot \mathbf{u})$$

$$cos\,\varepsilon = \boldsymbol{u}_r \cdot \boldsymbol{u}_v$$

$$\boldsymbol{a}_{PPN} = N\boldsymbol{\omega}_r \times \boldsymbol{v}$$

$$\boldsymbol{\omega}_r = \frac{\boldsymbol{r} \times \dot{\boldsymbol{r}}}{r^2}$$

Trajectory Shaping in Three Dimensions

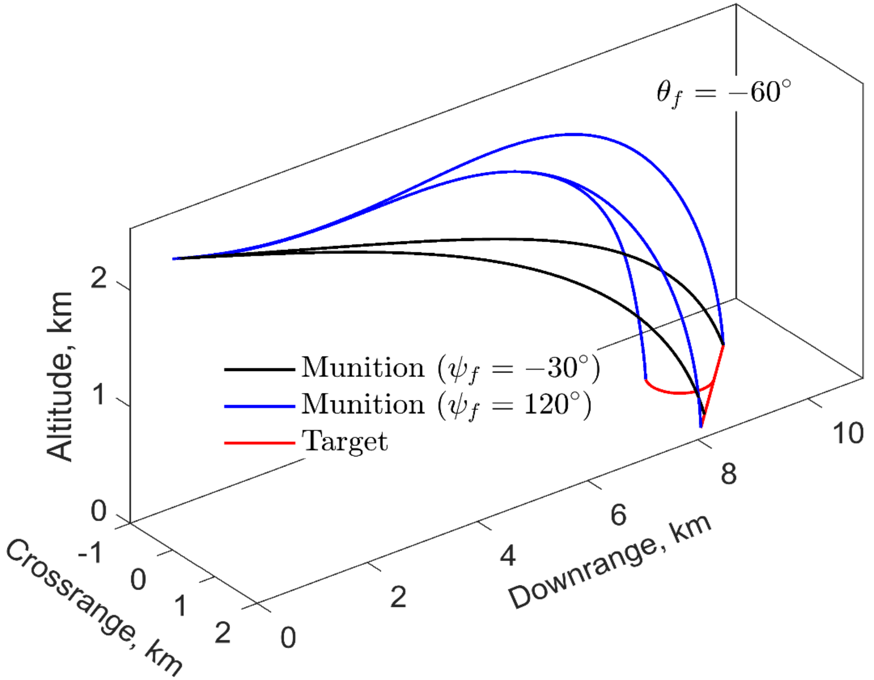
EPPN-based Impact Vector Guidance

$$\boldsymbol{a}_{\mathrm{IVG}-r} = v_c\left\{(N+1)\boldsymbol{\omega}_r + (N-1)\frac{v_c}{r}cos^{-1}(\boldsymbol{u}_f\cdot\boldsymbol{u}_r)\frac{\boldsymbol{u}_f\times\boldsymbol{u}_r}{|\boldsymbol{u}_f\times\boldsymbol{u}_r|}\right\}\times\boldsymbol{u}_v$$

$$\boldsymbol{a}_{\mathrm{IVG}-v} = v_c\left\{2N\boldsymbol{\omega}_r + (N-1)\frac{v_c}{r}cos^{-1}(\boldsymbol{u}_f\cdot\boldsymbol{u}_v)\frac{\boldsymbol{u}_f\times\boldsymbol{u}_v}{|\boldsymbol{u}_f\times\boldsymbol{u}_v|}\right\}\times\boldsymbol{u}_v$$

$$\boldsymbol{a}_{\mathrm{GENEX}} = \frac{v^2}{r}\{(n+2)(n+3)[\boldsymbol{u}_r - (\boldsymbol{u}_v\cdot\boldsymbol{u}_r)\boldsymbol{u}_v] - (n+1)(n+2)[\boldsymbol{u}_f - (\boldsymbol{u}_v\cdot\boldsymbol{u}_f)\boldsymbol{u}_v]\}$$

Trajectory Shaping in Three Dimensions

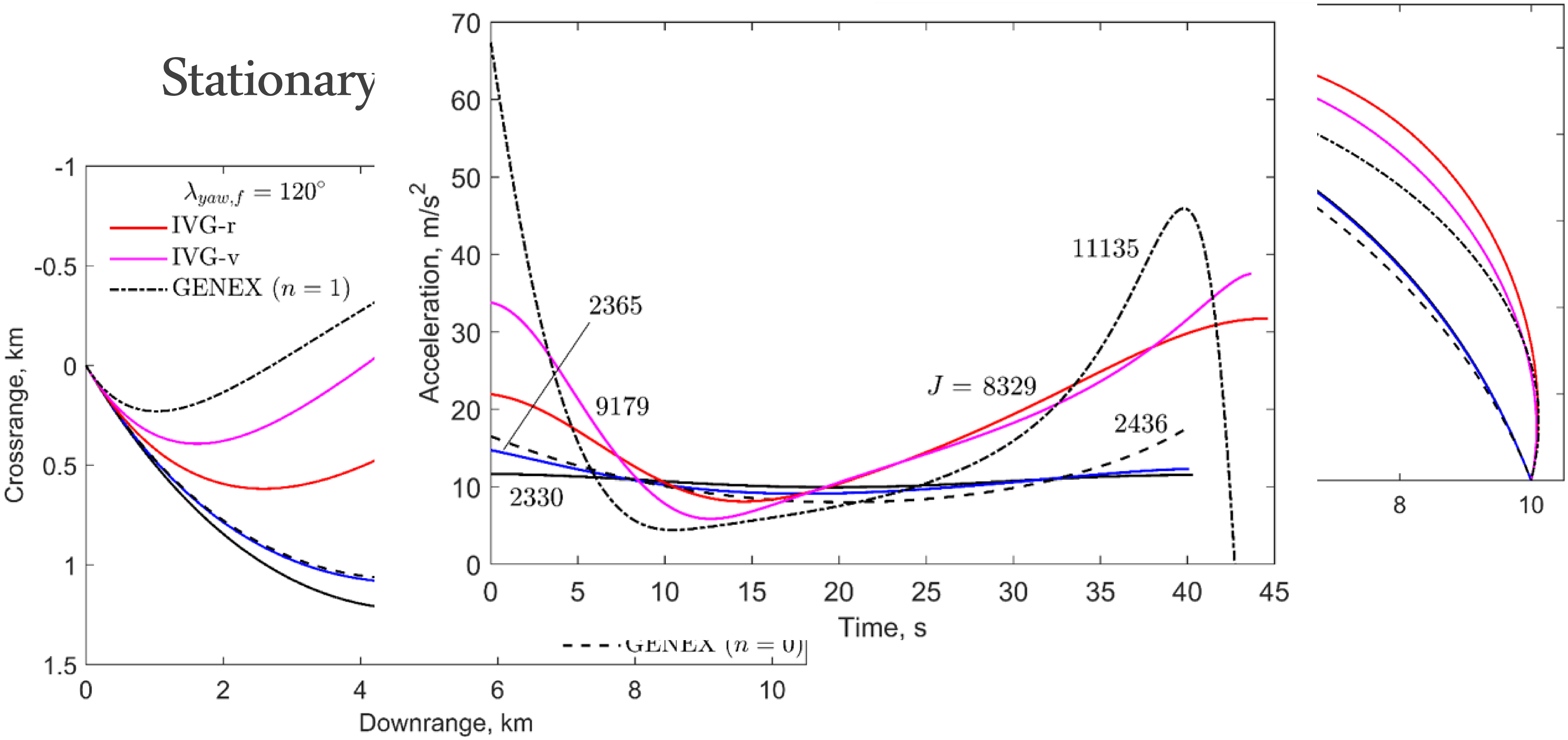


Simulation results against stationary target

Yaw Impact Angle	Guidance Law	Max. Acc., m/s^2	Total Control Effort, m^2/s^3
−30°	IVG-r	11.7	2330
	IVG-v	14.7	2365
	GENEX ($n = 0$)	17.3.	2436
120°	IVG-r	31.7	8329
	IVG-v	37.5	9179
	GENEX ($n = 1$)	67.4	11135

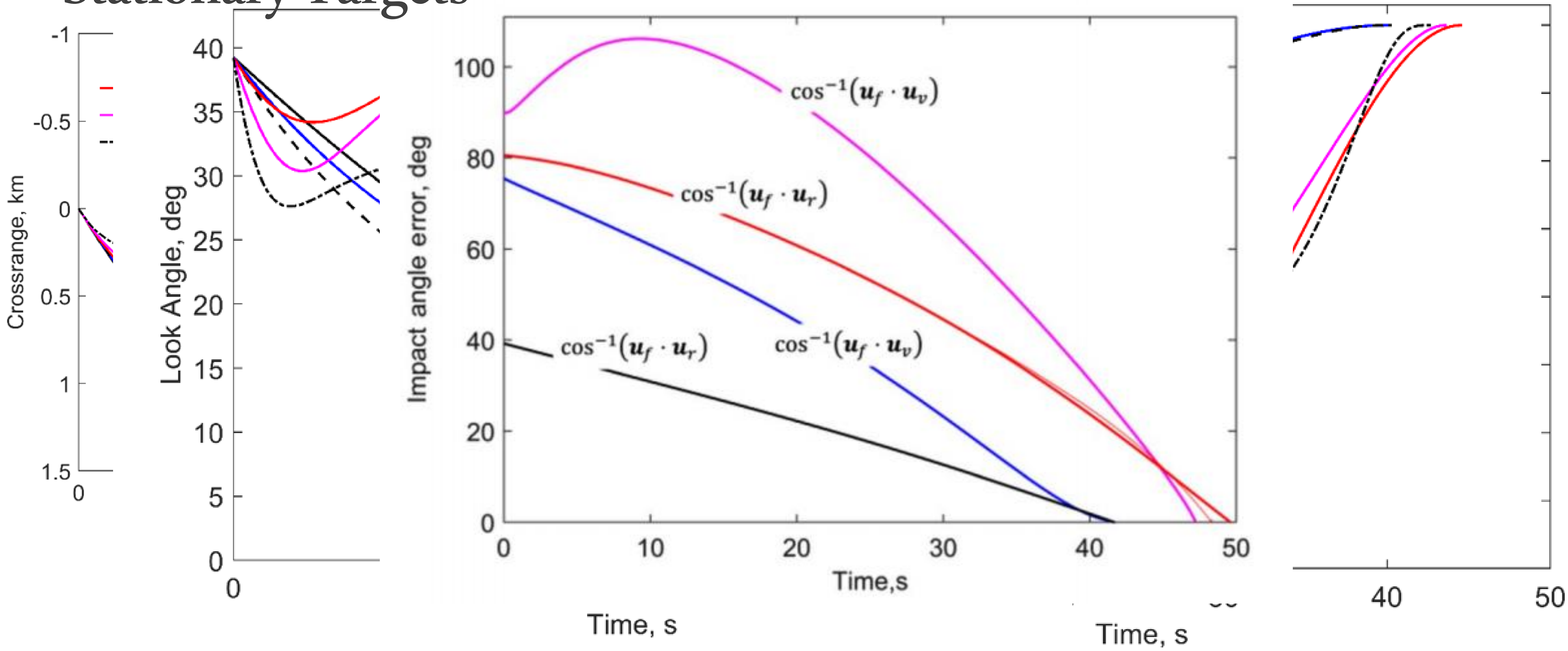
The missile is released horizontally from an altitude of 5 km with 300 m/s with a yaw angle of 30°
Target is at 10 km
The pitch angle of the desired impact vector is selected as $\theta_f = -60^\circ$ and yaw angle either $\psi_f = -30^\circ$ or $\psi_f = 120^\circ$
N = 4

Stationary



$\lambda_{yaw,f} = 120°$
IVG-r
IVG-v
GENEX ($n = 1$)
GENEX ($n = 0$)
Crossrange, km
Downrange, km
-1
-0.5
0
0.5
1
1.5
0
2
4
6
8
10
Acceleration, m/s^2
Time, s
70
60
50
40
30
20
10
0
5
15
20
25
30
35
40
45
11135
2365
9179
$J = 8329$
2436
2330
8
10

Stationary Targets



Moving/Maneuvering Targets/ Speed Change

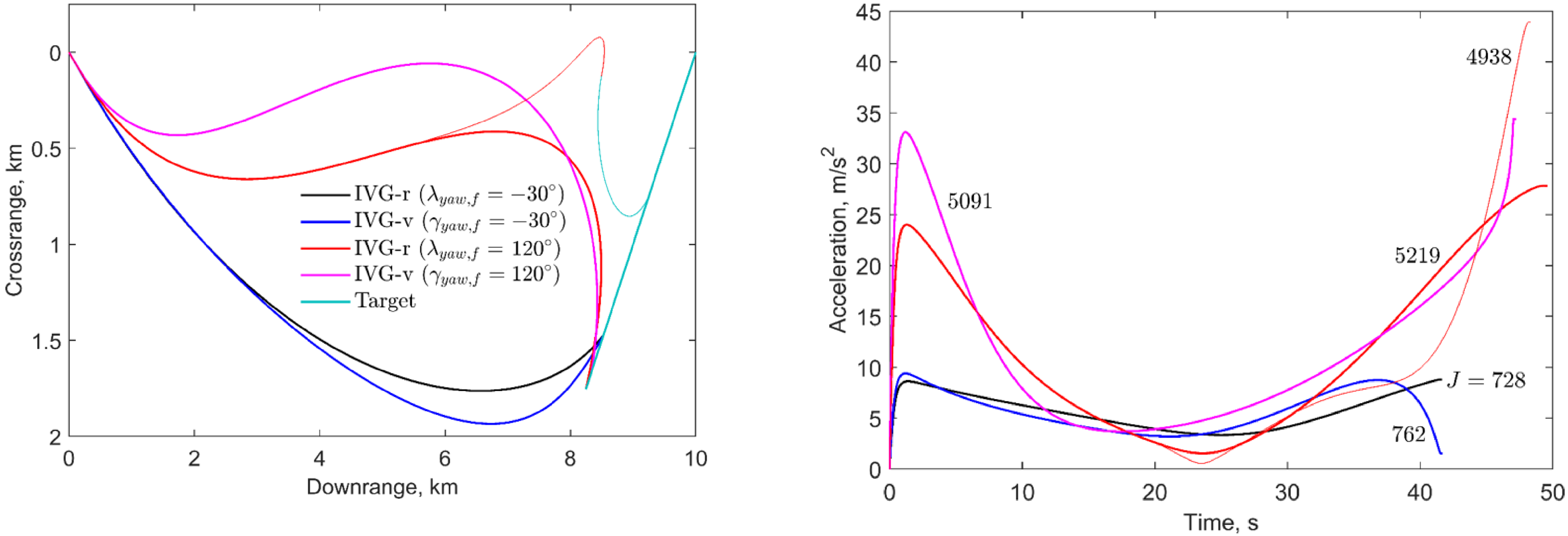
- The target is moving with a constant speed of 50 m⁄s and is capable of maneuvering with 5 m⁄s^2.
- There is gravity present, the deceleration due to drag is modeled as −7 × 10−5v^2.
- The autopilot is represented by a first-order lag of 0.3 s on acceleration response.
- The guidance command is held constant during the last 50 m to emulate a saturated seeker.

Summary of simulation results against moving target

Yaw impact angle	Guidance law	Maximum acceleration, m/s^2	Total control effort, m^2/s^3
−30°	IVG-r	8.8	728
	IVG-v	9.4	762
120°	IVG-r	27.9	5219
	IVG-v	34.4	5091
	IVG-r[a]	43.9	4938

[a]Maneuvering target.

Moving/Maneuvering Targets/ Speed Change



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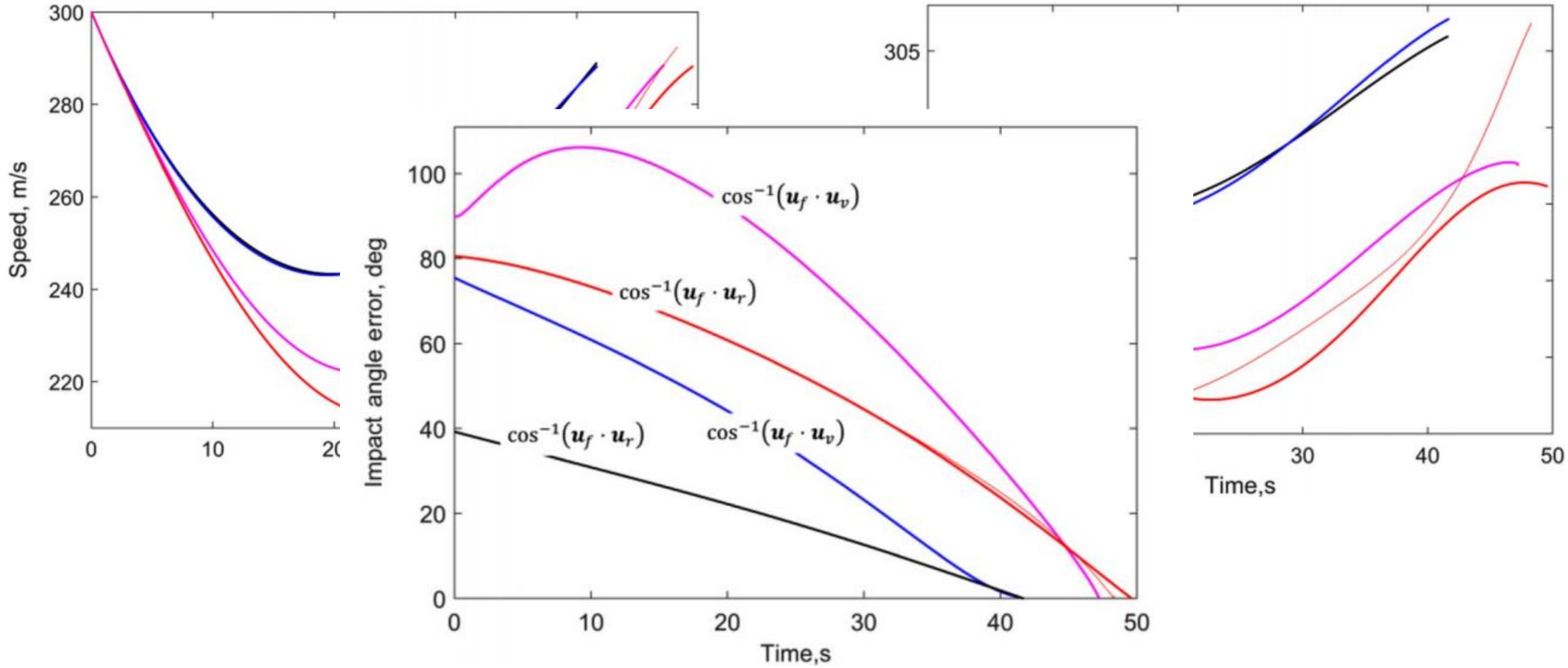

Impact Vector Guidance

- Two new guidance laws in 3D vector form to control the final impact direction are proposed.
- The effective pure PN constructs the acceleration command using the closing speed instead of the missile speed.
- The guidance laws are in essence 3D implementations of biased PN, they involve a unit vector to determine the bias direction.
- Either the LOS or the velocity vector rotates about this unit vector to reach the desired impact vector eventually.
- The proposed guidance laws can be used against stationary, moving, and maneuvering targets.

Thank you.