Impact Vector Guidance

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Impact Angle & Impact Vector

As their names imply, the objective of 2D methods is to achieve a specified impact angle whereas 3D methods aim to obtain a specified impact vector in space.

**Target Sets**

- Stationary
- Moving
- Maneuvering
State of the Art

Impact Angle

- Biased PN
- Lyapunov
- Sliding Mode
- SDRE
- NDI

Impact Angles in 3D

- GENEX
- Control Theory Based Methods
Structure

➢ Effective Pure PN (EPPN)

➢ Closed Form Solution of EPPN

➢ Comparison of EPPN & PPN

➢ Guidance Design

➢ Trajectory Shaping in 2D

➢ Trajectory Shaping in 3D

➢ Simulations

Effective Pure PN

Bias Vector
Effective Pure PN

\[ a_{PPN} = N v \dot{\lambda} \]

\[ a_{TPN} = N' v_c \dot{\lambda} \]

\[ \dot{a}_{PPN} = N' \dot{\lambda} \]

\[ \dot{a}_{EPPN} = N \frac{v_c}{v} \dot{\lambda} \]

\[ a_{EPPN} = N v_c \dot{\lambda} \]
Closed Form Solution of Effective Pure PN

As in the case of PPN, whose solution against a moving target is in the form of infinite series, but it is straightforward against a stationary target.

\[
r\dot{\epsilon} = v \sin \epsilon (1 - N \cos \epsilon)
\]

\[
\dot{r} = -v \cos \epsilon
\]

\[
r \dot{\lambda} = -v \sin \epsilon
\]

\[
\epsilon = \gamma - \lambda
\]

\[
v_c = -\dot{r} = v \cos \epsilon
\]

\[
\dot{\gamma} = N \cos \epsilon \dot{\lambda}
\]

\[
\int_{r_l}^{r} \frac{dr}{r} = \int_{\epsilon_l}^{\epsilon} \frac{d\epsilon}{N \sin \epsilon - \tan \epsilon}
\]

\[
\rho = \frac{r}{r_i}
\]

\[
\rho = \frac{(\tan(\epsilon/2))^{1/(N-1)}}{\{(N + 1)(\tan(\epsilon/2))^2 - (N - 1)\}^{1/(N^2-1)}}
\]

integration constant
Comparison of EPPN & PPN

The acceleration is normalized as

$$\alpha = \frac{N \sin \varepsilon \cos \varepsilon}{\sin \varepsilon_t \rho}$$

The values of the total control efforts normalized with respect to PPN with $N = 3$:

$$J = (\int \alpha^2 d\rho) / (\int \alpha^2 d\rho |_{N=3}^{PPN})$$

- Under certain circumstances, EPPN can be associated with lower maximum acceleration requirement and less total control effort.
- These circumstances are increased values of both the look angle and the navigation ratio, are also known to characterize trajectory-shaping guidance.

EPPN manifests itself as a powerful tool for impact angle control.
Effective Pure PN
Guidance Design

vertical separation between missile and target

\[ \lambda = -\frac{y}{v_c(t_f - t)} \]

horizontal separation between missile and target

\[ v_c = v - v_T \]

relative flight path angle

\[ \dot{\gamma} = \frac{\dot{y}}{v_c} \]

\[ \dot{y} = a = v_c(N\dot{\lambda} + b) \]
**Guidance Design**

\[ \dot{\gamma} = N\dot{\lambda} + b \]  
constant bias term

\[ b_r = \dot{\lambda} + (N - 1) \frac{\lambda - \lambda_f}{t_f - t} \]

\[ r \] indicates that the angle error is formulated with respect to the LOS.

\[ \lambda_f = \dot{\gamma}_f \]

\[ b = \frac{\dot{\gamma}_f - \dot{\gamma} - N(\lambda_f - \lambda)}{t_f - t} \]

\[ b_v = N\dot{\lambda} + (N - 1) \frac{\dot{\gamma} - \dot{\gamma}_f}{t_f - t} \]

\[ v \] indicates that it is formulated with respect to the velocity.

This fact might render this equation useless when the flight-path angle, rather than the relative one, is to have a desired final value against a moving target.
Trajectory Shaping in Two Dimensions

EPPN-based IA guidance laws

\[ \dot{\gamma}_r = \frac{v_c}{v} \left\{ (N + 1) \hat{\lambda} + (N - 1) \frac{v_c}{r} (\lambda - \lambda_f) \right\} \]

\[ \dot{\gamma}_v = \frac{v_c}{v} \left\{ 2N \hat{\lambda} + (N - 1) \frac{v_c}{r} (\gamma - \gamma_f) \right\} \]

\[ t_f - t = t_{go} \approx - \frac{r}{\dot{r}} = \frac{r}{v_c} \]

It is also possible to have their PPN-based counterparts. However, these are left out because their trajectory-shaping performances happen to be rather poor in comparison with these guidance formulations.
Trajectory Shaping in Two Dimensions

\[
\rho \frac{de}{dp} |_{r_*} = (N + 1) \sin \varepsilon - \tan \varepsilon - (N - 1) \cos \varepsilon \left( \int_1^\rho \frac{\tan \varepsilon}{\rho} \, dp + \lambda_i - \gamma_f \right)
\]

\[
\rho \frac{de}{dp} |_{r_*} = 2N \sin \varepsilon - \tan \varepsilon - (N - 1) \cos \varepsilon \left( e + \int_1^\rho \frac{\tan \varepsilon}{\rho} \, dp + \lambda_i - \gamma_f \right)
\]
The objective is to guide the missile in such a way that either the LOS vector $\mathbf{r}$ (with its unit vector $\mathbf{u}_r$) or the velocity vector $\mathbf{v}$ (with its unit vector $\mathbf{u}_v$) points in the same direction as $\mathbf{u}_f$ at the time of impact.

The desired impact vector $\mathbf{u}_f$ may be defined in an observation frame $\mathcal{F}_o$ with axes $\mathbf{u}^{(o)}_{1,2,3}$ in terms of the yaw angle $\psi_f$ and the pitch angle $\theta_f$.

$$
\mathbf{u}_f^{(o)} = \begin{bmatrix}
\cos \psi_f \cos \theta_f \\
\cos \theta_f \sin \psi_f \\
-\sin \theta_f
\end{bmatrix}
$$
Ultimate objective is to rotate $\mathbf{u}$ onto $\mathbf{u}_f$

$$\mathbf{a} = v_c \left\{ n\omega_r + (N - 1) \frac{v_c}{r} \delta \mathbf{u}_b \right\} \times \mathbf{u}_v$$

the direction of the bias term

$$\mathbf{u}_b = \frac{\mathbf{u}_f \times \mathbf{u}}{|\mathbf{u}_f \times \mathbf{u}|}$$

$$u = u_r \text{ or } u = u_\perp$$

$$\cos \varepsilon = \mathbf{u}_r \cdot \mathbf{u}_v$$

$$a_{PPN} = N\omega_r \times v$$

$$\omega_r = \frac{\mathbf{r} \times \dot{\mathbf{r}}}{r^2}$$

impact-angle error

$$\delta = \cos^{-1}(\mathbf{u}_f \cdot \mathbf{u})$$

$N + 1$ or $2N$
EPPN-based Impact Vector Guidance

\[ \mathbf{a}_{IVG-r} = v_c \left\{ (N + 1) \omega_r + (N - 1) \frac{v_c}{r} \cos^{-1}(u_f \cdot u_r) \right\} \frac{u_f \times u_r}{|u_f \times u_r|} \times u_v \]

\[ \mathbf{a}_{IVG-v} = v_c \left\{ 2N \omega_r + (N - 1) \frac{v_c}{r} \cos^{-1}(u_f \cdot u_v) \right\} \frac{u_f \times u_v}{|u_f \times u_v|} \times u_v \]

\[ \mathbf{a}_{GENEX} = \frac{v^2}{r} \left\{ (n + 2)(n + 3)[u_r - (u_v \cdot u_r)u_v] - (n + 1)(n + 2)[u_f - (u_v \cdot u_f)u_v] \right\} \]
The missile is released horizontally from an altitude of 5 km with 300 m/s with a yaw angle of 30°. Target is at 10 km. The pitch angle of the desired impact vector is selected as $\theta_f = -60^\circ$ and yaw angle either $\psi_f = -30^\circ$ or $\psi_f = 120^\circ$. $N = 4$. 

### Simulation results against stationary target

<table>
<thead>
<tr>
<th>Yaw Impact Angle</th>
<th>Guidance Law</th>
<th>Max. Acc., m/s²</th>
<th>Total Control Effort, m²/s³</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-30^\circ$</td>
<td>IVG-r</td>
<td>11.7</td>
<td>2330</td>
</tr>
<tr>
<td></td>
<td>IVG-v</td>
<td>14.7</td>
<td>2365</td>
</tr>
<tr>
<td>GENEX ($n = 0$)</td>
<td></td>
<td>17.3</td>
<td>2436</td>
</tr>
<tr>
<td>$120^\circ$</td>
<td>IVG-r</td>
<td>31.7</td>
<td>8329</td>
</tr>
<tr>
<td></td>
<td>IVG-v</td>
<td>37.5</td>
<td>9179</td>
</tr>
<tr>
<td>GENEX ($n = 1$)</td>
<td></td>
<td>67.4</td>
<td>11135</td>
</tr>
</tbody>
</table>
Stationary

$\lambda_{yov,f} = 120^\circ$

- IVG-$r$
- IVG-$v$
- GENEX ($n = 1$)

Acceleration, m/s$^2$

Time, s

Downrange, km

$J = 8329$

2365

9179

2436

11135

2330

Crossrange, km

0 1 2 3 4 5 6 7 8 9 10

-0.5 0 0.5 1 1.5

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Stationary Targets

Graph showing the relationship between crossrange, look angle, and impact angle error over time.
Moving/Maneuvering Targets/ Speed Change

➢ The target is moving with a constant speed of 50 m/s and is capable of maneuvering with 5 m/s².
➢ There is gravity present, the deceleration due to drag is modeled as \( -7 \times 10^{-5}v^2 \).
➢ The autopilot is represented by a first-order lag of 0.3 s on acceleration response.
➢ The guidance command is held constant during the last 50 m to emulate a saturated seeker.

<table>
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<tr>
<th>Yaw impact angle</th>
<th>Guidance law</th>
<th>Maximum acceleration, m/s²</th>
<th>Total control effort, m²/s³</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30°</td>
<td>IVG-r</td>
<td>8.8</td>
<td>728</td>
</tr>
<tr>
<td></td>
<td>IVG-v</td>
<td>9.4</td>
<td>762</td>
</tr>
<tr>
<td>120°</td>
<td>IVG-r</td>
<td>27.9</td>
<td>5219</td>
</tr>
<tr>
<td></td>
<td>IVG-v</td>
<td>34.4</td>
<td>5091</td>
</tr>
<tr>
<td></td>
<td>IVG-r⁻</td>
<td>43.9</td>
<td>4938</td>
</tr>
</tbody>
</table>

\( ^a \) Maneuvering target.
Moving/Maneuvering Targets/ Speed Change

![Graph 1: Crossrange vs Downrange](image)

- IVG-r ($\lambda_{gyve, f} = -30^\circ$)
- IVG-v ($\gamma_{pan, f} = -30^\circ$)
- IVG-r ($\lambda_{gyve, f} = 120^\circ$)
- IVG-v ($\gamma_{pan, f} = 120^\circ$)
- Target

![Graph 2: Acceleration vs Time](image)

- $J = 728$
- $J = 762$
- $J = 5219$
- $J = 5091$
- $J = 4938$
Moving/Maneuvering Targets/ Speed Change
Impact Vector Guidance

➢ Two new guidance laws in 3D vector form to control the final impact direction are proposed.

➢ The effective pure PN constructs the acceleration command using the closing speed instead of the missile speed.

➢ The guidance laws are in essence 3D implementations of biased PN, they involve a unit vector to determine the bias direction.

➢ Either the LOS or the velocity vector rotates about this unit vector to reach the desired impact vector eventually.

➢ The proposed guidance laws can be used against stationary, moving, and maneuvering targets.

Thank you.